Tax Policy and Human Capital Accumulation In a Resource Constrained Growing Dual Economy

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Abstract

This paper examines the role tax policy can play in fostering human capital accumulation in a resource constrained dual economy whose population is growing. The study shows how human capital accumulation, in turn, affects intersectoral terms of trade and the economic growth process of a dual economy. The dual economy is assumed to consist of two basic sectors, agriculture and manufacturing. Production in agriculture requires unskilled labor and capital whereas production in the manufacturing sector requires skilled and unskilled labor and capital. Schooling facilities are limited and access is rationed by the government. Moreover, schooling requires an investment of time. The paper demonstrates the existence of a unique short run equilibrium. It also demonstrates that the steady state equilibrium is unique and locally stable. Comparative steady state analysis suggests that a balanced budget increase in public investment in education (financed by a tax increase on capital income and/or incomes of skilled workers), alters the terms trade between agriculture and manufacturing sectors and favorably affects the economic growth process.

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I. INTRODUCTION

The role of human capital accumulation in economic growth has been the focus of considerable research recently. The rapidly expanding endogenous growth literature is evidence of this emphasis (see, for instance, Lucas (1988)). However, this literature has nothing to say about the impact of human accumulation in many poor LDCs - the economic structure of many of whom is still characteristically dual. Moreover, the role of the terms of trade between industry and agriculture has always been considered important in the development process of dual economies. While the endogenous economic growth literature, has emphasized the role of acquisition of skills and research and development in the process of economic growth, and *educational choice* by consumers/students, it has not, so far, shed any light on the effects of human capital accumulation the intersectoral terms of trade in dual economies. These terms of trade have, since long, been a crucial topic in the economics (and politics) of development of poor countries.[1]

Without much risk of oversimplification it may be stated that the endogenous growth literature has largely ignored the implications of human capital accumulation in developing countries where the issues mentioned above are extremely important. In developing countries with large populations, the issue of *choice of the amount* of education to be undertaken is secondary to the constraint that there almost always exists an excess demand for education and educational services have to be *rationed* among the students. The constraint on resources introduces a role that the government tax policy can play in fostering human capital accumulation and economic growth in poor LDCs.

In this paper we examine the effects of human capital accumulation on intersectoral terms of trade and economic growth of a resource constrained dual economy. In particular, we address the role government tax policy can play in the development process by increasing public investment in education.

We study a stylized version of a growing dual economy with human capital accumulation. Growth, however, is not endogenous. Some workers undertake human capital accumulation but the number of workers who can be trained at any point in time is limited by the existing capacity of schools. The capacity of the schools is, in turn, a result of government investment in education.[2] Government expenditure is financed by a tax[3] on capital income and/or the incomes of skilled workers. Moreover, education takes time and those who commit themselves to accumulating human capital[4] must forsake earning any income during the period of their education. Skilled labor can work specifically in the manufacturing (secondary) sector whereas unskilled workers are mobile between the agricultural and manufacturing sectors. We examine short run equilibrium of this economy and also study the dynamic and comparative static properties of steady state. We discover that, given the current investment in education, the steady state is unique, locally stable and is characterized by a unique ratio of skilled to unskilled workers. An immediate policy question that becomes relevant at this point is the impact of an increase in the investment in education (financed by an increase in the tax rate). We discover that such a measure would result in an increase in the ratio of skilled to unskilled workers and a drop in the relative price of manufactured goods and the capital intensity of agricultural operations. Such effects cannot be captured in a traditional dual economy type model (See Ranis (1989)).

The analysis in this paper is the first to address the important issue of the effects of human capital accumulation on economic growth in LDCs in the early stages of their development process. The analysis in this paper, therefore, is to be viewed as an alternate to the endogenous growth literature. We are of the view that whereas the assumptions, analysis and concerns of the endogenous growth literature better reflect the stylized facts of the more developed economies, the anlysis in this paper (which views access to schooling as being limted and necessarily rationed and which focuses on the importance of human capital on the terms of trade) is more suited to analyze the growth process in densely populated, dualistic, less developed economies. Our paper addresses some of the concerns of less developed economies more adequately than the endogenous growth literature, and explicitly incorporates the role the tax policy can play in the development process of these resource constrained economies.

The remainder of this paper is organized as follows. In section II, we outline the model. Section III studies the short run equilibrium and comparative static properties of the model. Long run steady state and its comparative static and dynamic properties are studied in section IV. Section V discusses the policy implications of the model and the last section offers some concluding remarks.

II. THE MODEL

Consider a closed economy with two sectors, A (agriculture) and M (manufacturing). In sector A, capital (K_A) and unskilled labor (N_A^u) produce output (Q_A) according to a twice continuously differentiable, linear homogeneous production function F_A:

$$Q_A = F_A(K_A, N_A^u) \tag{1}$$

In sector M, output (Q_M) is produced by unskilled workers(N_M^u), skilled workers (N^s) and capital (K_M) as per a twice continuously differentiable, linear homogeneous production function F_M:

$$Q_M = F_M(K_M, N_M^u, N^s) \tag{2}$$

Skills are acquired by accumulating human capital (through training) which takes time[5], τ, during which people undergoing training do not work. To simplify the analysis[6] we assume that it costs η units of commodity A to produce one skilled worker and that the government decides how much to spend on education[7]. This expenditure is financed by taxing capitalists and/or skilled workers[8]. The government's budget is always balanced.

Intersectoral migration of unskilled workers implies that their wage (w_A) in sector A must equal their wage (Pw_M) in sector M (where P is the price of commodity M in terms of commodity A). A further (purely simplifying) assumption is that sector A goods can only be consumed whereas sector M goods can be consumed or invested.

A worker in sector A is paid a fixed fraction[9], α, of per capita output. Intersectoral migration of unskilled workers ensures that

$$w_A = \alpha f_A, \text{ and} \tag{3}$$

$$\alpha f_A = P[f_M - k_M f_{M1} - n^s f_{M2}] \tag{3'}$$

where $f_A = F_A/N_A^u$, $f_M = F_M/N_M^u$, $k_M = K_M/N_M^u$, $f_{M1} = \partial F_M/\partial K_M$ = marginal product of capital in manufacturing, $n^s = N^s/N_M^u$, $f_{M2} = \partial F_M/\partial N^s$ = marginal product of skilled labor. In equation (3'), we have assumed that all factors of production are paid their respective marginal products in this sector.

A further simplifying assumption is that workers consume all their income whereas capitalists save and invest all their income. Hence in sector A, market clearing requires that the following equation be satisfied:

$$Q_A = C_A(P,w^u)N^u + C_A(P,f_{M2})N^s + \int_{t-\tau}^{t} I_E(v)dv \tag{4}$$

where $C_A(\,)$ is per worker demand for commodity A, w^u is the wage of unskilled workers, $I_E(v)$ is the government's investment in education at time v. Equation (4) says that the output of sector A is exhausted by the consumption of skilled and unskilled workers and those acquiring human capital[10]. Clearly $\partial C_A/\partial P > 0$, and the sign of $\partial C_A/\partial w^u$ = the sign of $\partial C_A/\partial f_{M2} > 0$.

Capital accumulation in sector A is given as:

$$\hat{K}_A \equiv \dot{K}_A/K_A = (1-\alpha)f_A(1-t_r)/(k_A P) \tag{5}$$

where it is implicit that all capital income in sector A is invested[11] and that investment goods have to be purchased from sector M. t_r is the proportional tax rate on capital income. A hat (ˆ) refers to rate of growth and a dot (.) above a variable signifies its time derivative.

Capital accumulation[12] in sector M is given by:

$$\hat{K}_M \equiv \dot{K}_M/K_M = f_{M1}(1-t_r) \tag{6}$$

Total population, N, consists of skilled and unskilled workers and those undergoing training. Hence we have

$$N = N^u + N^s + \int_{t-\tau}^{t} (I_E(v)/\eta)dv \tag{7}$$

where η is the cost (in terms of commodity A) of producing one skilled worker. Population grows at rate n, i.e., $\dot{N}/N = n$, whence

$$\dot{N}^u = nN - I_E(t)/\eta \tag{8}$$

Thus, population growth at time t consists of additions to unskilled labor and those beginning schooling.

III. SHORT RUN EQUILIBRIUM

Define the following five variables $l_A \equiv N_A^u/N^u$, $z_A \equiv K_A/N^u$, $z_M \equiv K_M/N^u$, $k_A = K_A/N_A^u$, and $k_M = K_M/N_A^u$ so that

$$z_A = k_A/l_A, \qquad 0 < z_A \leq k_A \quad \text{and} \tag{9}$$

$$z_M = k_M(1 - l_A), \qquad 0 < z_M \leq k_M \tag{10}$$

We can divide equation (4) by N_A^u to obtain

$$Q_A/N_A^u = f_A = C_A(P,w^u)/l_A + C_A(P,f_{M2})N^s/(N^u l_A) + \int_{t-\tau}^{t} I_E(v)dv/(N^u/l_A) \tag{11}$$

In the short run z_A and z_M are fixed whereas k_A and k_M can vary because of changes in the allocation of unskilled labor between sectors A and M. The short run equilibrium can be defined by the 5-tuple (k_A, k_M, l_A, P, w^u) which satisfies equations (3), (3'), (9), (10), and (11) for given values of z_A, z_M, N^s/N^u, and $\int_{t-\tau}^{t} I_E(v)dv$. We now proceed as follows. From equation (3') we write

$$P = \alpha f_A/[f_M - k_M f_{M1} - n_M^s f_{M2}] \tag{12}$$

Hence, using the fact that

$$n^s = [N^s/N^u]/(1 - z_A/k_A) \tag{13}$$

we have

$$\frac{\partial P}{\partial k_A} = \frac{[(f_M - k_M f_{M1} - n^s f_{M2})\alpha f_A' - \alpha f_A(-N^s/N^u(z_A/k_A^2))]}{[f_M - k_M f_{M1} - \{(N^s/N^u)/(1 - z_A/k_A)\}f_{M2}]^2} > 0 \tag{14}$$

Further,

$$\frac{\partial P}{\partial k_M} = \frac{[-\alpha f k_A \{-k_M f_{M1M1} - n^s f_{M2M1}\}]}{[f_M - k_M f_{M1} - \{(N^s/N^u)/(1-z_A/k_A)\} f_{M2}]^2} \tag{15}$$

Here $f_{M1M1} = \partial^2 f_M/(\partial k_M)^2$, $f_{M2M1} = \partial^2 f_M/(\partial k_M \partial n^s)$. Equation (15) points to the fact that response of P to changes in k_M depends upon the structure of production in sector M, whereas we learn from equation (14) that the response of P to changes in k_A does not depend on the value of k_A. At this point we make the assumption that $f_{M2M1} = 0$, i.e., the marginal product of skilled labor does not change with additions to capital. With this assumption $\partial P/\partial k_M$ is unambiguously negative.

We totally differentiate equation (11) to get:

$$\frac{dk_M}{dk_A} = \frac{(z_A/k_A^2)(k_A f'_A - f_A) - (\partial C_A/\partial P)(\partial P/\partial k_A)(1 + N^s/N^u) - (\partial C_A/\partial w^u)(\partial w^u/\partial k_A)}{(\partial C_A/\partial P)(\partial P/\partial k_M) + (\partial C_A/\partial P)(\partial P/\partial k_M)(N^s/N^u) + (\partial C_A/\partial f_{M2})(\partial f_{M2}/\partial k_M)(N^s/N^u)} \tag{16}$$

With the above assumption $dk_M/dk_A > 0$.

We have depicted the short run equilibrium in Figure 1. We have plotted equations (10) and (11) and termed them "the manufacturing output" and "market clearing" curves respectively. These schedules uniquely determine the short run equilibrium k_A^* and k_M^*.

Insert Figure 1 about here.

From the assumptions on $C_A(.)$ and the well-behavedness of the production functions, it can be seen that the graph of equation (11) stretches from the line $k_M = \bar{z}_M$ to $(k_A, k_M) = (\infty, \infty)$. Equation (11) cannot intersect the line $k_A = \bar{z}_A$. This is because if $k_A = \bar{z}_A$, $N_M^u = 0$ so that all output of sector A is consumed by sector A workers. But that cannot happen since their consumption is constrained to not exceed αf_A (where $\alpha < 1$ by assumption).

COMPARATIVE STATICS OF SHORT RUN EQUILIBRIUM

We totally differentiate equations (10) and (11) taking equation (12) into account and write in matrix form:

$$\begin{bmatrix} Y_1 & Y_2 \\ z_A k_M/k_A^2 & 1-z_A/k_A \end{bmatrix} \begin{bmatrix} dk_A \\ dk_M \end{bmatrix} = \begin{bmatrix} -f_A/k_A dz_A + C_A(P, f_{M2})dN^s \\ dz_M + (k_M/k_A)dz \end{bmatrix} \tag{17}$$

where:

$$Y_1 = f_A' z_A/k_A - f_A/(k_A^2)z_A - (\partial C_A/\partial P)(\partial P/\partial k_A) - (\partial C_A/\partial w^u)(\partial w^u/\partial k_A) - (N^s/N^u)(\partial C_A/\partial P)(\partial P/\partial k_A) \text{ and}$$

$$Y_2 = -(\partial C_A/\partial P)(\partial P/\partial k_M) - (N^s/N^u)(\partial P/\partial k_M) - (\partial C_A/\partial f_{M2})(\partial f_{M2}/\partial k_M).$$

We wish to calculate dk_A/dz_A, dk_A/dz_M, dk_A/dN^s, dk_M/dz_A, dk_M/dz_M and dk_M/dN^s. Let Δ be the determinant of the matrix on the left hand side of equation (17). We have now

$$\Delta = [1-z_A/k_A][(z_A/k_A^2)(k_A f_A' - f_A) - (\partial C_A/\partial P)(\partial P/\partial k_A) - (\partial C_A/\partial w^u)(\partial w^u/\partial k_A) - (N^s/N^u)(\partial C_A/\partial P)(\partial P/\partial k_A)] - (z_A/k_A^2)k_M[-(\partial C_A/\partial P)(\partial P/\partial k_M) - (N^s/N^u)(\partial P/\partial k_M) - (\partial C_A/\partial f_{M2})f_{M2M1}] < 0.$$

We can now prove the following proposition.

Proposition 1: Assume that the marginal product of capital in sector M does not depend upon the input of skilled labor ($f_{M2M1} = 0$). We then have (i) $dk_A/dz_A > 0$, (ii) $dk_A/dz_M > 0$, (iii) $dk_A/dN^s < 0$ (as more workers acquire skills, the capital intensity of agriculture drops), (iv) $dk_M/dz_A < 0$, (v) $dk_M/dz_M > 0$, (vi) $dk_M/dN^s > 0$ (capital labor ratio in sector M increases as more workers acquire skills).

Proof: See Appendix 1.

The above results may be compared to the standard results from the static models of human capital accumulation (e.g. Miyagiwa (1989)) and those of the dual economy literature (e.g. Amano (1980)). In Miyagiwa's model in the short run, N^s is a parameter and comparative static results refer to a shock that increases the stock of capital. Our model permits a much richer set of comparative static results since we treat z_A, z_M, and N^s as parameters and allow the capital-labor ratios in the two sectors to adjust. Moreover, our results are fundamentally more general since we combine human capital accumulation with dualistic growth. Amano's model, for instance, is unable to decipher the effect of changes in education since he does not model human capital accumulation. We are now in a position to examine the long run properties of the model.

IV. LONG RUN EQUILIBRIUM

The long run steady state of the economy is defined by the following three equations:

$$\hat{K}_A = [1/(k_A P)][(1-\alpha)f_A(1-t_r)] = \hat{N}_A^u \tag{18}$$

$$\hat{K}_M = f_{MI}(1-t_r) = \hat{N}_M^u \tag{19}$$

$$\hat{n}^s = \hat{N}^s - \hat{N}_M^u = 0 \tag{20}$$

The growth of skilled labor today is governed entirely by investment in education made τ periods earlier. Thus, using equation (20) we have

$$\hat{N}^s = \hat{N}_M^u = [1/(\eta N^s)][I_E(t-\tau)] \tag{21}$$

We can then prove the following proposition.

Proposition 2: Given current investment in education, the steady state is characterized by an *unique* ratio of skilled to unskilled workers. Further, there is a unique steady state k_A.

Proof: At time t, N^s is a datum. Using equations (8) and (21), we can define the unique steady state n^s consistent with current investment in education $I_E(t)$ as:

$$n^{s*} = [1/(\eta n N - I_E(t))][I_E(t-\tau)\eta] \tag{22}$$

where the star (*) refers to steady state. Using equation (19) and the well-behavedness of the M sector production function we can solve for k_M^*. We then have

$$P^* = [\alpha f_A][1/(f_M - k_M^* f_{M1} - n^s f_{M2})] \tag{23}$$

$$P^* = [1/(k_A I_E(t-\tau))][(1-\alpha)f_A(1-t_r)\eta N^s] \tag{24}$$

Using equations (23) and (24) it is possible to solve for the unique P^* and k_A^* consistent with steady state equilibrium.

We can depict P^* and k_A^* graphically as in Figure 2:

Insert Figure 2 about here.

Equation (23) describes a positive relation between steady state P and steady state k_A whereas equation (24) describes a negative relation between them. P^* and k_A^* are seen to exist uniquely. Current additions to the stock of skilled workers depends on expenditure on education τ time periods ago whereas the current stock of skilled workers is a datum. Hence the magnitude of the current investment in education determines the size of the unskilled labor force and, therefore, the steady state value of the capital-labor ratio in industry M. We can then solve for the steady state values of the price ratio and the capital labor ratio in sector A from two simultaneous equations. This completes the characterization of steady state as well as a proof of its existence and uniqueness.

LOCAL STABILITY OF STEADY STATE

We rewrite the steady state conditions as:

$$[1/(k_A P)][(1-\alpha)f_A(1-t_r)] - [1/(\eta N^s)][I_E(t-\tau)] = 0 \tag{25}$$

$$f_{M1}(1-t_r) - [1/(\eta N^s)][I_E(t-\tau)] = 0 \tag{26}$$

$$[I_E(t-\tau)]/\eta - \pi^e[nN - I_E(t)/\eta] = 0 \tag{27}$$

We can then prove the following proposition:

Proposition 3: The long run steady state is a locally stable equilibrium.

Proof: See Appendix 2.

COMPARATIVE STATICS OF STEADY STATE

We finally examine the effects of an increase in current investment in education on the steady state properties of the model. Investment in education in the past is a datum but the current investment in education can be varied. From the government's budget constraint we know that a change in government investment in education implies a corresponding change in the tax rate on capital income. We can prove:

Proposition 4: An increase in current investment in education (an increase in t_r) raises the steady state ratio of skilled workers to unskilled workers in sector M, and lowers the steady-state values of P and k_A.

Proof: See Appendix 3.

It is instructive to compare our results with the extant literature. The human capital literature associated with Findlay and Miyagawa is unable to analyze the dynamics of human capital accumulation. The dual economy literature associated with, among others, Marino and Amano, is unable to decipher the effects of human capital accumulation on the growth characteristics of the economy. Thus, the synthesis presented in this paper is a generalization of both branches of the literature.

V. POLICY IMPLICATIONS

The model studied in this paper leads to interesting implications for the role of tax policy in fostering economic growth in a resource constrained overpopulated poor countries with dualistic economic structures. First, it should be noted that in such economies, there necessarily exists an excess demand for education as schooling facilities are limited . The access to education thus has to be rationed by the government. Moreover, schooling requires an investment of time.

Second, current additions to the stock of skilled workers depends on investment in education made in the past (τ time periods ago) and the current stock of skilled workers is predetermined -- i.e., it is a datum.

Finally, it must be emphasized that although the growth rate in steady state is guided by the rate of growth of population tax, policy can certainly affect the characteristics of the steady and the transitions to it. In particular, government tax policy can affect split of the labor force (the stock of unskilled workers) into unskilled and skilled workers through investment in education. This human capital formation part is specially important since factors other than capital accumulation and population growth account for well over half of the growth in real national income.

The Proposition 4 is of key importance in demonstrating how an increase in the tax rate *(t_r)* alters the characteristics of the growth model analyzed in this paper and fosters economic growth. Comparative steady state analysis suggest that a balanced budget increase in public investment in education (an increase in t_r) raises the steady state ratio of skilled workers to unskilled workers in sector M, and lowers the steady-state values of P (price of commodity M in terms of commodity A) and k_A (the ratio of capital to unskilled workers in sector A).

A increase in the steady state value of the ratio of skilled to unskilled workers in the manufacturing sector suggests that this sector becomes more productive. Rising manufacturing output, consequently, leads to a decline in the relative price of manufacturing commodity (intersectoral terms of trade). On the other hand, capital to unskilled workers in the agricultural sector (the capital-labor ratio in this sector, as only unskilled workers are assumed to be used in agriculture) falls. This suggests that the agricultural sector releases resources. The aforesaid results strengthen the conclusion that an increase in investment in education, financed by a tax increase, fosters economic growth. Thus, government tax policy can play an important role in the development process of resource constrained LDCs whose economic structure is characteristically dual.

VI. CONCLUDING REMARKS

The existing literature on dualistic economic growth has neglected human capital formation. In this paper we have examined the dynamic long run and static short run properties of a dual economy with skilled and unskilled labor. Our analysis of the short run equilibrium revealed that the structure of production in the manufacturing sector is important. However, for a whole class of production technology -- one in which the marginal product of skilled workers in the manufacturing sector is unaffected by the changes in the stock of capital -- it was shown that a unique short run equilibrium will exist. A comparative static analysis suggests that the capital labor ratio in the agricultural sector would (i) rise when the ratio of capital to unskilled labor in agriculture or manufacturing rose, and (ii) fall when the stock of skilled workers rose. The capital labor ratio in the manufacturing sector would (i) rise when the ratio of capital to unskilled workers in manufacturing or the stock of skilled workers rose, and (ii) fall when the ratio of capital to unskilled workers in the agricultural sector rose.

We further discovered that given current investment in education, the ratio of skilled labor to unskilled labor and the capital labor ratios in the two sectors are uniquely determined in steady state. Moreover, if there is positive addition to the stock of skilled workers today, the steady state is locally stable. The comparative steady state properties of the model for changes in balanced budget increases in current investment in education were also analyzed. An increase in tax rate was shown to lead to a rise in the steady state value of the ratio of skilled to unskilled workers in manufacturing, and a fall in the realative price of manufacturing commodity and the capital-labor ratio in agriculture. These resulted in an interesting implication for the role of government tax policy in LDCs -- the government, through an increased investment in public education (financed by a tax on capital income and/or incomes of skilled workers), can foster economic growth in these countries.

The analysis of this paper can be extended to cover the case when skilled workers are required in the agricultural sector as well. An obviously interesting extension is the determination of the optimal current investment in education. These constitute a partial agenda for future research in this area.

FOOTNOTES

1. The role of human capital accumulation in developing countries has been examined in recent papers by Findlay (1984), Findlay and Kierzkowski (1983), and Miyagiwa (1989). Typically such studies have concentrated on two sector models and have analyzed the implications of human capital accumulation. These studies are, however, all static and can, therefore, shed very little light on the dynamics of human capital accumulation and economic growth.

   Moreover, recent models of economic growth in developing dual economies have typically disregarded human capital accumulation. This is true both of the traditional dual economy literature (see, for instance, Marino (1974),and Amano (1980)) and its Harris-Todaro variant (see, for instance, Jha and Lachler (1981)). Since factors other than capital accumulation and population growth account for well over half the growth of real national income, this exclusion is especially significant. The literature on economic growth with human capital accumulation in both its endogenous growth and non-endogenous growth versions seems inapplicable to developing countries that exhibit the salient characteristics associated with dual economies.

2. Past and current investment in education limit the production of skills. In an infinite horizon model like the present one, everyone would like to be a skilled worker since the wage of skilled workers is higher than that of unskilled workers. We imagine, therefore, some kind of rationing of available educational opportunities among applicants on the basis of a lottery.

3. The form of taxation plays no important role here. We work with proportional taxation and inelastic factor supplies. We may as well have worked with lump sum taxation without affecting the results.

4. The technology for producing skills in this paper is similar to that used by Miyagiwa (1989) and Findlay and Kierzkowski (1983). These authors have used a production of skills with time and commodity A (hence, indirectly, capital) as inputs. In a sense the analysis contained in this paper is like the endogenous technical progress literature associated with, among others, Conlisk (1969), and Drandakis and Phelps (1966).

5. The time, τ, for acquiring skills is assumed fixed and unalterable.

6. Relaxing this assumption and admitting a more general cost function clutters up the analysis without qualitatively affecting the results.

7. In Findlay and Kierzkowski (1983) the amount of time spent on education is a choice variable of the individual consumer. This stipulation is absent from Miyagiwa (1989). In the context of most developing countries where existing capacity severely restricts

availability of education it is, perhaps, more appropriate to assume that education is rationed among the various applicants.

8. In what follows we work with a proportional tax on capitalists. Since we assume inelastic supplies of factors of production, taxation plays no important role here. Hence it is assumed that the government decides how much to spend on education and finances this expenditure by taxing capitalists. the government's budget is always balanced.

9. It is easy to show that the following analysis would go through with only minor modifications even if we had assumed that workers in sector A get paid their marginal product, i.e., $w_A = f_A - k_A f_A'$. We follow the stipulation in equation (3) in order to conform more closely to the traditional dual economy literature.

10. In sector A market clearing requires that equation (4) be satisfied. Now write the equations of the disbursals of outputs of sectors A and M. Add them and subtract equation (4) of the text to realize that when the market for A clears so does the market for M. This is independent of the assumption made about investment.

11. It is not necessary to assume that all capital income is invested and all wage income consumed. This assumption is made purely to simplify the analysis.

12. Once again, the assumption being made here is that all capital income in sector M is invested.

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Appendix 1

COMPARATIVE STATICS OF SHORT RUN EQUILIBRIUM

Using (17) by Cramer's rule we can solve:

(i) $$dk_A/dz_A = [1/\Delta][(-f_A/k_A)(1-z_A/k_A) - (k_M/k_A)\{-(\partial C_A/\partial P)(\partial P/\partial k_M) - (N^s/N^u)(\partial P/\partial k_M)-(\partial C_A/\partial f_{M2})f_{M2M1}\}] > 0 \qquad \text{(A.1.1)}$$

(ii) $$dk_A/dz_M = [1/\Delta][(\partial C_A/\partial P)(\partial P/\partial k_M) + (N^s/N^u)(\partial P/\partial k_M) + (\partial C_A/\partial f_{M2})f_{M2M1}] > 0 \qquad \text{(A.1.2)}$$

(iii) $$dk_A/dN^s = [1/\Delta][(1-z_A/k_A)(C_A(.))] < 0 \qquad \text{(A.1.3)}$$

It follows from this that the capital-labor ratio in sector A declines as more workers become skilled.

(iv) $$dk_M/dz_A = [1/\Delta][(k_M/k_A)\{f_A' z_A/k_A - (f_A/k_A^2)z_A -(\partial C_A/\partial P)(\partial P/\partial k_A)(1+N^s/N^u)\} + (f_A/k_A)(z_A/k_A^2)] < 0. \qquad \text{(A.1.4)}$$

(v) $$dk_M/dz_M = [1/\Delta][f_A' z_A/k_A - (f_A/k_A^2)z_A -(\partial C_A/\partial P)(\partial P/\partial k_A)(1+N^s/N^u) - (\partial C_A/\partial w^u)(\partial w^u/\partial k_A)] > 0. \qquad \text{(A.1.5)}$$

(vi) $$dk_M/dN^s = [1/\Delta][-(z_A/k_A^2)(C_A)] > 0 \qquad \text{(A.1.6)}$$

Thus the capital-labor ratio in sector M increases as more workers acquire skills.

Appendix 2

LOCAL STABILITY OF STEADY STATE EQUILIBRIUM:

We can derive the Ruth-Hurwitz matrix of the system of equations (25), (26) and (27) using (23) as:

$$J = \begin{bmatrix} J_{AA} & J_{AM} & J_{AN} \\ J_{MA} & J_{MM} & J_{MN} \\ J_{NA} & J_{NM} & J_{NN} \end{bmatrix}$$

where:

$$J_{AA} = -[1/k_A^2][((1-\alpha)/\alpha)(1-t_r)][f_M - k_M f_{M1} - \pi^s f_{M2}] < 0,$$

$$J_{AM} = [1/k_A][((1-\alpha)/\alpha)(1-t_r)][-k_M f_{M1M1} - \pi^s f_{M2M1}] > 0,$$

$$J_{AN} = -[1/\alpha k_A][(1-\alpha)(1-t_r)f_{M2}] + [1/(\eta N_M^u (\pi^s)^2)][I_E(t-\tau)] > 0,$$

$$J_{MA} = 0,$$

$$J_{MM} = (1-t_r)f_{M1M1} < 0,$$

$$J_{MN} = [1/(\eta N_M^u (\pi^s)^2)][I_E(t-\tau)] > 0,$$

$$J_{NA} = 0,$$

$$J_{NM} = 0,$$

$J_{NN} = -[nN - I_E(t)/\eta] < 0$ (if total additions to population is greater than the current addition to the number of people undergoing training. In other words, there must be some increase in the size of the unskilled labor force).

A sufficient condition for *trace* $(J) < 0$ is that $J_{NN} < 0$. Moreover

$$\begin{vmatrix} J_{AA} & J_{AM} \\ J_{MA} & J_{MM} \end{vmatrix} > 0.$$

Further the determinant of J is negative when J_{NN} is negative. If this is the case the Ruth-Hurwitz condition for local stability are satisfied. (See Gandolfo (1980) pg. 276). From equation (27), however, $[nN - I_E(t)/\eta]$ will be positive (and hence J_{NN} will be negative), so long as there was some positive investment in education at time $(t-\tau)$. Hence the dualistic growth model is seen to possess a unique steady state which is locally stable.

Appendix 3

COMPARATIVE STATIC EFFECTS OF AN INCREASE IN INVESTMENT IN EDUCATION:

Current investment in education is $Pf_{M1}K_M t_r$. Substitute this into equation (22) and rewrite (22), (23), and (24) as:

$$\pi^s = [1/(\eta nN - Pf_{M1}K_M t_r)][I_E(t-\tau)\eta] \tag{A.3.1}$$

$$P = [\alpha f_A][1/(f_M - k_M f_{M1} - \pi^s f_{M2}] \tag{A.3.2}$$

$$Pk_A = [1/(I_E(t-\tau))][(1-\alpha)f_A(1-t_r)\eta N^s] \tag{A.3.3}$$

To examine the comparative steady state properties we recall that a change in current education expenditure is tantamount to an increase in the tax rate on capitalists t_r. We totally differentiate equations (A.3.1) and rearrange to get (assuming that the cross products $f_{M1M2}=f_{M2M1} = 0$):

$$d\pi^s = [1/(\eta nN - Pf_{M1}K_M t_r)^2][I_E(t-\tau)\eta\{f_{M1}K_M t_r dP$$
$$+ PK_M t_r f_{M1M1}dk_M + Pf_{M1}t_r dK_M + Pf_{M1}K_M dt_r\}] \tag{A.3.4}$$

Upon totally differentiating equation (A.3.2) we have

$$dP = [1/(f_M - k_M f_{M1} - \pi^s f_{M2})][\alpha f_A' dk_A]$$
$$+ [1/(f_M - k_M f_{M1} - \pi^s f_{M2})][\alpha f_A\{k_M f_{M1M1}dk_M + \pi^s f_{M2M2}d\pi^s\}] \tag{A.3.5}$$

Upon totally differentiating (A.3.3) we have

$$k_A dP + Pdk_A = [(1-\alpha)\eta/I_E(t-\tau)][f_A(1-t_r)dN^s$$
$$- f_A N^s dt_r + (1-t_r)N^s f_A' dk_A] \tag{A.3.6}$$

The equation system given by (A.3.4), (A.3.5), and (A.3.6) is solved for $d\pi^s/dt_r$, dP/dt_r, and dk_A/dt_r.

We now assume that all total derivatives except those associated with $d\pi^s$, dP, dk_A, and dt_r are zero. (We cannot assume, however, that $dN^s=0$ since $\pi^s=N^s/N_M^u$. For simplicity we assume that all changes in π^s, i.e., $d\pi^s$ comes from changes in N_M^u).

Using these assumptions we define the following notations:

$$z_{11} = [\eta n N - P f_{M1} K_M t_d]^2 > 0,$$

$$z_{12} = -[I_E(t-\tau)\eta f_{M1} K_M t_r] < 0,$$

$$z_{13} = 0,$$

$$R = [I_E(t-\tau)\eta P f_{M1} K_M] > 0,$$

$$z_{21} = -[\alpha f_A r^s f_{M2M2}] > 0,$$

$$z_{22} = [f_M - k_M f_{M1} - r^s f_{M2}]^2 > 0.$$

$$z_{23} = -[\alpha f'_A (f_M - k_M f_{M1} - r^s f_{M2}] < 0,$$

$$z_{31} = 0,$$

$$z_{32} = [k_A I_E(t-\tau)] > 0,$$

$$z_{33} = [P I_E(t-\tau) - (1-\alpha)\eta(1-t_r)N^s f'_A] = ?,$$

$$S = -[(1-\alpha)\eta f_A N^s] < 0.$$

We can now write equations (A.3.4) through (A.3.6) in matrix form as:

$$\begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{bmatrix} \begin{bmatrix} dr^s \\ dP \\ dk_A \end{bmatrix} = \begin{bmatrix} R \\ 0 \\ S \end{bmatrix} \qquad \text{(A.3.7)}$$

Let the determinant of the matrix on the left hand side of (A.3.7) be δ. Applying Cramer's rule:

$$dr^s/dt_r = (1/\delta) \begin{vmatrix} R & z_{12} & 0 \\ 0 & z_{22} & z_{23} \\ S & z_{32} & z_{33} \end{vmatrix} = \delta_1/\delta \qquad \text{(A.3.8)}$$

$$dP/dt_r = (1/\delta) \begin{vmatrix} z_{11} & R & 0 \\ z_{21} & 0 & z_{23} \\ 0 & S & z_{33} \end{vmatrix} = \delta_2/\delta \qquad \text{(A.3.9)}$$

and

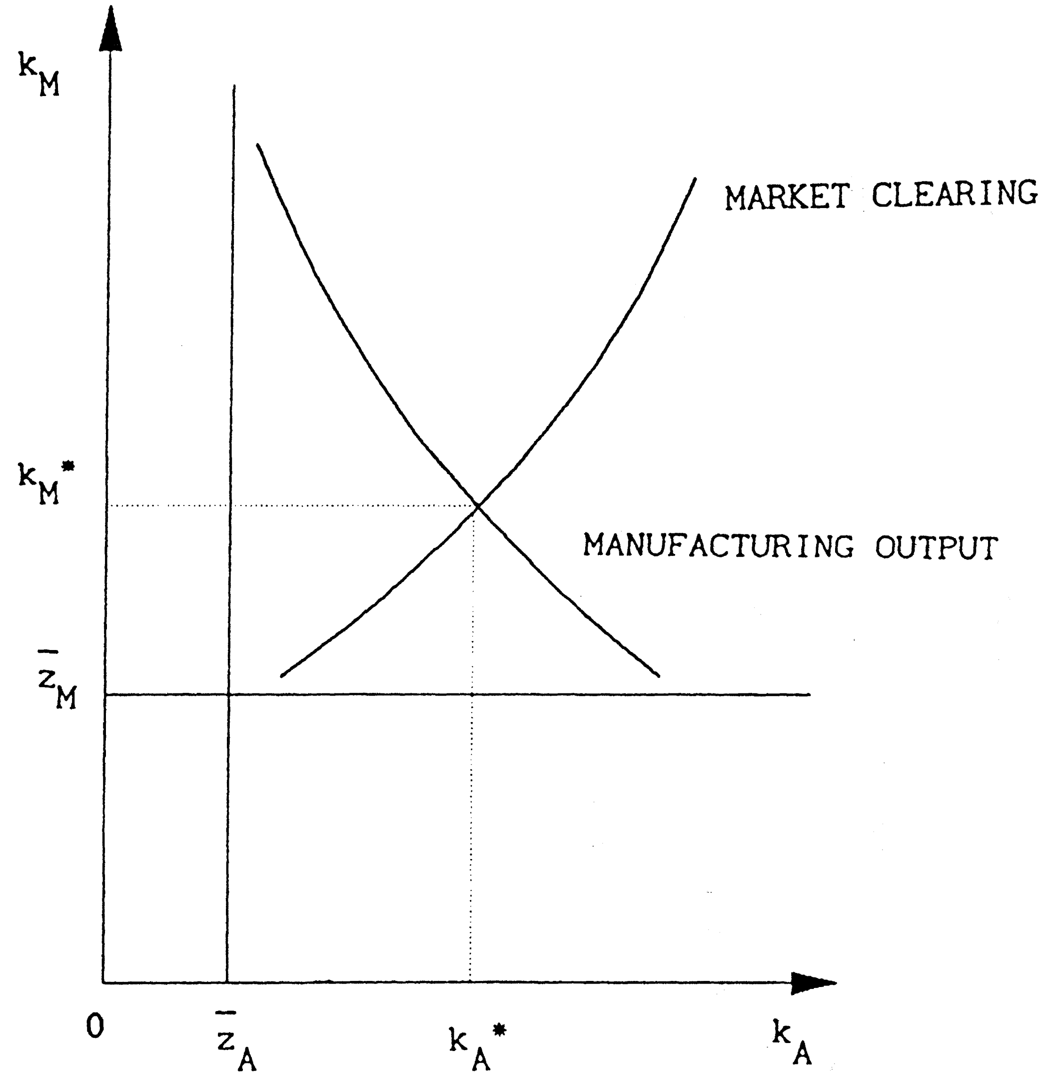
$$dk_A/dt_r = (1/\delta) \begin{vmatrix} z_{11} & z_{12} & R \\ z_{21} & z_{22} & 0 \\ z_{31} & z_{32} & S \end{vmatrix} = \delta_3/\delta \qquad \text{(A.3.10)}$$

Now $\delta = z_{11}(z_{22}z_{33}-z_{23}z_{32}) - z_{21}(z_{12}z_{33})$. The only sign that needs to be worked out is that of z_{33}. Now, it can be checked that a sufficient condition for z_{33} to be positive is that $(1-\alpha)(1-t_r)$ is small in relation to $[1/(\eta N^s f_A'(f_M-k_M f_{M1}-\pi^s f_{M2}))][(\eta N^s f_A')(\alpha f_A I_E(t-\tau))]$. We make this assumption to get $\delta > 0$.

Now, $\delta_1 = R(z_{22}z_{33}-z_{23}z_{32}) + S(z_{12}z_{23}) > 0$ (given that $\alpha<1$, $t_r<1$, and assuming $k_A>f_A$). Hence $d\pi^s/dt_r > 0$.

Evaluating δ_2, we have $\delta_2 = -z_{11}Sz_{23} - z_{21}Rz_{33} < 0$. Hence $dP/dt_r < 0$.

Now evaluating δ_3, we have $\delta_3 = z_{11}z_{22}S - z_{21}(z_{12}S - z_{32}R)$. We can rewrite this as $\delta_3 = z_{11}z_{22}S - z_{21}z_{12}S + z_{21}z_{32}R$. The first two terms seem to outweigh the third. If this is the case then $\delta_3 < 0$, yielding $dk_A/dt_r < 0$. This establishes Proposition 4.



k_M
k_M^*
$\bar{z}_M$
MARKET CLEARING
MANUFACTURING OUTPUT
0
$\bar{z}_A$
k_A^*
k_A

Figure 1

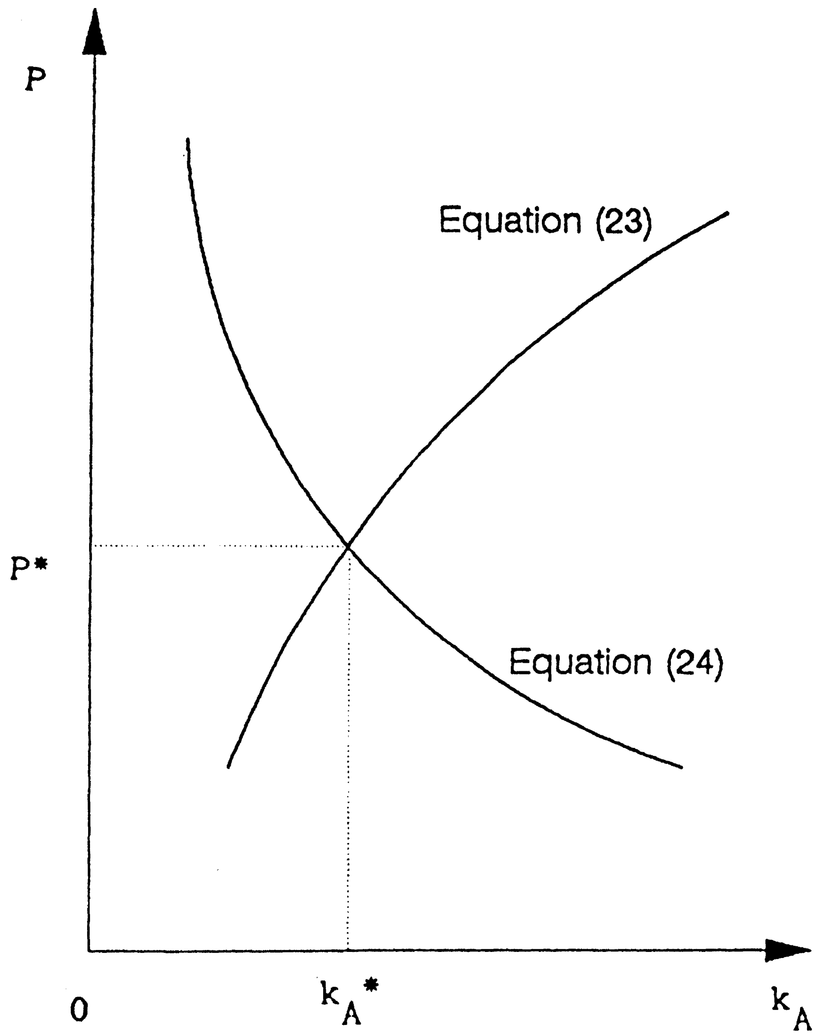


Figure 2