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A Stochastic Programming with Recourse Model for Supply Chain and Markeing Planning of Short Life Cycle Products

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Abstract

A firm that deals with a short life-cycle product needs to make strategic supply chain decisions such as capacity planning and initial marketing decisions such as advertising well in advance under uncertain future demand. The firm initially can only generate a set of demand scenarios and provide a realistic assessment of the demand only after observing actual demand during the initial periods. In this paper, we provide an integrated marketing and supply chain framework and suggest the use of a two stage stochastic programming with recourse to analyze this situation. The original model is a stochastic non linear model with integer constraints, which is extremely difficult to solve. So, we suggest an innovative approach to convert it to a linear mixed integer stochastic program. In the numerical study, we discuss in detail the behavior of optimal pricing and advertising policies in the presence of a supply constraint and the benefits of including stochsticity and marketing decisions that can shape the demand process while planning for the new products.

Keywords: Marketing, stochastic programming with recourse, innovation diffusion, supplies chain and capacity management under uncertainty, advertising and pricing.

1. Introduction

The firms that deal with short life-cycle products need to make strategic supply chain decisions such as capacity planning well in advance under an uncertain demand. In the presence of a short life cycle, the capacity decision is irreversible because the lead time required to adjust the capacity is often too long (Ho et al. 2002). As a result, in industries such as semiconductor, video game consoles when the demand exceeds the supply, the firm looses sales (Business Week 2007). On the other hand, when the capacity is excess, the capacity costs that include plants, mortgage payments, rents, wasted training etc. considerably erode profits (Shapiro 1977). Cohen et al. (2000) suggested that there should be an explicit link between the demand diffusion and capacity planning decisions. Cantamessa and Valentini (2000) enhanced the demand model by using a Bass diffusion process (Bass 1969) and investigated the benefits associated with implementing backorder, delayed product launch and lost sales strategies.

This research stream assumed that the demand for an innovation is deterministic. Nonetheless, it is a common knowledge that the demand is stochastic in nature (Kurawarwala and Matsuo 1996). For example, Nintendo revised the forecast for its wii console from 14 millions to 17.5 millions (Marketing Week 2007). Also, innovation and imitation factors that influence the diffusion process dynamics depend upon social and customer behaviors such as individualism, uncertainty avoidance, power distance, masculinity, and peer pressure (Yaveroglu and Donthu 2002, Van den Bulte and Stremersh 2004) that can not be predicted accurately. For example, Sega experienced a slower than anticipated diffusion pattern for its console (Thomke 1999). Therefore, it is a strong assumption to model the diffusion parameters as deterministic. Hence, Kurawarwala and Matsuo (1996) and Kumar and Swaminathan (2003) suggested that there is a need to include the stochasticity in the Bass diffusion model while making integrated production and marketing decisions for innovations.

Stochastic programming with a recourse model is well suited to solve such problems (Bienstock and Shapiro 1988). It is been often used to plan the capacity under uncertain situations (Ahmed et al. 2003). The underlying problem is modeled as a two (multi) stage stochastic programming with recourse where the first stage decides the capacity for the entire horizon while the second stage takes recourse actions to correct infeasibilities. Because the focus of this research was on improving computational efficiencies, it neither considered the diffusion effects nor integrated the marketing decisions with operations planning and investigated their impact on the profits.

The main objective of this work is to extend the prior work by explicitly modeling the uncertainty inherent in the innovation diffusion process and propose an enhanced two stage stochastic programming model with recourse to compute robust supply chain plans for the innovations. The original model is a nonlinear binary stochastic programming model that does not guarantee optimality, and so we suggest an innovative approach that converts it to a stochastic mixed integer program that can now be solved to optimality.

Researchers increasingly understand the need to integrate marketing and supply chain decisions (Ulusoy and Yagzac 1995, Shah 2004). The prior research (Cantamessa and Valentini 2000) relies on passive options such as incurring backorders and lost sales to handle the demand explosion occurring during the diffusion process. Nonetheless, firm can adjust its pricing and advertising policies to manage the demand growth. For example, Apple Inc. slashed iPhone price by $ 200 to boost sales after observing low demand (Michaels and Dalrymple 2007). Therefore, in our model, we explicitly include marketing variables namely price and advertising that can influence the diffusion process as recourse actions in addition to backorders and lost sales. This approach is thus motivated by Ulusoy and Yagzac (1995) and Khmelnitsky and Kogan (1996) that combine capacity and tactical marketing decisions. However, unlike our work, their research neither considered the diffusion effects nor did they incorporate the uncertainty in the demand process in their optimization models. Quick response manufacturing literature suggests the importance of adjusting sourcing and inventory decisions after observing initial sales for new products (Fisher and Raman 1996). In the paper, we demonstrate the importance of adjusting marketing decisions in response to actual sales behavior.

The rest of the paper is organized as follows. In the next section, we propose both nonlinear and mixed integer stochastic programming models with recourse. In Section 3, we report experimental design and report the results of the numerical study. Finally, conclusion and managerial implications are discussed in Section 4.

2. Stochastic Programming Model with a Recourse

2.1 Stochastic non linear programming model with recourse

The demand for an innovation typically follows a diffusion process. Bass (1969) proposed an innovation diffusion model, which is parsimonious and has found a good empirical fit across several product categories (Mahajan et al. 2000). The model is intuitive and is based on the theory that the diffusion is first accepted by innovators who subsequently motivate potential adopters to purchase the innovation. In our paper, we use the Bass model because of these strengths. Bass (1969) modeled the demand process using the following differential equation.

$$d(t)=\left\{p+\frac{q_1}{m}D(t)\right\}[m-D(t)] \tag{1}$$

where $d(t)$ is instantaneous demand at time t, p and q are constants$\in$ (0,1), which represent the effects of mass media and the adopters on the potential adopters respectively. D(t) is the cumulative number of adopters at time t while m is the market potential.

Now, we extend the original Bass model to include marketing variables namely price and advertising. Among the various ideas proposed to model the effects of pricing and advertising, we have decided to use the approach that suggests that price affects the overall market size and advertising affects the shape of adoption curve. It implies that price influences a consumer's decision to buy the product and so affects the potential market but not the timing of purchase. On the other hand, advertising does not affect the decision to buy the product but the timing of purchase. Empirical study by Jain and Rao (1990) have shown that price at time t effects the overall market size at time t as shown below

$$m(t) = mP_t^{\varepsilon} \tag{2}$$

where ε is the price elasticity of demand. Figure 1 shows how price can influence the diffusion process.

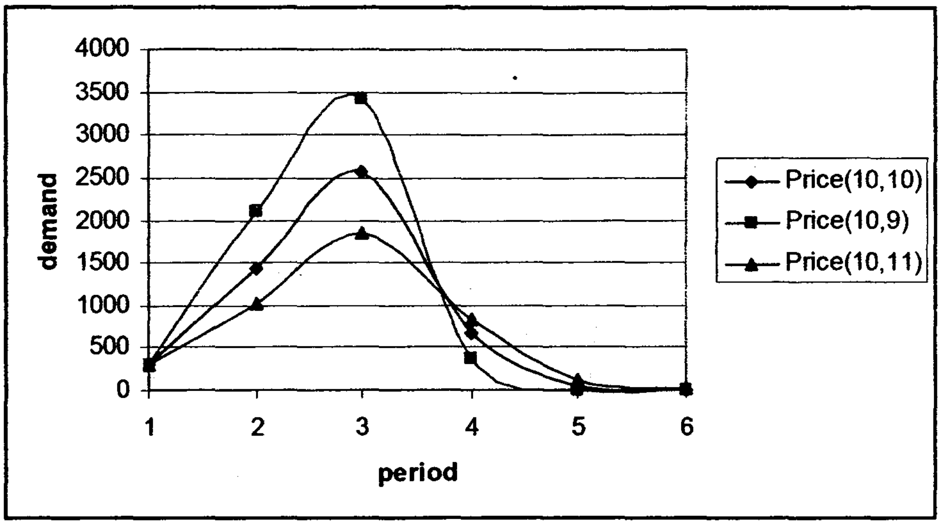

Figure 1. Demand dynamics as a function of price

Several advertising models have been proposed in the literature that attempt to influence imitators and innovators in different ways (Mahajan et al. 2000). An empirical study by Horskey and Simon (1983) has shown that p, the coefficient of an external influence could be represented as a function of advertising expenditure with diminishing returns.

$$p(t) = p_1 + p_2 \log A(t) \qquad (3)$$

where p_1 is the effect of the mass media as in the Bass model while p_2 denotes an advertising effectiveness. A(t) denote advertising expenses incurred in time t. Note that $p(t) = p_1$ when the advertisement expenditure is zero. Figure 2 graphically illustrates how advertising can bring forward the future demand. Based on the past experience with similar products, the firm would be in a position to estimate the value of p_2. Now, we rewrite the Bass model (equation 1) to capture the pricing and advertising effects by using equations 2 and 3 as

$$d(t)=\left\{p_1+p_2\log A(t)+\frac{q_1}{m}D(t)\right\}\left[mP_t^{\varepsilon}-D(t)\right] \qquad (4)$$

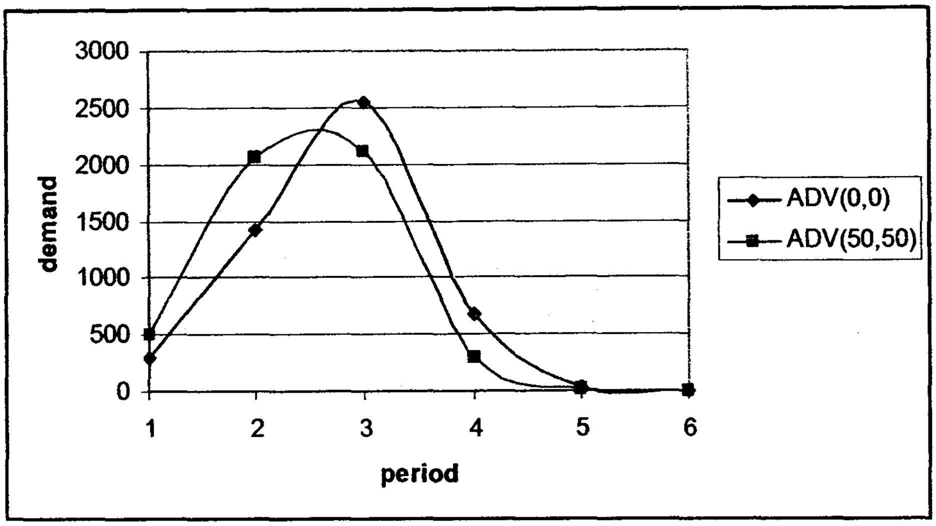

Figure 2. Demand growth as a function of advertising

Revenue can then be written as

$$R(t)=\sum_{t=1}^{T}d(t)P(t)-A(t) \qquad (5)$$

Equation 4 assumes that the firm can correctly estimate the parameters of the diffusion process. But, as discussed in the introduction section, these parameters are subject to randomness. We believe that the impact of pricing and advertising can be measured from past experience (experience with similar markets and similar category of products), but it is very unlikely that past performance will be able to estimate the original Bass model parameters in a precise manner. At best, by using the past data on similar products and the feel of the market, initially, the firm can generate a set of demand scenarios. For example, management can consider scenarios where the market potential is likely to be low, moderate and high. Similarly, the timing of peak can be early or late and peak demand during peak can be high or low. This information can be subsequently used to estimate the Bass model parameters (Shah 2004). Now, we rewrite the demand equations in the presence of the uncertainty as below. We assume that p_2 and ε can be estimated correctly. Note that our stochastic demand model (equation 6) can also represent the uncertainty in the price elasticity and advertising effectiveness if it exists in certain situations. Also, our demand model can also easily incorporate influence of advertising on imitators. This offers more flexibility in modeling realistic situations.

$$d^s(t)=\left\{p_1^s + p_2 \log A(t) + \frac{q^s}{m^s} D^s(t)\right\}\left[m^s P_t^\varepsilon - D^s(t)\right] \qquad (6)$$

$$R^s(t) = \sum_{t=1}^{T} d^s(t)P(t) - A(t) \qquad (7)$$

Where s represents a scenario. The firm can provide a realistic assessment of the demand only after observing actual demand during initial periods. For example, both Fisher and Raman (1996) and Bass (1969) have shown that the demand process (subsequently its parameters) can be predicted with certainty after observing the actual initial demand. So, we divide the product life cycle in two stages where the demand process is random in stage I while it becomes deterministic in the second stage once the firm obtains the actual demand data and uses it to correctly estimate the parameter values. Figure 3 shows the sequence of events and decisions taken in both stages.

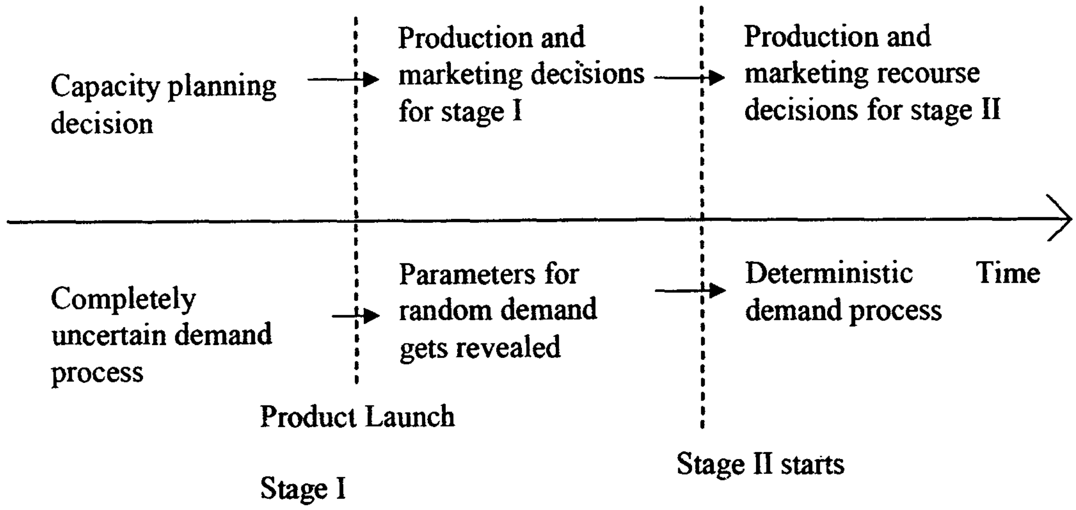

Figure 3. Sequence of events occurred and decisions made in both stages

Two-stage stochastic programming model is based on the following assumptions. First, random events related to the demand process occur with probabilities that are independent of the first stage decisions. Second, the truth regarding the diffusion process is revealed before the start of the second stage so that the firm can use an appropriate contingency plan in the second stage. Third, the overall objective is to maximize the expected profits over the two stages. Note that because there always exist a contingency plan, which is a combination of advertising, pricing, backorders, and inventory decisions, the feasibility is always ensured. The two stages only conceptually divide the product life cycle into two parts based on demand uncertainty and therefore each stage can consist of several periods. In our model, we assume that the first stage consists of the first period while the second stage consists of the remaining periods.

Let,

Indices

t time, 1…T

s	scenarios, 1…S

c	capacity options available, 1…C

Parameters

D_t^s demand in period t under scenario s as a function of price and adv (equation 6)

α production cost per unit

h_t inventory holding cost per unit in period t

w_t backorder cost per unit in period t. For the final period, it denotes lost sales cost

F_c plant size if the capacity option c is selected

I_c capacity investment cost if the capacity option c is selected

β discounting factor

ϕ_s probability of occurrence of scenario s

Variables

First stage variables

Y_c binary variables would take value 1 when the capacity of size c is selected

X_1 production quantity in stage 1 (period 1)

P_1 price in stage 1

A_1 advertising in stage 1

i_1^s inventory at the end of stage 1(period 1) in scenario s

b_1^s backorder at the end of stage 1(period 1) in scenario s

Second stage variables

P_2^s price in stage 2 under scenario s

A_2^s advertising in stage 2 under scenario s

X_t^s production quantity for the period t (in stage 2) under scenario s

i_t^s inventory at the end of the period t (in stage 2) under scenario s

b_t^s backorder at the end of the period t (in stage 2) under scenario s

Maximize

$$\begin{aligned}&\sum_{s=1}^{S}\phi_s(P_1D_1^s)-A_1-\alpha X_1-\sum_{c=1}^{C}I_cY_c-\sum_{s=1}^{S}\phi_s(h_1 i_1^s+w_1b_1^s) \quad \{\mathit{first\ stage}\}\\ &+\sum_{s=1}^{S}\sum_{t=2}^{T}\phi_s\beta^{t-1}(P_2^sD_t^s-A_2^s-\alpha X_t^s-h_ti_t^s-w_tb_t^s) \quad \{\mathit{second\ stage}\}\end{aligned} \quad \mathbf{(8)}$$

Subject to

First stage constraints

$$X_1 \le \sum_{c=1}^{C}F_cI_c \quad (9)$$

$$\sum_{c=1}^{C}I_c=1 \quad (10)$$

$$X_1-i_1^s+b_1^s=D_1^s \qquad s=1,\dots,S \quad (11)$$

$$X_1,P_1,A_1,i_1^s,b_1^s\ge 0 \quad \mathbf{(12)}$$

Second Stage Constraints

$$X_t^s \le \sum_{c=1}^{C}F_cI_c \qquad t=2,\dots,T \;\; s=1,\dots,S \quad (13)$$

$$X_t^s+i_{t-1}^s-b_{t-1}^s-i_t^s+b_t^s=D_t^S \qquad t=2,\dots,T \;\; s=1,\dots,S \quad (14)$$

$$P_2^s,A_2^s,X_t^s,i_t^s,b_t^s\ge 0 \qquad \forall t=1,\dots,T, s=1,\dots,S \quad \mathbf{(15)}$$

The objective function (eq. 8) maximizes the discounted expected profits by subtracting investment costs, expected production costs, expected inventory and backorder costs from expected revenues. Constraints (eq. 9 and 13) keep production in each period below the manufacturing capacity under all scenarios. Constraints (eq. 11 and 14) are inventory and backlog equations for each period under each scenario. We assume that the demand is completely backlogged until the last period where it results in the lost sales due to insufficient capacity. So, backlog cost per unit in the period T is the lost sales cost per unit and inventory holding cost per unit in the period T is the salvage value per unit. We also do not take into account the learning effects.

Modeling Extensions

Similar to Shah (2004), one can add the sourcing decisions in our model by including the sourcing constraints and the sourcing costs in the objective function for each scenario in both stages. Similar to Cantamessa and Valentini (2000), we can also incorporate the lost sales and learning effects in each period for each scenario in our model and relax the above assumption. These extensions would however increase the complexity of the model. Our main objective to limit the model complexity is to effectively bring out the insights regarding marketing decisions under the supply constraints.

Both the revenue function in the objective function (eq. 8) and the constraints (eq. 11 and 13) are non linear in nature. Also, the model contains integer variables such as Y_c. Non linear stochastic programming with recourse with integer constraints is an extremely difficult problem to solve. It is also impossible to analytically prove the optimality and compute the closed form expressions for the optimal decision variables. Meta-heuristics methods such as tabu search, genetic algorithms can be used to find a good solution. But, it is not possible to prove the optimality of the best solution found in the search process. Hence, in the next section, we suggest an innovative modeling approach to convert this problem to a mixed integer stochastic programming model that can be solved to optimality in a reasonable amount of time.

2.2 *Mixed Integer Stochastic Programming Model with Recourse*

Instead of treating pricing and advertising variables as continuous, we suggest the firm should identify a discrete feasible set of both pricing and advertising options. This modeling approach has several advantages. First, it allows us to convert a non linear model into a linear model as discussed below. Second, the optimal solution sometimes suggests the use of extravagant prices when non linear demand functions are used (Welam 1977). This happens because the demand remains positive even at the arbitrary high prices, which subsequently results in infinite profits. When the firm uses the feasible set, this problem is automatically resolved. Third, it allows us to incorporate the valuable judgment and experience of a marketing manager in the optimization model. Fourth, in practice, the firm often uses innovative pricing ideas such as psychological pricing, which can result in discrete pricing (Holdershaw et al. 1997). Our approach allows us to include the outcomes of such strategies in the model.

Let, θ denote the feasible set of the pricing policies while ϖ denote the feasible set of advertising options available to the firm. We define a marketing strategy as combination of pricing and advertising options. We define u $=\theta \times \varpi$ and w$=\theta \times \varpi$ as the marketing strategies that are available in stage I and stage II respectively. Let, $Z1_u$ be a binary variable that takes value 1 when the marketing strategy u is selected in stage 1 while $Z2^s_w$ denotes the binary variable that takes value 1 when the marketing strategy w is selected in stage 2 under scenario s. Let, Z^s_{uw} be a binary variable that takes value 1 when the strategy u is selected in stage 1 and the strategy w is selected in stage 2. Because these binary variables together completely define both the diffusion path and price and advertising in each period under each scenario s, we can write both revenue and demand (equations 7 and 6) for each period under each scenario as parameters. This allows us to convert the non linear terms in the previous formulation (equations 8, 11, 13) into linear equations.

Let, R^s_{uw} be the revenue generated when the firm selects the strategies u and w in the first and second stages respectively under scenario s. D^s_u be the demand in period 1 under scenario s when the firm selects the strategy u. D^s_{uwt} be the demand in the period t (stage 2) when the firm selects strategies u and w respectively. We now write the mixed integer stochastic program:

Maximize

$$\begin{aligned}&\sum_{s=1}^{S}\sum_{u=1}^{U}\sum_{w=1}^{W} \phi_s(R^s_{uw}Z^s_{uw}) - \alpha X_1 - \sum_{c=1}^{C} I_c Y_c - \sum_{s=1}^{S}\phi_s(h_1 i^s_1 + w_1 b^s_1)\{first\ stage\} \\ &-\sum_{s=1}^{S}\sum_{t=2}^{T}\phi_s\beta^{t-1}\left(\alpha X^s_t + h_t i^s_t + w_t b^s_t\right) \qquad \{second\ stage\}\end{aligned} \tag{16}$$

Subject to

First stage constraints

$$X_1 \le \sum_{c=1}^{C} F_c I_c \quad (17)$$

$$\sum_{c=1}^{C} I_c = 1 \quad (18)$$

$$X_1 - i_1^s + b_1^s = \sum_{u=1}^{U} Y1_u D_u^s \qquad s = 1,\dots,S \quad (19)$$

$$\sum_{u=1}^{U} Z1_u = 1 \quad (20)$$

$$X_1 , i_1^s , b_1^s \ge 0 \quad \textbf{(21)}$$

Second Stage Constraints

$$X_t^s \le \sum_{c=1}^{C} F_c I_c \qquad t = 2,\dots,T \;\; s = 1,\dots,S \quad (22)$$

$$X_t^s + i_{t-1}^s - b_{t-1}^s - i_t^s + b_t^s = \sum_{u=1}^{U}\sum_{w=1}^{W} Z_{uw}^s D_{tuw}^S \qquad t = 2,\dots,T \;\; s = 1,\dots,S \quad (23)$$

$$\sum_{w=1}^{W} Z2_w^s = 1 \qquad s = 1,\dots,S \quad (24)$$

$$Z1_u + Z2_w^s - 2 Z_{uw}^s \le 1 \qquad s = 1,\dots,S \;\; u = 1,\dots,U \;\; w = 1,\dots W \quad (25)$$

$$\sum_{u=1}^{U} \sum_{w=1}^{W} Z_{uw}^s = 1 \qquad s = 1,\dots,S \quad (26)$$

$$X_t^s , i_t^s , b_t^s \ge 0 \qquad \forall t = 1,\dots,T, s = 1,\dots,S \quad \textbf{(27)}$$

Constraint (20) implies that a single marketing strategy is selected in stage 1. Constraint (24) ensures that under each scenario, the firm selects one marketing strategy in stage 2.

Constraints (25 and 26) together imply that the firm selects a single demand stream over the product life cycle for each scenario.

3. Numerical Study

The objective of this study is to show how the optimal capacity, pricing and marketing decisions can vary with the uncertainty in the diffusion process, operations and market characteristics. In this numerical study, we consider a firm that wants to determine the joint capacity, production and marketing plans for a new product with a life cycle of around 1.5 years.

3.1 Experimental Set Up

In the base experiment, we solved the SMIP as an expected value problem where the random parameters were replaced with their expected values that resulted in a deterministic mixed integer program. In the base experiment, though capacity decision was included as a variable, we excluded the pricing and advertising decisions and so used passive strategies such as backorders and lost sales to manage the demand growth. Our base experiment was similar to Cantamessa and Valentini (2000) so that we could show the benefits of including the stochastic behavior and marketing decisions in the new product planning models. Demand, revenue and production data were prepared using a spreadsheet while the mathematical programs were modeled using AMPL and were subsequently solved using CPLEX.

In the first experimental setup, we assumed that there is no discounting while in the second set up, we assumed discounting effects. In all experimental set ups, we kept parameters, inventory cost per unit per unit time, backorder cost per unit per unit time, discounting at high and low levels. For inventory and backorder costs, we used \$.25 and \$1 values while for discounting, 1 and 0.6 values were used as low and high levels.

We assumed six scenarios for the diffusion process which are shown in Table 1. We refer to the first three (last three) scenarios as imitative (innovative) because their p/q ratio is lower (higher) respectively. For scenario distribution, we used three situations: in a symmetric case, we used the probability distribution (0.166, 0.166, 0.166, 0.166, 0.166, 0.166), in a slight asymmetric case, we used (0.1, 0.3, 0.1, 0.1, 0.3, 0.1) while for a complete asymmetric case, we used (0.025, 0.45, 0.025, 0.025, 0.45, 0.025).

The production cost per unit is \$4. We assumed that all the unfulfilled orders are backlogged. At the end of the life cycle, the firm would incur a lost sales cost of \$ 7 while the salvage value of the inventory per unit was \$ 3. The inventory holding cost per unit per unit time was set at \$0.5. The capacity expansion decisions were modeled by assuming that four levels of capacities were available. The capacity size and investment costs vectors could be represented as (650, 850, 1050, 1250) and (6500, 8500, 10500, 12500) respectively. The price elasticity of demand was -2 while the advertising effectiveness was 0.001. Based on the firm's prior experience and judgment, the firm has suggested the following three pricing options (9, 10, and 11) and two advertising options (0, 50). We have decided to work with a unit period of three months that resulted in six periods during the product life cycle. We however found that the influence of inventory holding rate was insignificant. The other parameters are kept at a constant level as we believed that their influence of the optimal policies would be insignificant. The parameter values were somewhat extreme and had been chosen to bring out the characteristics of the optimal policies with greater evidence.

Parameters	S1	S2	S3	S4	S5	S6
m	5000	4000	6000	5000	4000	6000
p	0.005	0.005	0.005	0.025	0.025	0.025
q	0.45	0.45	0.45	0.25	0.25	0.25

Table 1. Diffusion process scenarios used in the study

3.2 Results and Discussion

3.2.1 Base Case

In the base experiments, we exclude the marketing variables and hence the firm uses increased backorder, inventory build up and lost sales options to manage the demand. From an operations perspective, these strategies are intuitive and are consistent with the prior research. Also, because these experiments do not take into account the demand uncertainty, the firm is not able to handle the extreme situations effectively. As a result, the profits erode considerably in the base case. We quantify the benefits of using our model as $\frac{\pi_e - \pi_b}{\pi_b}$ where π_e denote profits under experiments (1, 2) and π_b denote the profits under the base case, which are shown in Figure 4. The Figure 4 suggests that the magnitude of the benefits increase as per unit backorder costs and discounting increase and also when scenario distribution becomes more symmetric. In the subsequent sections, we discuss the reasons behind these improvements. As the Figure 4 shows, there was not much difference between slight and complete asymmetric results and hence have excluded them in the results (Tables 2 and 3) for parsimony.

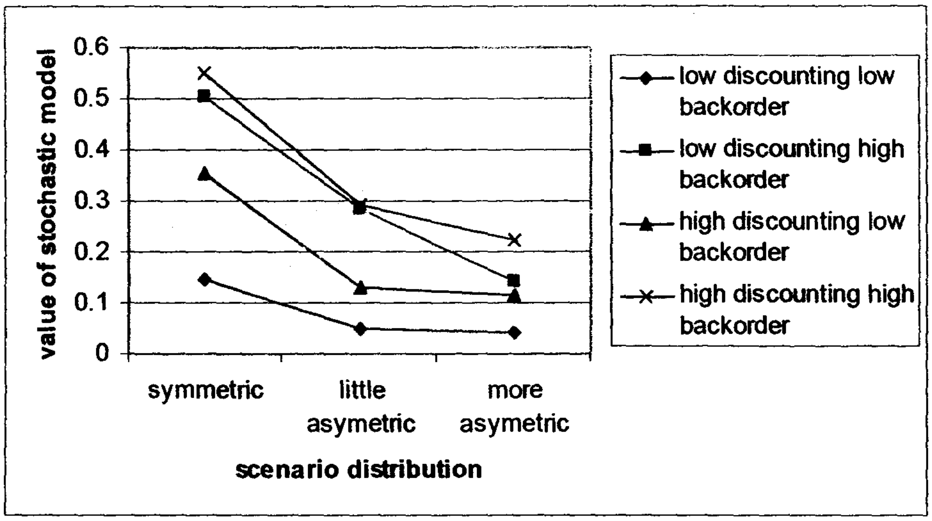

Figure 4. Increase in profits as a function of our model

3.2.3 Experiment 1

Table 2 reports the results of the experiment 1. The firm selects a higher capacity level under the symmetric case than the asymmetric case. This happens because under asymmetric case, the firm attempts to reduce the "excess" capacity expansion costs as it anticipates that the lower market demand situation is more likely.

Behavior of inventory and backordering strategies is similar to the prior research where the inventory is gradually increased during the initial period which is subsequently depleted during the peak demand while backorders occur mainly during and for some time after the peak demand. This happens because of the property of the diffusion process where the demand is below the capacity level initially which exceeds the capacity during the peak time and finally decreases during the decline phase. Hence, we do not report these results in detail (in Tables 2 and 3). The important difference is that these variables are strategically adjusted with our model by shaping the demand process.

In the first stage, the firm selects base price (10) to hedge against the uncertainty in the diffusion process. Advertising decision for this stage however depends upon the per unit backorder cost. When the backorder cost is low, then the firm uses advertising to accelerate the diffusion process to minimize the inventory holding costs in the initial periods. However, as the backorder costs increase, the firm aims to build inventory in the early stages that can be used to meet the demand during the peak demand and hence does not accelerate the product growth by increasing the advertising expenditure in this situation.

Scenario	Backorder	First stage Decisions			Second Stage Recourse Decisions						
		capacity	price	Adv		S1	S2	S3	S4	S5	S6
Symmetric	Low	1050	10	50	price	10	9	10	10	9	11
					Adv	0	0	0	0	0	0
	High	1050	10	0	price	11	10	11	11	10	11
					Adv	0	0	0	0	0	0
Asymmetric	Low	850	10	50	price	11	10	11	11	10	11
					Adv	0	0	0	0	0	0
	High	850	10	0	price	11	11	11	11	11	11
					Adv	0	0	0	0	0	0

Table 2. First and second stage decisions for experiments 1

We observe that in the second stage, the firm has chosen a lower advertising level. When there is no discounting, it is not beneficial for the firm to encourage customers to purchase the product early from a revenue perspective. As discussed earlier, higher advertising tends to reduce the inventory holding costs if the inventory increases in the absence of advertising. However, in the second stage, because the inventory does not increase much, there is no incentive for the firm to increase advertising. As a result, the firm prefers zero advertising option.

The optimal pricing in the second stage depends upon both scenario and backorder costs. In general, when the demand turns out to be high (low), then the firm tends to increase (decrease) prices to match supply with demand. This behavior is similar to revenue management notion that charges higher prices when demand exceeds the supply. This is consistent with Apple Inc. decision to cut iPhone price after observing low demand. Also, when the backorder costs are high, then the firm chooses higher prices to decelerate the demand growth in order to reduce the backorder costs.

3.2.4 Experiment 2

Table 3 shows the results of the experiment 2. When the discounting is high, there is an incentive for the firm to advertise in the first stage because it results in demand acceleration which subsequently results in the increased discounted revenues. Hence, the firm selects a higher advertising level in the first stage. Similar to set up 1, the firm chooses the base price to hedge against the uncertainty in the demand process.

Scenario	Backorder	First stage Decisions			Second Stage Recourse Decisions						
		capacity	price	Adv		S1	S2	S3	S4	S5	S6
Symmetric	Low	1050	10	50	price	9	9	10	10	9	11
					Adv	50	50	50	0	0	0
	High	1050	10	50	price	9	9	10	10	9	11
					Adv	50	50	50	0	0	0
Asymmetric	Low	850	10	50	price	10	9	11	10	9	11
					Adv	50	50	50	0	0	0
	High	850	10	50	price	10	9	11	11	10	11
					Adv	50	50	50	0	0	0

Table 3. First and second stage decisions for experiments 2

Second stage advertising decision depends upon whether the diffusion curve exhibits the innovative or imitative behavior. When the process follows the imitative behavior, then the firm is able to build inventory in the first stage that is available to meet the increased demand resulting from higher advertising in the second stage. As a result, the firm is able to increase the discounted revenue by spending more on advertising. Figure 2 showed the demand growth as a function of advertising in the imitative case in both stages.

On the other hand, when the diffusion pattern is innovative, the firm is not able to build enough inventories in the first stage as the demand for the product is high. In this situation, when the firm increases advertising, then it only results in increased backorders as the firm does not have enough supply to meet the demand. Consequently, the firm decides not to advertise when the process follows the innovative behavior. This finding can be supported by Nintendo's decision to stop advertising for its wii console due to insufficient supply (Marketing Week 2007).

Similarly, when the process is imitative, the firm accelerates the diffusion process by reducing prices so as to increase the discounted revenue. In contrast, the firm decides not to decrease them in the innovative case because there are not enough inventories to meet the increased demand. Figure 1 showed the diffusion process dynamics as a function of price in both stages. It can be seen that both advertising and pricing decisions in the second stage behave in the same fashion in the high uncertainty situation.

4 Conclusions and Future Research

In this paper, we provided an integrated framework for linking marketing and supply chain decisions for a short life cycle product. We suggested the use of a two stage stochastic programming with recourse to model the problem. Original model that included price elasticity and logarithmic advertising function resulted in a stochastic non linear model with integer constraints which was extremely difficult to solve. So, we used an innovative approach to convert it to a linear mixed integer stochastic program.

Our computational experiments demonstrated the importance of including the stochastic diffusion process and marketing decisions in the production planning model of the new product. Optimal advertising and pricing decisions depended upon the operations characteristics. For example, when the discounting was low, then in the first stage, the optimal advertising was low (high) when backorder costs were high (low). Also, the firm increased the price as the backorder costs increased.

As the discounting factor increased, the firm used the option of high advertising and low pricing when the diffusion process followed the imitative path. However, for the innovative diffusion process, the firm decided not to advertise and mainly chose the pricing policies that did not accelerate the product growth. These contrasting behaviors were the consequences of the supply constraints. Thus, our results demonstrated the importance of including the supply constraints while deciding optimal pricing and advertising strategies.

Our research could be extended in several directions. Because the objective of the study was to propose an enhanced model and show the benefits of integrating marketing and supply chain decisions, we have not focused on improving computational speed. Stochastic mixed integer programs of moderate size can be solved to optimality in a reasonable amount of time and also because of the strategic nature of the problem, we believe that computational time is not a major issue. Still, it may be worthwhile to develop a computational algorithm that solves our model efficiently. We consistently found that advertising depends upon operations costs and constraints. An analytical model can be developed to prove the optimality of some of our findings and draw further insights on advertising under supply constraints.

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