Vehicle Routing And Scheduling For Postal Delivery Vans

By

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### 1. Introduction

This paper attempts to identify algorithms and heuristics best suited for finding solutions to the problem of routing and scheduling postal delivery vans in a city given a set of operational constraints and requirements. The problem in its simplest form is of routing and scheduling vans based at a central depot through post offices that are geographically dispersed over the city, for delivering and collecting mail within desired time windows. It is referred to as the Vehicle Routing Problem with Time Windows (VRPTW) and is an important extension of the basic Vehicle Routing Problem (VRP).

The VRPTW appears in a large number of practical situations and is a topic researched extensively. The basic VRP problem can be described as 'finding optimal routes for a fleet of vehicles, starting from and returning to the depot, such that all customer demands are met and the total travel cost is minimized.' The term VRP is often used for routing as well as scheduling problems. In practice there are many problem specific requirements that are to be met while planning the routes and schedules. Some of these for the postal delivery problem can be issues such as time windows for visiting the post offices, the possibility of each van operating more than one route in the given time period, there being multiple depots, time windows for delivery and collection, drivers' schedule constraints such as maximum route duration, consideration of loading and unloading times, whether deliveries and collections can be coupled or not, customer (post office) priorities, precedence relationships among the POs, multiple commodities such as mail and cash collection and van capacity. Other issues could be size of the fleet available, homogeneous or heterogeneous fleet, underlying network being directed, undirected or mixed and multiple objectives. The issues pertinent to the MMS problem are discussed in further detail in section 6. These extensions and constraints have to be incorporated in the specific formulation of the problem under consideration or handled through heuristic methods.

Solution procedures for the VRP with suitable extensions are complex even for moderately large problems and are usually tailored to exploit the resulting underlying structure of the problem at hand. The scheduling requirements, when added to the routing problem that admits a better defined structure, tend to destroy the simpler structures. Finding a feasible solution to the VRPTW is itself an NP-complete problem (Savelsbergh 1986).

The following section is a survey of the literature on VRPTW. The various formulations of VRPTW are discussed in section 3. Solution procedures including both exact algorithms and heuristics are surveyed in sections 4 and 5. Section 6 describes the Bangalore city postal delivery system and the pertinent issues to be considered The mathematical formulation of the problem is given in Section 7. The existing routes and schedules currently used by the postal department and an improved solution obtained by generating additional columns are given in section 8. The concluding section outlines the direction for further research.

# **2. Literature Review**

The VRP represents an area of extensive research due to its wide applicability and degree of difficulty. Detailed surveys including the ones by Christofides (1985) and Fisher (1995) document optimization as well as heuristic methods proposed for the problem.

The VRP with time windows and a variety of practical applications have been researched actively. Several surveys including the ones by Magnanti (1981), Bodin et al. (1983), Desrochers et al. (1988), Solomon et al. (1988) and Laporte (1992) give comprehensive overviews of related work and provide insights into various methodologies used. Desrochers et al. (1990) attempt to classify models with various characteristics in order to bring uniformity on the subject. Bodin (1990) gives a very interesting account of the experiences with various vehicle routing and scheduling problems that the author encountered in practice over a period of twenty years.

Most recently, Desrosiers et al. (1995) give a concise and comprehensive exposition of basic time constrained routing, scheduling and the related problem types. The paper focuses on decomposition methods as solution methodology. The bibliography lists above 200 references and is a rich source of information on advances made in the area.

# **3. Formulations of the VRP**

The basic VRP can be formulated in several ways. Dynamic programming formulations have been used for variants of the problem with some degree of success. Integer programming based formulation approaches such as set covering, set partitioning and travelling salesman have been used often to develop solution methods. The article by Cullen et al. (1981) is one of the earlier ones that reports using set partitioning based heuristics for routing.

In case scheduling constraints and time windows are also present, the formulations can be extended to include these using set covering, set partitioning or travelling salesman based formulations for the VRPTW.

# *The Set Partitioning Based Formulation*

A set-partitioning problem is a binary integer programming problem of the form:

Min $CY$ s.t. *AY=1* where *Y* is a binary decision vector and the coefficients in *A* take the value 0 or 1.

A set-covering problem is a set-partitioning problem with the covering constraints being relaxed to  $AY \geq 1$ . For the VRPTW, each row of the matrix A represents a customer to be serviced by a vehicle and each column represents a route that is feasible for the given

constraints. If a route j services customer i, then the coefficient  $a_{ij}$  of the matrix A is 1, otherwise it is 0. Any feasible solution to the set-partitioning problem ensures that each customer is serviced by one route. In case of the set-covering problem it ensures that each customer is serviced by at least one route. If all feasible routes under the problem constraints could be possibly enumerated, the optimal solution for the set-partitioning formulation would be the optimal solution for the VRPTW. However, an exhaustive enumeration of all feasible routes is not practical as their number becomes very large with an increase in the number of customers.

### **4. Solution Procedures**

Given the degree of difficulty of the VRP, a considerable amount of work has been done to develop heuristics for arriving at good solutions. Solution methods for time constrained routing and scheduling problems have evolved progressively from ad-hoc methods to simple heuristics and optimization-based exact and heuristic methods.

Heuristics for the VRP can be classified as constructive, improvement and two-phase procedures, incomplete optimization methods, and meta-heuristics such as tabu search, genetic search and simulated annealing. Solomon (1987) extended constructive heuristics for the VRP, such as nearest neighbour, insertion, savings and sweep, to the VRPTW and reported their comparative performance on problems sets generated by them.

Kolen et al. (1987) proposed a branch-and-bound method but the problems tested are very small in size. Desrosiers et al. (1988) proposed a Lagragian relaxation approach for the problem.

Gilmore and Gomory (1961) first applied column generation techniques or generalized linear programming decomposition principle (Dantzig and Wolfe (1959)) to an integer programming formulation of the cutting stock problem. However, their approach did not produce optimal integer solutions to the problem. Desrosiers et al. (1984) reports the relatively successful application of column generation technique to the VRPTW with a single depot and capacity constraints but no precedence relations and schedule duration constraints. The problem considers only delivery (or collection) requirements but not both simultaneously. Desrosiers et al. (1986) compares the performance of the column generation approach with a branch-and-bound approach using network relaxation of the problem. They report that the former performs better on the problem set tested, especially when the width of the time windows increases. Ribeiro et al.(1989) combined column generation techniques with branch-and-bound and solved some very large scale set partitioning problems in traffic assignment to optimality and reported that application of the same approach to other problems including VRPTW held promise. Dumas et al. (1991) incorporate precedence and vehicle capacity constraints and use a column generation technique to solve some relatively small instances of the pick up and delivery problems (PDPTW). These problems arise when satisfying transportation requests, each requiring pick up and delivery at pre-specified origins and destinations. Desrochers et al.

(1992) update their earlier work on using a column generation technique and report that larger problems are solved to optimality.

Gendreau et al. (1994) and Potvin et al. (1996a) use branch exchange improvement, procedures such as 2-opt and 3-opt, along with tabu search for the VRPTW and report that the approach outperforms the best existing heuristics on standard test problems. Further work using this approach is likely to continue. Potvin et al. (1996b) use a genetic search approach and report obtaining competitive solutions using this approach as well.

## **5. Column Generation Technique**

Algorithms used for optimization are based on the formulation of the basic problem. The classical decomposition principle used on set covering and set partitioning formulations of the VRPTW has been reported to work well. This would imply optimizing over the set of all feasible routes to select a subset that would yield the optimal objective function value. However, enumerating all possible routes is a formidable task even for moderately sized problems. To overcome this, the linear programming relaxation of the set partitioning formulation is solved using column generation techniques that have become popular for solving large-scale integer programs. Columns in the feasible set of routes are generated as needed by solving shortest path problems appropriately constrained to define an admissible route for the problem under consideration. Thus only those routes that are promising are generated as the algorithm iterates towards optimality for the linear programming relaxation of the problem. At each iteration a master problem, consisting of a partial set of all feasible routes generated so far, is solved. This is a linear programming relaxation of usually only the set covering constraints. The values of the optimal dual variables obtained are used to determine if any routes currently not included in the partial set can improve the solution. This entails solving a constrained shortest path problem, which usually includes the scheduling and side constraints if present. If a solution with negative marginal cost is found it is added to the set of routes in the master problem which is then re-optimized. The iterative procedure is repeated until no additional routes with negative marginal cost can be found which implies that the optimal solution to the linear programming relaxation is found.

The bound on the original problem, obtained in this manner, shows empirical evidence of being usually tight. Bramel et al. (1997) attempt to analytically characterize the effectiveness of set covering formulations for the VRPTW. They show that the relative gap between fractional and integer solutions of the set covering formulation of the VRPTW becomes arbitrarily small as the number of customers increases. A branch-andbound method can, therefore, be attempted to close the gap between the linear and integer solutions.

Furthermore, when the constraints are restrictive, the column generation approach has been reported to work well. This is attributed to the fact that the number of feasible columns will not be too large.

We propose to investigate the effectiveness of this approach for the postal mail van routing and scheduling problem with the given characteristics under consideration. The problem with simultaneous delivery and collection requirements as is the case here is more complex than one where collections (deliveries) are followed by deliveries (collections). Not much work is found in the literature pertaining to this specific problem.

# **6. Postal mail van routing and scheduling problem (Mail Motor Service** - **MMS)**

This section gives a comprehensive description of the Bangalore City postal delivery (MMS) system. The available resources, specific requirements, features and peculiarities of the problem to be modelled are presented.

The postal department has to facilitate the delivery and collection of mail to and from about 100 post offices both in the morning and the afternoon for most of the post offices. Some post offices may only have a delivery requirement. In addition to the bags of mail, bags containing money (accounts bags) also have to be collected from the POs. Currently, the department has 40 vans and 50 drivers at its disposal. These vans ply on thirty-four routes that the postal department has generated to fulfil the delivery and collection requirements. The extra vans are used as standby vehicles.

# *The Post Offices (Customers)*

The post offices are located throughout the city. The distances between certain pairs of POs that are serviced consecutively on an existing route are available. The remaining can be obtained either by acquiring a suitable geographical information system (GIS) or by extrapolating from the ones available. Some that are not available can be ignored due to practical considerations such as their locations not being connected by a direct and easily pliable road or being obviously too far apart. The travelling times between pairs of post offices on existing routes are available. The remaining can be obtained either by a rough estimation or by actually sending a vehicle between the pair of POs under consideration.

The post offices can be classified roughly, based on the volume of transactions, into three categories namely large, medium and small. Post offices handling bags between 1-20 are categorized as small, between 21- 40 as medium and above 40 as large. There are around 5 large and 15 medium post offices, the remaining being small.

# *Operational Details*

Mail comes into and goes out of the city by bus, train or air. Mail for within the state is sent and received by bus, that for out of state by train and some by air. The Bangalore City Transit Mail office (TMO) serves as the nodal point for inbound and outbound mail and could be considered to be the central depot. However, due to space constraints the mail is sorted and bags are closed and marked for a specific destination at the Bangalore City Transit Delivery Sorting (TDS) office. The sorted mail has to be then transferred to the TMO. The TMO and TDS are located in close proximity to each other at a distance of **two Kms. Part of these transfers from the TDS to the TMO take place in the morning just before the commencement of the schedules. Once all the mailbags are collected at the TMO in the morning, time has to be allowed at the TMO for segregation of bags with local mail according to the routes they have to be assigned to.**

**Apart from the TDS there are other sorting offices as well. For example mail arriving by air is sorted at the Air Sorting Office and the Airport Transit Mail Office (APTMO) serves as the nodal point for this mail.**

**In addition some post offices called the 'nodal' POs are equipped to sort local mail and close the bags for the desired destination. Such mail arrives sorted at the TMO and can be distributed directly to the local POs. The GPO serves as one of the major nodal POs and consolidates sorted mail from other nodal post offices as well. This mode of operation was introduced a few years ago to lessen the burden on the TMO and TDS in order to improve service. For this reason the nodal POs and the GPO must be visited before the closing of a schedule on several routes.**

**The mode of operation as described above implies that though the TMO in principle functions as the depot and can be taken as the starting and terminating point for all routes, the TDS, nodal post offices or the GPO may have to be visited preceding this for dropping or picking bags. This will entail precedence constraints in the formulation of the problem. It also implies that each post office is not necessarily visited only once as in a typical VRPTW. This will have to be handled as well in the formulation.**

### *Routing and Scheduling Requirements*

**Mail is delivered and collected in two shifts, the first in the morning and the second in the afternoon. Morning mail delivery to the POs can start at 6.30AM and should be completed by 9.30AM for timely distribution. The larger POs should receive their mailbags earlier to provide adequate time for mail processing and distribution.**

**Morning mail is available for collection at the POs starting 8 AM onwards. This is the mail collected at the pust office after the previous day's last clearance. Afternoon mail has to be delivered between 1.30PM and 3.30PM and can be collected after 4PM. In general, the schedules are drafted close to the post office opening hour for delivery of mails in the morning and close to the closing hour in the evening. The account bags have to be collected only in the morning after 9AM from around half of the POs.**

**Each schedule is to be no longer than 8 hours within which a break of 30 minutes must be provided. Any extra time is to be compensated for at overtime rates.**

**Capacities on vehicles are reported to be nonrestrictive as the loads are observed to be lower than the vehicle capacities on existing routes. For our purpose we can start by ignoring the capacity constraints and verify that the routes in the solution thus obtained do not violate the existing vehicle capacities. If they do, vehicle capacity constraints will also have to be included.**

Routes must be generated such that all mails can be delivered and collected as required in the morning and the afternoon from the post offices. Certain precedence relationships as mentioned earlier must be included and adhered to by the routes generated. The schedule on each route must be no longer than 8 hours and should incorporate time required for delivery and collection along with the time required for travel and a break. Standard time needed for loading/unloading can be assumed to be approximately 5 minutes for every 20 bags. Collection and delivery time windows must be adhered to and large post offices should be served earlier.

The morning delivery and collection time windows are neither completely overlapping nor disjoint. This will have to be incorporated in the formulation. One possibility is to duplicate the nodes as collection and delivery nodes. For the evening routes, since collection starts only after delivery, routes can be generated as 'backtracking' routes, though existing precedence relationships will have to be included.

Currently the routes are scheduled such that each driver completes his schedule within eight hours including a break of half an hour. This is called a *continuous schedule.* Another option available is of *split scheduling*, whereby a driver can be given a schedule that is split into 4 and 3 hours duration within a period of 12 hours with a half an hour break. We will find optimal routes under both types of schedules for comparison. It is felt that introducing split schedules may reduce the number of routes and drivers needed.

It is also possible to stagger the opening time for the post offices if this results in greater efficiency. Currently a few post offices do start earlier than the rest but this aspect needs to be looked into carefully.

# *Objectives*

The objectives that can be considered in finding the set of optimal routes and schedules that satisfy the demand (delivery and collection) within specified time windows taking into account the travelling and loading/unloading times, schedule duration constraints, driver schedule requirements, and precedence relation constraints are:

- Minimize the number of routes, thus minimizing the number of vans and drivers needed.
- Minimize the total distance travelled (or time needed)
- Minimize the idle time on the routes.

# 7. Mathematical Formulation

The MMS problem has been formulated using a set-partitioning based formulation. The following notation is used:

Let  $G = (N, A)$  be a network, where N the set of nodes representing post offices 1, 2,...,n. The TMO which serves as the central depot is replicated as nodes 0 and 2n+k+l to distinguish between the origination and termination of a route. Each route will, therefore, **originate from node 0 and end at node 2n+k+l. The post offices 1,2,..., n have also been replicated as nodes n+1, n+2,..., 2n assuming that each of them has a delivery and collection requirement. This can be modified if some of the post offices do not have a collection requirement.**

**Each node is serviced exactly once. If a post office serving as a nodal office has to be visited after collections are made, a node is created to handle this. Precedence relationships are imposed for this node to ensure that it is visited after all the nodes representing post offices attached to this nodal post office have been visited on the** collection route. Let there be k such nodes and let  $N_i \subset N$ ,  $j = 1, 2,...$ , k denote the set of **nodes representing post offices attached to the nodal post office j.**

*A* is the set  $\{(i,j), i,j \in N\}$  of arcs connecting nodes i and j. Associated with each arc  $(i,j)$  $\in$  *A* is a a travel duration t<sub>ij</sub> and a travel cost  $c_{ij}$  which is taken to be the distance between **i and j. The fixed cost of using a vehicle can be included by adding it to all arcs (0,j) originating from the depot. We assume that the loading and unloading time at post office** i is included in the duration  $t_{ii}$  of arc  $(i, j)$ .

**The deliveries and collections are to be made within a time window defined by the earliest time, ai and the latest time, bi. Since the time windows are identical for delivery ai**  $=$  al and  $b_i = b1$  for  $i = 1, 2, \ldots, n$ . Similarly, since time windows for collection are also identical,  $a_i = a2$  and  $b_i = b2$  for  $i = n+1, n+2,..., 2n+k$ . The loading or unloading at the post offices can begin at time T<sub>i</sub> within the time window defined by the earliest time, a<sub>i</sub> and the latest time,  $b_i$ . If a van arrives at a post office earlier than  $a_i$ , it will be permitted to **wait. Furthermore, vehicles must be scheduled such that the duration of the trip is no more than a total time T.**

**In addition let,**

- **Q is be the set of feasible routes for the Mail Motor Service VRPTW**
- $\delta_{ir}$  is a binary constant that takes value 1 if route  $r \in \Omega$  visits post office  $i \in N \setminus \{0, 2n+k+1\}$  and 0 otherwise.
- **Cr is the cost of route r. The cost of a route is defined as the sum of the cost of the arcs on the route.**
- **yr is a binary variable that takes the value 1 if route r is used and 0 otherwise.**

**A set partitioning based formulation of the VRPTW on the network G can be given as follows:**

$$
\begin{aligned}\n\text{Min} \quad &\sum_{r \in \Omega} c_r y_r \\
\text{s.t.} \\
&\sum_{r \in \Omega} \delta_{ir} y_r = 1, \quad i \in \mathbb{N} \setminus \{0, 2n + k + 1\} \\
&\quad r \in \Omega \\
&\text{y}_r \text{ binary}, \quad r \in \Omega\n\end{aligned} \tag{1}
$$

**The set partitioning formulation given above selects a minimal cost subset of feasible routes in** *Q.*

New columns  $y_r$  in  $\Omega$ , of minimum marginal cost are generated by solving the constrained shortest path problem  $(4) - (11)$ . The objective function coefficients depend on the dual variables  $\pi_i$  associated with the LP relaxation of the above set partitioning **problem. Two kinds of variables are needed to define the subproblem. The flow variables**  $x_{ij}$ ,  $(i, j) \in A$ , on the arcs and the time variables  $T_i$ ,  $i \in N$  for the start time of service at **node i. The constrained shortest path subproblems to be solved are:**

$$
\begin{array}{ll}\n\mathbf{Min} & \Sigma \quad \Sigma \quad (\mathbf{c}_{ij} - \pi_i) \mathbf{x}_{ij} \\
(i, j) \in A\n\end{array} \tag{4}
$$

**s.t.**

$$
\sum x_{ij} - \sum x_{ji} = 0
$$
  
j \in N / {0,2n+k+1} (5)

$$
\sum x_{0j} = \sum x_{j2n+k+1} = 1
$$
 (6)  
j \in N / {0,2n+k+1} = i \t(6)

$$
x_{ij} > 0 \implies T_i + t_{ij} \leq T_j \quad (i, j) \in A \tag{7}
$$

$$
T_i + t_{i2n+j} \leq T_{2n+j} \qquad i \in N_j, j = 1,2,...,k \qquad (8)
$$

$$
a_i \leq T_i \leq b_i \qquad \qquad i \in N \tag{9}
$$

$$
0 \leq T_i \leq T \qquad \qquad i \in N \tag{10}
$$

$$
x_{ij} \text{ binary} \qquad (i, j) \in A \qquad (11)
$$

**The objective function (4) finds a minimal cost path subject to constraints (5)-(l 1). Flow constraints are given by (5) and (6). Constraints (7) and (9) ensure feasibility of the schedule. (8) specifies existing precedence relationships. Finally constraint (10) ensures that the duration of the route is no more than the permitted time T.**

**Constraint (7) is non-linear in its present form but can be written in the following linearlized form:**

$$
T_i + t_{ij} - T_j \leq (1 - x_{ij})M_{ij} \quad (i, j) \in A \tag{12}
$$

where  $M_{ij} \ge b_i + t_{ij}$  -  $a_j$ . The formulation with constraint (12) replacing (7) is equivalent as long as  $x_{ii}$  is required to be binary.

#### **8. Existing and Improved Solutions**

**The routes and schedules followed by MMS Bangalore were reorganized during 1993.**

**Currently there are 14 morning and 20 afternoon routes and schedules. The morning routes and schedules are attached as Appendix 1 as CURSOL.**

**The existing MMS routes have been incorporating the growth in the number of post offices over the years more by observation than by reviewing and re-structuring the entire set of routes. This has resulted in various inefficiencies. Some routes cover much larger distances than others. Moreover, some schedules have a considerable amount of idle time that seems to have been filled by shuttling the mail van between points located close to each other such as the TMO, TDS and GPO or by making the vans wait at post offices for longer duration than necessary. Some post offices are visited by more than one route for delivery and/or collection within a short time period, which is unnecessary in many cases. In cases where there are practical considerations that require several visitations, such as deliveries for sorting at nodal POs, the nodes can be duplicated and precedence relations imposed for this to be incorporated in the mathematical formulation.**

**Currently the loading and unloading times do not take into account the number of mailbags that have to be handled. Information on average loads is available and schedules should be drawn taking this into consideration.**

**Since the revision of schedules, there have been changes in vehicular traffic such as increased traffic in the city and imposition of traffic rules like one way streets, installation of signals at different junctions. The schedules seem to have enough slack time to be able to absorb these changes without affecting the deliveries and collections.**

**Three 'cash schedules' facilitate the collection of account bags. It is necessary to segregate the ordinary mail and account bag collection since the latter needs the presence of security personnel. The cash schedules are also thought to be operating with idle time and can cover a larger number of post offices. Tying up the banking hours with these schedules also needs to be looked into.**

# *Re-structuring of the MMS Schedules*

**On a careful study and analysis of the existing routes and schedules it is found that certain constraints could be present merely for simplifying manual scheduling and have become part of the system. We believe that the postal delivery system is functioning** suboptimally and requires re-structuring. The routes can be redefined and alternate routes **can be found such that a smaller number of vehicles are required and the total distance travelled by them is reduced. To confirm this, the set covering formulation of the problem was solved using CPLEX with a set of routes in the existing solution and additional routes generated by observation.**

**The solution obtained is given in Appendix 2 as NEWSOL. The number of routes in NEWSOL has reduced to 12 from 14 in CURSOL. This directly implies that the number of vans needed is reduced by 2, which is a 14% improvement. In addition, the current routes are all 8-hour routes while 3 of the new routes are 4-hour routes. This can lead to further savings as the half routes can be combined with 3-hour routes in the evening to generate a foil schedule. The total distance covered is 887 Kms. by the old routes and 788** Kms. by the new routes. This reduction by over 11% is a significant improvement over the current solution.

# 9. Directions and Conclusion

There is scope for improvement in the operation of the postal department's MMS problem. This paper serves as an interim report on further work. Generation of routes to satisfy simultaneous delivery and collection requirements, as solutions to the shortest path sub problems with scheduling, duration, precedence and other constraints mentioned above is the next step. Dynamic programming approaches have been used for solving constrained shortest path problems that have originated as sub problems of various VRPTW scenarios. We propose to use a similar approach adapted to handle the problem at hand. Methods and basis for effectively reducing the state space and time windows for more efficient handling of the problem under consideration will be investigated. To generate routes that will be close to the desired schedule duration, we propose to experiment with using lower as well as upper limits on the duration. This should also result in reducing the number of columns that can be generated and make the problem more tractable. However, the time windows are not very 'narrow' which may affect the performance of the column generation approach.

In the present context the time window and schedule duration constraints are hard constraints and most of the research has been directed towards hard time window constraints. For the problem under consideration they may be treated as soft, that is a penalty can be imposed for violating the constraints within a certain limit, as this should be practically viable if it results in a superior solution. It would be of interest to look into the benefit, if any, of treating the constraints as soft.

GIS software is becoming increasingly accessible in India. There are a number of vendors offering road maps and related information. It will be desirable and useful to obtain accurate inputs using a suitable software package. The comprehensiveness and ease of use of data being made available is to be assessed for our requirements. GIS software can also be used for visual presentation of solutions for easy comprehensibility. This would make it easier for the user to identify drawbacks, if any, in the solution from the point of view of practical implementation. This can prove to be beneficial as the formulation can be suitably modified, if possible, to provide a solution with the desired characteristics.

Given the abundance of applications of the VRPTW, the framework for the MMS routing and scheduling problem can be extended and applied to other related problems.

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# **Appendix 1: CURSOL (Current Morning Routes)**











## **Appendix 2: NEWSOL (New Morning Routes)**













**\* The number in parentheses is the old route number if changed**