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ECONOMIC SIZING OF WAREHOUSES WHEN THERE ARE RESTRICTIONS ON HIRING OF WAREHOUSE SPACE: AN INTEGER PROGRAMMING APPROACH.

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ABSTRACT

Determining the optimal size for a private warehouse is a planning problem which can affect the overall operations of a firm. Some authors presented a linear programming formulation which determines the optimal size warehouse to construct when demand is highly seasonal and public warehouse space is available for lease on a monthly basis. In this paper, the situation where the public warehouse space should be leased only for a long period of time is considered. Integer programminmg formulations are presented for single lease and multiple lease situations. Computational experiences will be presented later.

1. INTRODUCTION

In logistic system, the managements of business organisations frequently encounter the problem of sizing of a warehouse after its location is determined. A privately owned warehouse requires capital investement and administrative overheads. A public warehouse may be hired but may have higher cost per unit of storage. When demand for storage space is seasonal, one must decide on the most economical combination of both private and public warehouses. In some situations, especially in fertiliser industry,the demands for various types of fertilisers are highly seasonal. The distribution of fertiliser consists of movement of fertiliser from factories to specified intermediate storage points to destinations. The intermediate storage points are usually situated near the railway lines to facilitate ease in movement. The State Warehousing Corporation and others constructed a number of warehouses at important locations and the warehousig space is hired out. Usually, the warehousing space is not hired for short durations, like a month. The public warehouses can be leased for a long period of time, e.g. one year. The leasing usually starts from April of a year to March of next year. In any month, depending on the availability of public warehouse space, one may lease certain amount of storage space, but the lease will be valid upto the end of some fixed period. The leasing costs will be incured whether the storage space leased is utilised or not and it will be effective from the month of hiring. The problem is to determine the optimal size of the private warehouse, the amount of storage space to be leased from a public warehouse and the month of starting the lease. This is formulated as a 0-1 integer programming problem. Computational experience will be reported seperately.

2. REVIEW OF WORK DONE IN THE PAST.

Two types of warehouse sizing problems are discussed in literature - static and dynamic. In the static problem, once the size of the private warehouse is determined, it remains constant throughout the planning horizon. In the dynamic problem, the private warehouse size may change from one period to another during the planning period. In both cases, it is assumed that any required amount of public warehouse space can be hired in any period.

Ballou [1] gave a method for determining the most economical combination of private warehouse space and public warehouse space for a static problem. Hung and Fisk [3] presented a linear programming formulation for a static problem to determine the optimal size of the warehouse to be constructed when demand is highly seasonal and public warehouse space is available on a monthly basis. The model is extended for the dynamic sizing problem in which the warehouse size is allowed to change over time. Computational experience using UNIVAC FMPS package for a sample problem was also reported by Hung and Fisk [3].

3. DESCRIPTION OF THE MODEL.

We consider a planning horizon of T periods. It is assumed that the location for private warehouse is already determined. Any amount of public warehouse space can be leased in any month t, and the lease will be effective from that month to the end of planning horizon T i.e. the lease duration will be (T-t+1) periods.

First it is assumed that a decision on the amount of public warehouse space to be leased is made only once in a planning horizon. This is extended to the case when such decisions can be made any number of times in the planning horizon.

For each period t in the horizon, demands for the warehouse space are estimated. Assuming, in general, there are n estimates, and for each estimate the probability of occurrence is P_j, $j=1,2,...,n$ and $\Sigma_j P_j = 1$.

Ballou [1] showed that warehousing cost for period t can be computed from the following formula:

$$C_{tj} = C_0 X + C_v Y_{tj} + C_p (D_{tj} - Y_{tj}) \qquad (1)$$

where

C_{tj} = warehousing cost in period t under demand estimate schedule j;

C_0 = overheads and amortized capital expenditure. per sq.ft per period;

X = size of private warehouse, in ft^2 ;

C_v = variable private warehousing cost, per sq.ft of storage per period;

C_p = variable public warehousing cost,per ft^2 of storage per period;

Y_{tj} = amount of private warehouse space used in period t, under estimate j;

D_{tj} = demand for storage space, in ft^2 in period t, under estimate j.

It is also assumed that only a fraction f of the private warehouse space can be used for storage. Thus

$$Y_{tj} = f.\,X \quad \text{if } D_{tj} > fX \qquad (2)$$
$$= D_{tj} \quad \text{if } D_{tj} \leq fX$$

The total expected cost for the planning horizon is:

$$EC = \sum_{t=1}^{T}\sum_{j=1}^{n} P_j C_{tj} \qquad (3)$$

Thus the problem of sizing a private warehouse is to determine warehouse size X and the allocation of storage Y_{tj}'s such that EC is minimized.

A simple alternative to Ballou's method of finding optimal warehouse size is given by Hung and Fisk [3]. They use linear programming formulation. They first replace, for each demand period t, the set of demand estimates and their corresponding probabilities of occurrence with the expected value of demand D_t :

$$D_t = \sum_{j=1}^{n} P_j D_{tj}$$

Similarly, the amount of private warehouse space used in each period t under estimate j is replaced by Y_t , the expected value of warehouse space used in period t.

The linear programming formulation developed for the static problem is as follows:

$$\text{Minimize } EC = \sum_{t=1}^{T} [\, C_0 X + C_v Y_t + C_p (D_t - Y_t) \qquad (4)$$

subject to:

$$Y_t \le f.X , \qquad t=1,2,\ldots,T \qquad (5)$$

$$Y_t \le D_t \qquad t=1,2,\ldots,T \qquad (6)$$

$$X \ge 0 ,\ Y_t \ge 0 \qquad t=1,2,\ldots,T \qquad (7)$$

In this model, the amount of public warehouse space hired in month t is (D_t - Y_t), which can vary from month to month.

In this paper, we consider the situation where the public warehouse space could be leased only for a long period of time. When it is decided to lease public warehouse space in time period t, then hire for the space should be paid for periods t through T, irrespective of whether the leased space is utilized or not.

This type of situation frequently arises in the case of a product which has seasonal demand, eg. fertilizers. The management of public warehouses would like to enter into long-term lease rather than renting on a month-to-month basis. Usually, they enter into leases at the beginning of the year to last for one year. If any warehouse space is available within a year, it can be leased from that period to the end of the year.

First we develop a model assuming that we can enter into a lease utmost once in a planning horizon. The model can be extended to the case of multiple leasings in a planning horizon. The multiple leasings could be from different public warehouses, where the variable public warehousing costs could be different.

4. INTEGER PROGRAMMING FORMULATION

Single Lease

In the case of one time lease, let

Z_t = binary variable taking values 1 or 0 depending on whether public warehouse space is leased or not leased in time period t; $t = 1,2,...T$

U_t = size of leased public warehouse space, in ft^2, in time period t, $t = 1,2,....T$

The definition of other variables will be as stated above.

We now look for a formulation to ensure that if public warehouse space of size U_t is to be leased in time period t, then that space U_t should be leased for periods t through T

i.e. $U_{t+1}, U_{t+2}, ..., U_T$ should be all equal to U_t .

Two integer programming formulations are given below:

FORMULATION 1

$$Minimize\ EC = \sum_{t=1}^{T} [\ C_0 X + C_v Y_t + C_p U_t\ ] \qquad (8)$$

subject to:

$$Y_t \le f.\ X\ , \qquad t = 1,2,...,T \qquad (9)$$

$$Y_t \le D_t\ , \qquad t = 1,2,...,T \qquad (10)$$

$$U_t \ge D_t - Y_t\ , \qquad t = 1,2,...,T \qquad (11)$$

$$U_t \le M\ Z_t\ , \qquad t = 1,2,...,T \qquad (12)$$

$$U_{t+1} \ge U_t\ , \qquad t = 1,2,...,(T-1) \qquad (13)$$

$$U_{t+1} - U_t \le M\ (1 - Z_t)\ , \qquad t = 1,2,...,(T-1) \qquad (14)$$

$$Z_t \leq Z_{t+1} , \qquad t = 1,2,\ldots,(T-1) \qquad (15)$$

$$X \geq 0 , Y_t \geq 0 , U_t \geq 0 , \qquad t = 1,2,\ldots,T \qquad (16)$$

$$Z_t = 0 \text{ or } 1 \qquad t = 1,2,\ldots,T \qquad (17)$$

The value of M is assumed to be a large positive number. Constraints (9) and (10) respectively ensure that the projected amount of private warehouse space used in each given period is no greater than available warehouse space and estimated demand for storage space.

Constraint set (11) defines the amount of public warehouse space hired in time period t.

Constraint set (12) ensures that a positive amount of public warehouse space may be hired in time period t, only if a decision is made to hire.

Constraints sets (13), (14) and (15) ensure that if U_t sq.ft of public warehouse space is hired in time period t, then it should be leased from period t through period T.

Constraint set (16) ensures that private warehouse size, all projected requirement, and space leased in public warehouse are non negative.

FORMULATION 2

$$Minimize\ EC = \sum_{t=1}^{T} [\ C_0 X + C_v Y_t + C_p (T-t+1) U_t\ ] \qquad (8)$$

subject to:

$$Y_t \leq f.\ X , \qquad t = 1,2,\ldots,T \qquad (19)$$

$$Y_t \le D_t \quad , \qquad t = 1,2,\ldots,T \qquad (20)$$

$$U_t \le M\ Z_t \quad , \qquad t = 1,2,\ldots,T \qquad (21)$$

$$\sum_{i=1}^{t} U_i \ge D_t - Y_t \ , \qquad t = 1,2,\ldots,T \qquad (22)$$

$$\sum_{t=1}^{T} Z_t \le 1 \qquad (23)$$

$$X \ge 0 \ , \ Y_t \ge 0 \ , U_t \ge 0 \ , \qquad t = 1,2,\ldots,T \qquad (24)$$

$$Z_t = 0 \text{ or } 1 \qquad t = 1,2,\ldots,T \qquad (25)$$

Constraint sets (11), (13), (14) and (15) in formulation 1 are replaced by constraint sets (22) and (23). These ensure that the public warehose space is leased utmost once in tha planning period.

MULTIPLE LEASES

Assume that there are K public warehouses from which one can lease warehouse space. Let C_p^k stand for the variable public warehousing cost per ft^2 of storage per period for public warehouse k, k = 1,2,...,K. In any period t, let U_t^k be the amount of space leased in public warehouse k. The lease will be valid from period t through T. It is assumed that space in a specific warehouse can be leased utmost once in a planning period.

Let Z_t^k be the binary variable taking values 1 or 0 depending on whether space in public warehouse k is leased or not leased in time period t.

The integer programming formulation is as follows:

$$Minimize\ EC = \sum_{t=1}^{T} [C_0 X + C_v Y_t + (\sum_{k=1}^{K} C_p^k (T-t+1) U_t^k)] \tag{26}$$

subject to:

$$Y_t \le f.X\ , \quad t=1,2,\ldots,T \tag{27}$$

$$Y_t \le D_t\ , \quad t=1,2,\ldots,T \tag{28}$$

$$\sum_{i=1}^{t} \sum_{k=1}^{K} U_i^k \ge D_t - Y_t\ , \quad t=1,2,\ldots,T \tag{29}$$

$$U_t^k \le MZ_t^k\ , \qquad t=1,2,\ldots,T \qquad k=1,2,\ldots,K \tag{30}$$

$$\sum_{t=1}^{T} Z_t^k \le 1\ , \qquad k=1,2,\ldots,K \tag{31}$$

$$X \ge 0,\ Y_t \ge 0,\ U_t^k \ge 0, \quad t=1,2,\ldots,T \quad k=1,2,\ldots,K \tag{32}$$

$$Z_t^k = 0\ or\ 1\ , \qquad t=1,2,\ldots,T \qquad k=1,2,\ldots,K \tag{33}$$

Constraint set (29) ensures that the total amount of space leased in all public warehouses is atleast as much as the requirement.

5. **SOLUTION PROCEDURE**

The number ofconstraints in the formulation is large and large scale mixed integer programming algorithms have to be used to arrive at a solution. Considering the structure of constraints, it may be possible to arrive at a solution using some heuristic methods. It is observed that in the case of single lease, the optimal value of X will be either 0 or equal to one of D_t. We can make use of this to arrive at a simpler method of arriving at the optimal solution. Different solution methods are being tried and the results will be reported seperately.

6. REFERENCES

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