

Value of Information in a Serial Supply Chain under a Nonstationary Demand Process

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In this paper, we consider a two level supply chain with a manufacturer supplying the product to a retailer. The retailer faces nonstationary demand that follows an ARIMA(0,1,1) process. In the absence of information sharing, the retailer only conveys his order quantity to the manufacturer, and with information sharing, he also conveys the demand level in each period. The model based on nonstationary demand process is significant; given the empirical findings that the demand processes observed in industries such as FMCG, grocery, apparel, and auto components are autocorrelated and nonstationary. In our model, we consider a periodic review system for both retailer and manufacturer and derive the expressions for the optimal order upto level for the players. We also obtain the expressions for the cost functions of the two players and conduct a numerical study to evaluate the value of information sharing. We find that both the retailer's ordering process and the manufacturer's demand process are independent of the inertia of the external demand process, and they depend only on the delivery lead time of the retailer from the manufacturer. We observe that both variance of the manufacturer's demand process and his cost decrease due to information sharing. However, the savings in the manufacturer's cost with information sharing decrease as the external demand process becomes less stable or more transitory, and they increase as the demand uncertainty increases. Lastly, we note that the cost reduction is more by reducing the manufacturer's rather than the retailer's lead time.

Key words: supply chain management; information sharing; nonstationary demand process; inventory model

1. Introduction

A supply chain is efficient only when supply matches demand as closely as possible with minimum cost. In this regard, the importance of information sharing among the supply chain members has been widely acknowledged by both practitioners and academia. In practice, it is observed that number firms have made significant investments in EDI (Electronic Data Interchange), by which information about end customer demand is shared quickly among the supply chain members (see, e.g., Srinivasan et al. 1994, Bourland et al. 1996). Also, due to EDI there is very little delay in

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transmitting orders to upstream members. Manufacturers and retail chains in organized retail have joined hands to implement CPFR (Collaborative Planning Forecasting and Replenishment) (see, e.g, Stank et al. 1999). In this case, managers of both firms form a team and jointly develop forecasts of end customer demand for different SKUs. This way there is a common agreement on what demand to expect, and then the manufacturers and suppliers organize their production plans to meet the demand. The firms make such significant investments in information sharing systems in order to lower the cost of mismatch between supply and demand.

In the supply chain management literature, the topic of benefits and modes of information sharing in supply chains has been well studied (e.g., Cachon and M. 2000, Lee and S. 2000, Chen 2003, etc.). Information sharing typically improves visibility of not only the supply chain inventory but also demand from the end consumers. The increased visibility of inventory and consumer demand for the players reduces supply chain inventory costs by enabling the players in developing improved and efficient operations plans at each level in the supply chain (see Lee et al. 1997a, 1997b). For example, for the products in industries such as FMCG, grocery, apparel, and auto components that are offered in large number of SKUs, demand forecasts are typically based on past demand/sales, and hence, accuracy of these forecasts is always doubtful. Moreover, factors such as seasonality and marketing efforts by the retailer also demonstrate significant influence on the demand levels for these products. In this case, by sharing the inventory and demand related information regularly with the manufacturer, the retailer can facilitate the manufacturer in improving its production and inventory planning that can also be efficient from the supply chain's perspective. Information sharing essentially helps retailer and supplier both in closely aligning supply with demand by positioning the right SKU at the right point in the supply chain in the right amount, and more so inventory management benefits from matching supply with demand by information sharing among the players (see Cachon and C. 2005). However, it is commonly observed that retailers in particular hesitate to share demand related information with their suppliers. Therefore, highlighting the benefits for the retailer from sharing demand information with the supplier is important.

In most of the industries, demand forecasting is a complex job. In particular, if the demand process for a product is evolving over time and demand across different time periods is correlated, then accuracy of the forecasts developed can be easily questioned. Furthermore, irrespective of whether products are functional or innovative, product life cycles are getting shorter resulting into increased unpredictability of these demand processes. In spite of these known limitations, practitioners typically rely on forecasts based on time series of prior demand in their inventory planning. Inventory planning for matching supply with demand becomes further challenging if

the underlined demand process is nonstationary (see Lewis 1998). In this case, understanding the implications of nonstationarity of the demand process for inventory planning by the supply chain members is essential.

In addition to demand forecasting and inventory management, logistics is also an important aspect of matching supply with demand. At the tactical level, each supplier decides which products to be shipped to which retailer and how soon. At the operational level, the supplier also decides the quantity and timing of various products to be shipped to the retailers so that supply meets demand. Such planning requires due consideration for constraints on transportation modes available (such as number and capacity of trucks available for delivery) as well as constraints due to delivery time windows desired. In this regard, delivery lead time is a critical factor in controlling the supply chain costs, and sharing of demand related information can result in increased profits for the supply chain and the individual players (see, e.g., Gavirneni et al. 1999). These delivery lead time related concerns are identical for both supplier and retailer. Therefore, from the supply chain perspective, analyzing the impact of delivery lead time on the supply chain costs is necessary.

To counter the bullwhip effect arising out of information asymmetry among the supply chain members, sharing demand/sales information has been viewed as a major strategy (see Lee et al. 1997a). In this case, quantifying the value of demand information is also essential from the perspective of the cost of adopting information sharing systems such as EDI, CPFR. In their seminal paper on the value of information sharing, Lee et al. (1997b) provide a model for the most basic time-series demand process (AR(1)) to calculate the value of demand information. In this paper, we consider one such nonstationary demand process that is more general in nature and develop an analytical model to measure the value of information sharing in the supply chain and to analyze the impact of nonstationarity of the external demand process and the supply chain delivery lead times on the supply chain cost.

In the supply chain management literature, number of studies exist that evaluate the value of information sharing in the supply chain. For instance, Gavirneni et al. (1999) study the capacitated setting in a typical supply chain with partially and completely shared information related to inventory policy between a supplier and a retailer. They estimate the savings for the supplier due to information sharing and identify the conditions under which information sharing is more valuable. Cachon and M. (2000) investigate the value of sharing demand and inventory data between one supplier and multiple retailers to find that substantial savings can be achieved by reducing delivery lead time and batch size. Similarly, Yao et al. (2005) estimate the value of information sharing in the case of a supply chain consisting of a mix of traditional retail channel and a direct channel when

the manufacturer accepts product returns. Bourland et al. (1996) examine the changes brought about by the exchange of timely demand information in inventories and service levels at both supplier and customer. They show that inventory related benefits are particularly sensitive to demand variability, the service level provided by the supplier, and the degree to which the order and production cycles are out of phase. Yu et al. (2002) quantify the benefits of information sharing based supply chain partnerships by deriving the optimal inventory policies for the manufacturer and the retailer in a two-level decentralized supply chain under different information sharing scenarios. In their paper, Yee (2005) investigate the impact of information sharing of the demand mix on the supply chain performance, under increasing product customization, by changing customer demand pattern and production capacity. Using a simulation model, they demonstrate that a demand mix can be determined that produces the best supply chain performance. However, time series demand models are commonly adopted in this field.

The models of based on time-series demand processes gain importance in the supply chain management literature, given that Erkip et al. (1990) and Lee et al. (1997a) have empirically found that demands of consumer products are autocorrelated over time. Lee et al. (2000), have used panel data to examine the weekly sales pattern of 165 SKUs at a supermarket over a two year period. They found that 150 out of 165 SKUs analyzed demonstrate statistically significant autocorrelated demand processes. The authors suggest that the reason for this autocorrelated demand in more than 90% of SKUs analyzed is the repeat purchase behavior of consumers. Moreover, most of the autocorrelated demand processes are observed to be nonstationary (see Box et al. 1994, Lewis 1998). In this regard, motivated by Lee et al. (1997b), researchers started explicitly modeling time-series demand processes in their studies (see, e.g., Lee et al. 2000, Raghunathan 2001, Graves 1999, Gaur et al. 2005, etc.).

Raghunathan (2001) show, analytically and through simulation, that the manufacturer's benefit is significant when the parameters of the AR(1) process are known to both supplier and retailer, as in Lee et al. (1997b). Moreover, Raghunathan in particular shows that the manufacturer can reduce the variance of its forecast by using the entire order history to which it has access. Gaur et al. (2005) consider a two-stage supply chain model in which a retailer serves an ARMA demand and show how the time-series structure of the demand process affects the value of information sharing in a supply chain. Graves (1999) examines multistage stage supply chains in which the external demand process is assumed to be ARIMA. He shows that the order at all upstream processes are also ARIMA processes. Gilbert (2005) extends the results of Graves to show that the orders and inventories at each stage are ARIMA. He also shows that sharing point-of-sales information may

be redundant, and hence, may not mitigate the bullwhip effect. However, though they provide managerial insights, Graves and Gilbert do not estimate the value of information sharing in their model.

Our modeling approach parallels that of Lee et al. (1997b). In this paper, we consider a two level supply chain with a single manufacturer supplying a single retailer. The retailer faces nonstationary demand that follows an ARIMA(0,1,1) process. Without information sharing in the supply chain, the retailer only conveys his order quantity to the manufacturer. However, with information sharing, in addition to conveying his order quantity the retailer also conveys his demand realization in each period. We consider a periodic review system for both retailer and manufacturer and derive expressions for the retailer's and the manufacturer's demand over their respective lead times. We also derive expressions for the manufacturer's optimal order upto level with and without information sharing. Finally, we calculate the retailer's cost and the manufacturer's cost with and without information sharing. In this case, the value of information is equal to the reduction in the supply chain cost due to information sharing. We next conduct a numerical study to determine the value of information sharing. We find that the reduction in the manufacturer's cost with information sharing decreases, as the demand becomes less stable or more transitory over time. Also the reduction in the manufacturer's cost with information sharing increases as the uncertainty of demand increases. Lastly, we find that in order to reduce the supply chain cost, the players should focus more on reducing the manufacturer's lead time from the external supplier rather than the retailer's lead time from the manufacturer. In this case, we also show that the retailer's ordering process, and hence, the manufacturer's demand process are independent of the inertia of the external demand process and they depends only on the delivery lead time of the retailer from the manufacturer.

The remainder of the paper is organized as follows. In Section 2 we develop and analyze the model. In particular, we first derive the retailer's ordering process and later develop the manufacturer's ordering process. For the latter, we consider two separate cases: with and without information sharing. In Section 3 we provide the closed-form expressions for the relevant cost functions. Further in Section 4, we perform a numerical study using a representative data set. We mention the key findings in Section 5 and conclude the paper.

2. The Model

Consider a serial supply chain with one manufacturer and one retailer. The external demand process for a single item occurs at the retailer and the underlying demand process is assumed to be nonstationary. In particular, the demand process considered is autoregressive integrated moving

average (ARIMA) of order (0,1,1). This process is discussed in detail in Box et al. (1994). One of the two reasons of using this particular model of nonstationary demand process is that it has been used previously by Graves (1999) for a single item inventory model and by Gilbert (2005) in a multi stage supply chain. Second, many of the specific models (e.g., AR model, ARMA model, etc.) that have been used in the existing studies on value of information sharing are special cases of ARIMA model (see Lee et al. 2000, Raghunathan 2001, Gaur et al. 2005). Moreover, the ARIMA model has also been extensively used to explain the demand processes of many of the industrial products such as plastic containers (Montgomery and L.A. 1976), semiconductor industry (Niu et al. 2007), packaging (Gijo 2011), etc. This demand process can be represented as shown below:

$$D_1 = \mu + \epsilon_1 \quad (1)$$

$$D_t = D_{t-1} - (1 - \alpha)\epsilon_{t-1} + \epsilon_t; \quad t = 2, 3, \dots \quad (2)$$

Here D_t is the observed demand in period t , and $\mu \geq 0$ and $0 \leq \alpha \leq 1$ are the known parameters. Also, $\{\epsilon_t\}$ is a time series of i.i.d. random variables that we assume to be the normally-distributed random noise with $E[\epsilon_t] = 0$ and $Var[\epsilon_t] = \sigma^2$. We assume that $\left[(1 - \alpha)^2 + 1\right]\sigma^2$ is significantly smaller than μ so that the probability of a negative demand in any period is negligible.

By expanding (2) we obtain the following:

$$D_t = \epsilon_t + \alpha\epsilon_{t-1} + \alpha\epsilon_{t-2} + \dots + \alpha\epsilon_1 + \mu \quad (3)$$

According to Muth (1960), the shock associated with the current time period has a weight of unity. The weight of the shock in successive time periods is constant and somewhere between 0 and 1. In this case, according to Graves (1999), α can be viewed as a measure of inertia in the process; larger value of α results in a less stable or more transitory demand process.

In this paper, we consider a periodic review system in which both retailer and manufacturer review their inventory levels and in every period they replenish their inventory from the upstream member in the supply chain. In our model, we assume that the replenishment lead times from the manufacturer to the retailer and the external supplier to the manufacturer are in constant periods l and L , respectively. (In this paper, parameters represented in upper case and lower case are designated for the manufacturer and the retailer, respectively.)

What follows is the description of the ordering process of the retailer and the manufacturer. First, the retailer observes demand D_t in period t , $t = 1, 2, 3, \dots$, and before the end of period t , he reviews the inventory level. The retailer places an order of size Y_t with the manufacturer and this order is received at the beginning of time period $t + l + 1$. Similarly, at the end of time period t ,

the manufacturer receives and ships the retailer's order quantity Y_t and places an order with his supplier. For both retailer and manufacturer, we assume that excess demand is backlogged.

We assume that no fixed order cost is incurred by any of the players in the supply chain. Let h and p be the retailer's unit holding and shortage cost, respectively, for each time period. Similarly, let H and P be the manufacturer's unit holding and shortage cost for each time period. These cost parameters are assumed to be constant over time. We also assume that the retailer and the manufacturer adopt the order-up-to level policy. This policy minimizes the total (discounted) holding and shortage cost over the infinite horizon (see Heyman and M. 1984, Nahmias 1993).

Our approach for evaluating the value of information sharing in the serial supply chain parallels that of Lee et al. (2000). For the given external demand process we evaluate the retailer's ordering decision. The retailer's order is the manufacturer's demand. We then analyze the manufacturer's ordering process over the lead time L . We analyze the manufacturer's ordering process for two cases: (i) no information sharing: the retailer only conveys his order to the manufacturer, and (ii) with information sharing: in addition to conveying his order, the retailer also conveys the observed demand level to the manufacturer in each period. We compare the manufacturer's order quantity and the related cost for the two cases and evaluate the value of information sharing.

2.1. Retailer's Ordering Process

First, we consider the retailer's ordering process. Let S_t , $t = 1, 2, 3, \dots$ be the retailer's order-up-to level, and at the end of period t the retailer orders Y_t from the manufacturer. Here,

$$Y_t = D_t + (S_t - S_{t-1}) \quad (4)$$

In each period, the retailer's order quantity is equal to the demand in that period and the change in the order-up-to level in relation to the preceding period. Under the assumption that $\left[(1 - \alpha)^2 + 1\right] \sigma^2$ is significantly smaller than μ , the probability of $Y_t < 0$, $\forall t$, is negligible (see Lee et al. 1997b).

When l is the replenishment lead time from the manufacturer to the retailer, the total demand over the lead time, denoted by $\sum_{i=1}^{l+1} D_{t+i}$, can be expressed as follows:

$$\sum_{i=1}^{l+1} D_{t+i} = (l+1) [D_t - (1 - \alpha) \epsilon_t] + \sum_{i=1}^{l+1} [1 + (l+1-i) \alpha] \epsilon_{t+i} \quad (5)$$

(In this paper, we assume the convention $\sum_a^b (\cdot) = 0$ if $a > b$. Also, $(\cdot)_t = 0$ if $t < 0$.)

Note that the demand over the lead time is a function of the demand realized in period t , i.e., D_t , and also the error in demand, i.e., ϵ_t . Define m_t and v_t as the conditional expectation and

variance, respectively, of the total demand over the lead time where, $m_t = E\left(\sum_{i=1}^{l+1} D_{t+i} | D_t\right)$ and $v_t = Var\left(\sum_{i=1}^{l+1} D_{t+i} | D_t\right)$. Therefore, from (5) we obtain

$$m_t = (l+1) D_t \quad (6)$$

$$v_t = v\sigma^2 \quad (7)$$

where,

$$v = (l+1)^2 (1-\alpha)^2 + \sum_{i=1}^{l+1} [1 + (l+1-i)\alpha]^2 \quad (8)$$

In this case, the retailer's optimal order-up-level, denoted by S_t^* , is given as:

$$S_t^* = m_t + k\sigma\sqrt{v} \quad (9)$$

where, $k = \Phi^{-1}[p/(p+h)]$ for the standard normal distribution function Φ . From (4) and (9), the retailer's order quantity can be given as:

$$Y_t = D_t + (l+1)(D_t - D_{t-1}) \quad (10)$$

From (10), it can be noted that the retailer's order process conditioned on the observed demand in each time period is independent of α , the measure of inertia in the demand process observed by the retailer. Note that the manufacturer's demand process is same as the retailer's ordering process, and hence, the manufacturer's demand process is also independent of the inertia of the external demand process. Moreover, both these processes depend only on the delivery lead time of the retailer from the manufacturer.

2.2. Manufacturer's Ordering Process

Now consider the manufacturer's ordering process. We assume that the manufacturer is aware of the fact that the retailer's demand process is ARIMA(0,1,1) and the parameters α and σ are known to the manufacturer.

Let T_t , $t = 1, 2, 3, \dots$ denote the manufacturer's order-up-to level. The manufacturer determines the order-up-to level by anticipating the total demand over the lead time L . Since, the manufacturer's demand process is the retailer's ordering process, the total demand over the lead time, which we denote by B_t , is equal to the total order placed by the retailer over the time period $t+1, t+2, \dots, t+L+1$. Hence, $B_t = \sum_{i=1}^{L+1} Y_{t+i}$. What follows is description of the conditional mean and variance of the manufacturer's ordering process, given the retailer's order quantity Y_t . From (2) and (10) the manufacturer can deduce that:

$$Y_{t+i} = Y_{t+i-1} + (l+2)\epsilon_{t+i} + [\alpha(l+2) - (l+1) - (l+2)]\epsilon_{t+i-1} + (l+1)(1-\alpha)\epsilon_{t+i-2};$$

$$\text{for } i = 1, 2, \dots \quad (11)$$

Moreover, conditioned on the fact that Y_t is known, we can show that

$$Y_{t+i} = Y_t + (l+2)\epsilon_{t+i} + [\alpha(l+2) - (l+1)]\epsilon_{t+i-1} + \alpha \sum_{j=2}^{i-1} \epsilon_{t+i-j} + (\alpha - l - 2)\epsilon_t + (l+1)(1-\alpha)\epsilon_{t-1} \\ \forall i \geq 2 \quad (12)$$

(For $i = 1$, refer (11).) Therefore, the manufacturer's demand over the lead time L for any given Y_t is as shown below:

$$B_t = \sum_{i=1}^{L+1} Y_{t+i} \\ = (L+1)Y_t + (l+2)\epsilon_{t+L+1} + \sum_{i=1}^L [1 + (l+1+i)\alpha]\epsilon_{t+L+1-i} \\ + [(L-1)(\alpha - l - 2) + \alpha(l+3) - (3l+5)]\epsilon_t + (L+1)(l+1)(1-\alpha)\epsilon_{t-1} \quad (13)$$

In order to find the optimal order-up-to level T_t , the manufacturer needs to find the distribution of B_t . We derive the distribution of B_t for the two cases mentioned earlier: (i) no information sharing: the retailer only conveys his order to the manufacturer, and (ii) with information sharing: in addition to conveying his order, the retailer also conveys the observed demand level to the manufacturer in each period.

2.2.1. No Information Sharing

When the retailer does not share the external demand information in each period with the manufacturer, the manufacturer receives only information about the retailer's order quantity Y_t . In this case, the error terms ϵ_{t-1} and ϵ_t have already been observed by the retailer but are unknown to the manufacturer. Therefore, from (13), the manufacturer infers that his demand process is normally distributed with mean M_t and variance $V\sigma^2$, where

$$M_t = (L+1)Y_t \quad (14)$$

$$V = (l+2)^2 + \sum_{i=1}^L [1 + (l+1+i)\alpha]^2 + [(L-1)(\alpha - l - 2) + \alpha(l+3) - (3l+5)]^2 \\ + [(L+1)(l+1)(1-\alpha)]^2 \quad (15)$$

It can be observed that the variance of the the manufacturer's demand process is time invariant and it is increasing in L . In this case, the manufacturer's optimal order-up-to level, denoted by T_t^* , is given by:

$$T_t^* = M_t + K\sigma\sqrt{V} \quad (16)$$

where $K = \Phi^{-1}[P/(P+H)]$ for the standard normal distribution function Φ .

2.2.2. Information Sharing

When the retailer shares the external demand information in each period with the manufacturer, the manufacturer now knows ϵ_{t-1} and ϵ_t in addition to the retailer's order quantity Y_t . Therefore, from (13), the manufacturer infers that his demand process is normally distributed with mean M'_t and variance $V'\sigma^2$, where

$$M'_t = M_t + [(L-1)(\alpha-l-2) + \alpha(l+3) - (3l+5)]\epsilon_t + (L+1)(l+1)(1-\alpha)\epsilon_{t-1} \quad (17)$$

$$V' = (l+2)^2 + \sum_{i=1}^L [1 + (l+1+i)\alpha]^2 \quad (18)$$

It can be observed that V' is time invariant and it is increasing in l , L and α . Moreover, we also note that $V' \leq V$. This implies that information sharing reduces the variance of the total shipment quantity over the manufacturer's lead time and the demand process faced by the manufacturer. In this case, the manufacturer's optimal order-up-to level, denoted by T'_t^* , is given by:

$$T'_t^* = M'_t + K\sigma\sqrt{V'} \quad (19)$$

From (13), it can be noted that the random variable B_t is a linear function of ϵ_k for $k = t-1, t, t+1, \dots, t+L+1$. From the ARIMA(0,1,1) demand process, we know that ϵ_k determines the demand realization for period k . In the case of no information sharing, the demand realization in period t , i.e., ϵ_t , is not known to the manufacturer. Hence, the manufacturer calculates $M_t = E(B_t)$ and $V = Var(B_t)$. However, when the retailer shares the demand realization with the manufacturer, the manufacturer knows the realization ϵ_t in each period and correctly determines the mean and variance of the distribution of B_t . That is the manufacturer determines the conditional mean $M'_t = E(B_t|\epsilon_t, \epsilon_{t-1})$ and variance using $V' = Var(B_t|\epsilon_t, \epsilon_{t-1})$.

The random variable B_t , representing the total retailer order over the lead time l , always has the same normal distribution (with parameters M'_t and V'). In the case of no information sharing, the manufacturer does not know the correct normal distribution for B_t , and hence, assumes the parameters M_t and V . The manufacturer's optimal decision for the order-up-to level is based on incorrect parameters of the normal distribution of the random variable B_t . However, in the case of information sharing, the manufacturer's optimal decision is based on correct parameters of the normal distribution, and hence, it would result in lower cost. The optimal decision in the case of no information sharing is based on the wrong distribution and would result in higher cost.

In the following sections we show the above described implications of information sharing on the manufacturer's cost function and the variance of the demand process faced by the manufacturer.

3. Cost Functions

In this section, we show how to determine the manufacturer's holding and shortage cost, both in the case of no information sharing and information sharing. We also determine the retailer's holding and shortage cost. The analytical formulae in this section are based on the results of Lee et al. (2000), and the interested reader is advised to refer to Lee et al. (2000) for more details on the derivation of these cost functions.

Let $X_t \sim N(M'_t, V')$. Let C_{tM} be the expected holding and shortage cost of the manufacturer with no information sharing, and it is expressed as:

$$C_{tM} = E_\epsilon \left[P \cdot E(X_t - T_t)^+ + H \cdot E(T_t - X_t)^+ \right] \quad (20)$$

By substituting for X_t and T_t and simplifying further, we obtain

$$C_{tM} = E_\epsilon \left[\sigma \sqrt{V'} \left((H + P) \cdot L(\hat{K}) + H\hat{K} \right) \right] \quad (21)$$

where, $L(\cdot)$ is the right loss function for the standard normal distribution. Here,

$$L(z) = E(Z - z)^+ = z\Phi(z) + \phi(z) - z \quad (22)$$

where, Φ and ϕ are the cumulative distribution and the density function, respectively, of the standard normal distribution function. Also,

$$\hat{K} = \frac{M_t - M'_t}{\sigma \sqrt{V'}} + K \sqrt{\frac{V}{V'}} \quad (23)$$

Similarly, let C'_{tM} be the expected holding and shortage cost of the manufacturer in the case of information sharing. Then,

$$C'_{tM} = E_\epsilon \left[P \cdot E(X_t - T'_t)^+ + H \cdot E(T'_t - X_t)^+ \right] \quad (24)$$

By substituting for X_t and T'_t and simplifying further, we obtain

$$C'_{tM} = \sigma \sqrt{V'} \left((H + P) \cdot L(K) + HK \right) \quad (25)$$

To obtain the retailer's holding and shortage cost function, which we denote by C_{tR} , using v , we calculate the conditional variance of the retailer's demand over the lead time l . Then,

$$C_{tR} = \sigma \sqrt{v} \left((h + p) L(k) + hk \right) \quad (26)$$

where, $k = \Phi^{-1}[p/(p+h)]$ as stated earlier. Since, the retailer's cost is a function of ϵ_t which is observed by the retailer himself, there is no gain or loss for the retailer from demand related information sharing with the manufacturer.

Note that the supply chain cost is equal to the sum of the cost of both manufacturer and retailer. Since, information sharing has no impact on the retailer's cost function, henceforth we focus only on the manufacturer's cost function in order to evaluate the impact of information sharing on the supply chain cost function.

4. Numerical Study

In this section, we perform a numerical study to verify our analysis and to illustrate the magnitude of cost savings for the manufacturer due to information sharing. We also measure the reduction in variance of the manufacturer's demand process due to information sharing. In particular, we consider the characteristics of the external demand process such as α and σ and the characteristics of the supply process, i.e., the manufacturer's and the retailer's lead times, i.e., L and l , and analyze their impact on the manufacturer's cost.

First, we consider the impact of the demand process characteristics on the manufacturer's cost and the variance of his demand process. In our example, we consider the demand process with $\mu = 0$. The retailer's cost parameters considered are $p = 50$, $h = 2$, and the manufacturer's cost parameters considered are $P = 25$, $H = 1$. The replenishment lead time for the retailer, l , equals 10 and that for the manufacturer, L , equals 5. When we illustrate the impact of α , we set $\sigma = 40$ and we vary α from 0 to 1. Moreover, when we analyze the impact of σ , we set $\alpha = 0.5$ and we vary σ from 0 to 100. For the given set of parameters, we generate demand levels for 2000 consecutive time periods and compute the simulated average cost of the manufacturer when the retailer does not share demand information with the manufacturer. Note that when the retailer shares demand information with the manufacturer, the average cost, C'_{tM} , has a closed form expression as given in (25).

α	% reduction	α	% reduction
0.0	85.9626	0.6	63.2784
0.1	84.1046	0.7	57.7509
0.2	81.3080	0.8	52.1501
0.3	77.6966	0.9	46.6729
0.4	73.4013	1.0	41.5086
0.5	68.5512		

Table 1 Reduction in The Manufacturer's Cost Due to Information Sharing

Figure 1 shows the manufacturer's cost with and without information sharing as a function of α . Firstly, the manufacturer's cost with information sharing is always lower than the cost without

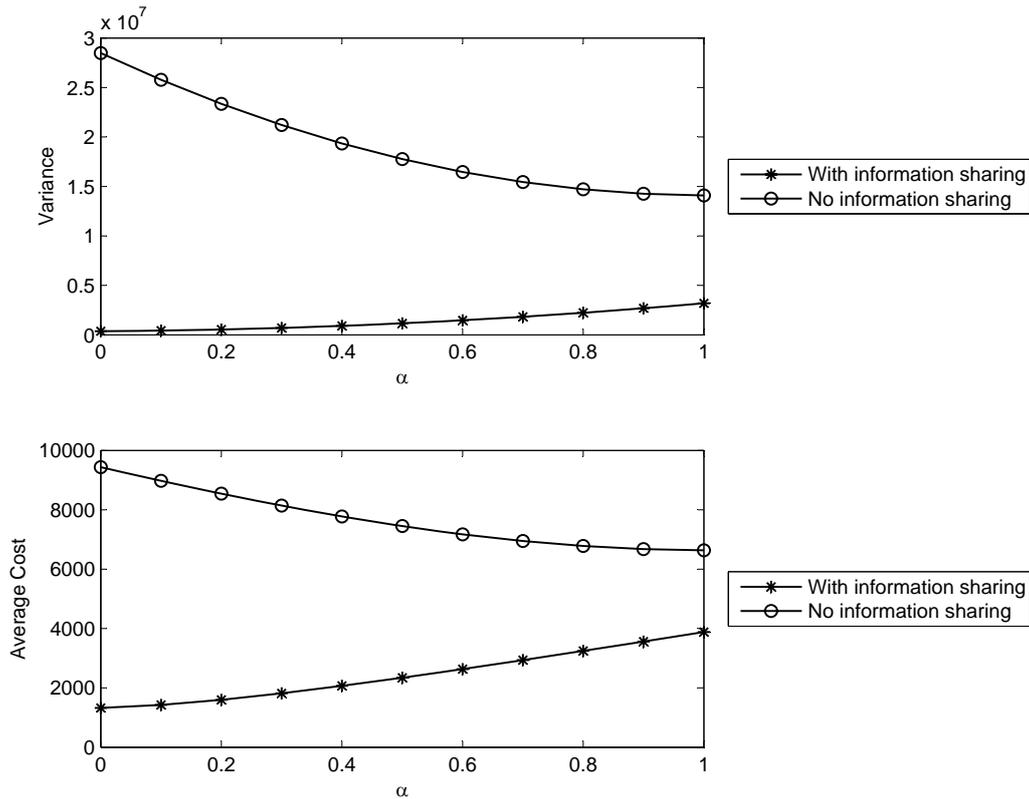


Figure 1 Impact of α on The Manufacturer's Demand Process and Average Cost

information sharing. This implies that information sharing is always beneficial to the manufacturer, as we would expect. We also find that as α increases, the percentage decrease in the manufacturer's cost with information sharing decreases (see Table 1). This means that the manufacturer's cost with and without information sharing are closer with the increasing α as explained further. The main reason why the manufacturer's cost with information sharing decreases is that the manufacturer uses the correct distribution of demand over his lead time to determine the optimal inventory position. On the contrary, in the case of no information sharing, the manufacturer uses the incorrect distribution of demand over the lead time, and hence, the optimal ordering decision is based on the incorrect distribution. The manufacturer's decision in the case of no information sharing is suboptimal for the right distribution of demand over the lead time resulting in higher cost in the case of no information sharing. Therefore, the further apart the wrong and right distributions get, the more will be the difference between the manufacturer's cost with and without information sharing. Since each distribution is normal, it is completely specified by its mean and variance. We know that $V \geq V'$, but V is a decreasing function of α . Therefore, as α increases, V gets closer to V' (see Figure 1). If the variances of the two normal distributions (the wrong and the right

distribution of demand over the manufacturer's lead time) get closer, the optimal decisions also get closer and so does the manufacturer's cost in the two cases.

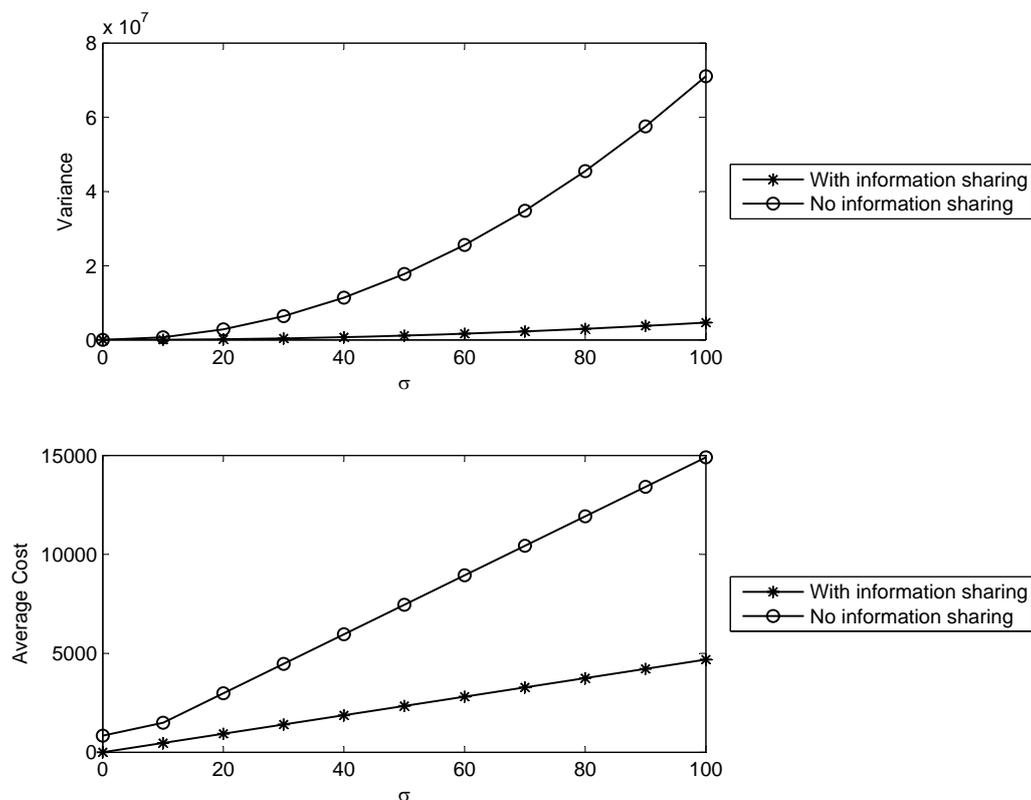


Figure 2 Impact of σ on The Manufacturer's Demand Process and Average Cost

Figure 2 reports both the variance of the manufacturer's demand process and the corresponding cost when σ is varied from 0 to 100 with and without information sharing. We find that both variance and cost increase with σ , irrespective of whether or not the external demand information is shared by the retailer with the manufacturer. We see that higher is the external demand uncertainty, higher is the uncertainty in the manufacturer's demand process. This results in higher cost of the manufacturer. However, for the manufacturer the variance of his demand process and the cost under information sharing are lower than that under no information sharing. This shows that information sharing is certainly beneficial to the manufacturer. Finally, when the uncertainty of demand is higher, the value of information sharing is higher, as we would expect. That is the decrease in the manufacturer's cost with information sharing is higher when σ is higher.

Similarly, in Figures 3 and 4, we show the impact of lead time of both retailer and manufacturer (l and L), respectively, on the variance of the manufacturer's demand process and his cost. It is

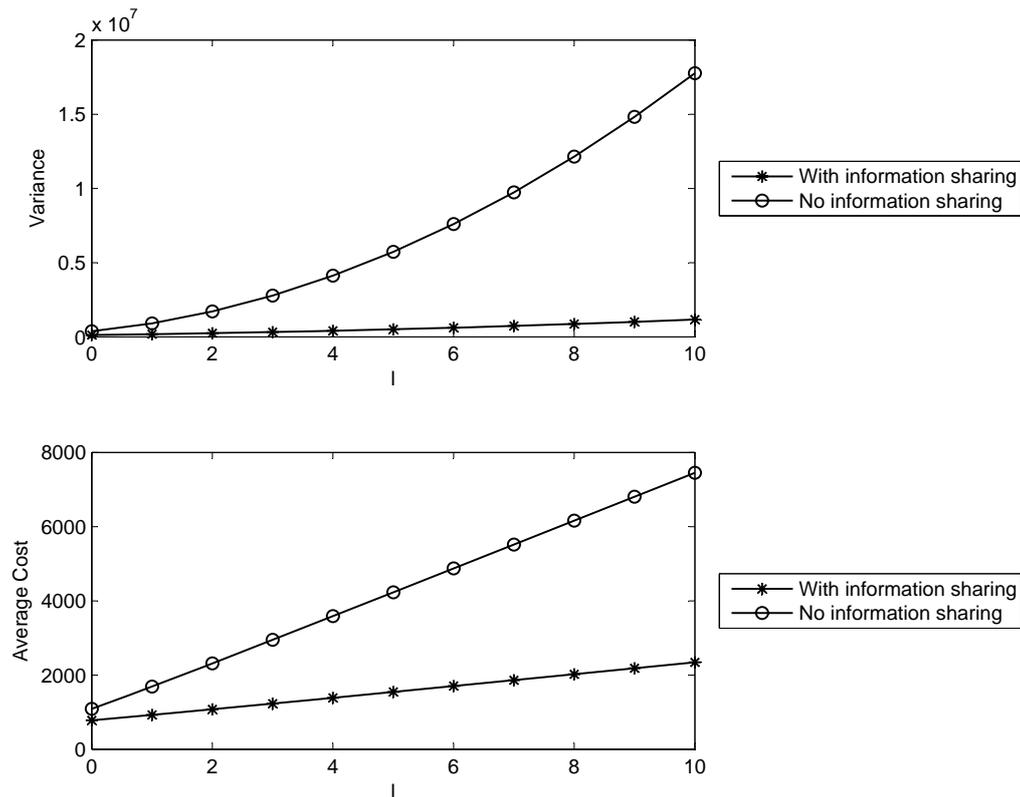


Figure 3 Impact of The Retailer's Lead Time on The Manufacturer's Demand Process and Average Cost

evident from the figures that both variance and cost increase with the increasing lead time of both retailer and manufacturer. Also, we note that in the case of no information sharing, the variance and cost are relatively higher if the manufacturer's lead time is higher than the retailer's lead time. The managerial implication of this observation is that given a choice, the supply chain players should focus on reducing the lead time of the manufacturer from the external supplier, rather than focusing on reducing the retailer's lead time from the manufacturer. This will result in a greater decrease in the manufacturer's cost, and hence, the total supply chain cost.

5. Conclusions

In this paper, we consider a two level supply chain consisting of a manufacturer supplying the product a retailer. In our model, both players follow a periodic review system for managing their inventories. Our aim is to develop a model for a nonstationary demand process and evaluate the value of information sharing in a supply chain. In the absence of information sharing in the supply chain, the retailer only conveys his order quantity to the manufacturer. However, with information sharing, in addition to conveying his order quantity the retailer also conveys the observed demand to the manufacturer in each period. The value of information in this case is equal

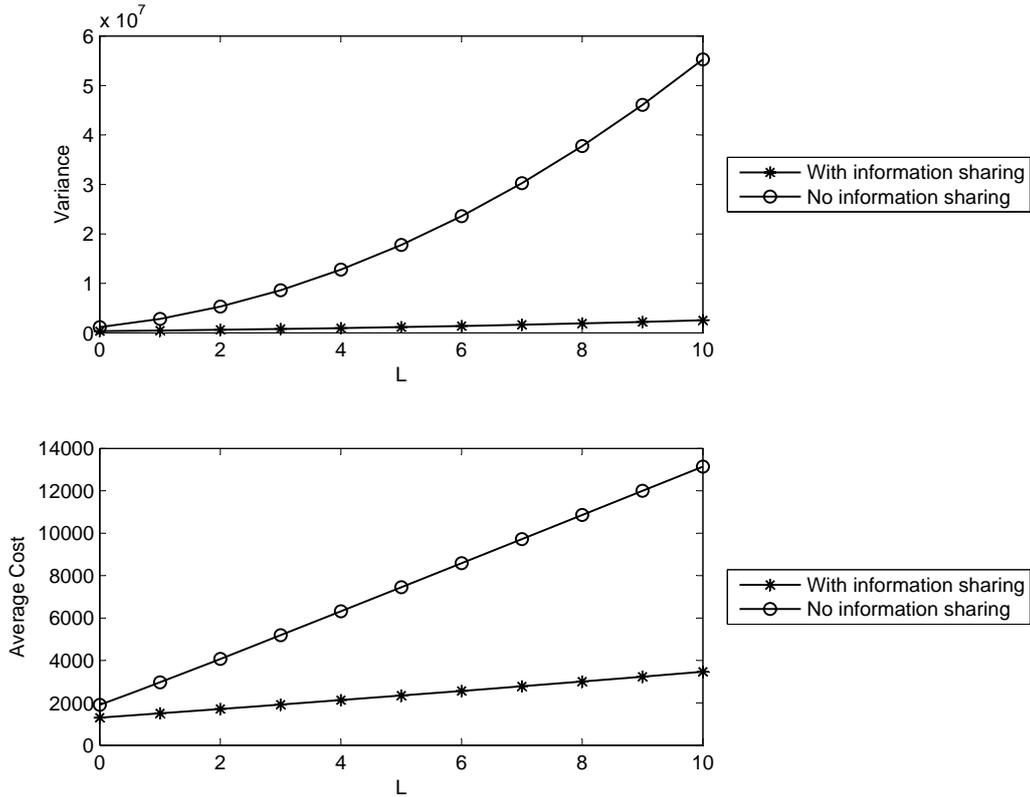


Figure 4 Impact of The Manufacturer's Lead Time on The Manufacturer's Demand Process and Average Cost

to the reduction in the supply chain cost due to additional demand related information available with the manufacturer. In our model, we assume that the retailer faces a nonstationary demand process of ARIMA(0,1,1) kind. The primary reason of using the ARIMA demand process is that it generalizes autocorrelated demand processes such as AR and ARMA that are typically considered in the existing studies on value of information sharing in serial supply chains. Moreover, ARIMA demand processes have been widely observed in practice.

The analysis of the model consists of obtaining the optimal ordering processes for both retailer and manufacturer and providing corresponding expressions for the retailer's and the manufacturer's inventory positions. In particular, we identify for the manufacturer the optimal order upto levels with and without information sharing. Finally, we calculate the cost of both players for each of the two cases: with and without information sharing. In both the cases we observe that the retailer's cost function has no implications of sharing demand information with the manufacturer. However, the manufacturer's cost, and hence, the supply chain cost decrease due to sharing of demand information by the retailer. We also show that the variance of the manufacturer's demand process decreases due to information sharing. Moreover, we show that both the retailer's ordering process

and the manufacturer's demand process are independent of the inertia of the external demand process and they depends only on the delivery lead time of the retailer from the manufacturer.

In this paper, we also present a numerical study to evaluate the value of information sharing. We observe that the reduction in the manufacturer's cost with information sharing decreases, as the external demand becomes less stable or more transitory over time. Also, the reduction in the manufacturer's cost with information sharing increases as the uncertainty of the external demand increases. Lastly, we find that in order to reduce the supply chain cost, the players should focus more on reducing the manufacturer's lead time from the external supplier rather the retailer's lead time from the manufacturer. It remains to be seen if the benefits of information sharing are as significant, for other models of nonstationary demand. We believe our model provides a framework for analyzing the value of information sharing in other nonstationary demand processes.

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