WORKING PAPER No.208

Joint Life Insurance Policies with Differential Benefits And Premiums to the Policyholders

by

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June 2003

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Abstract

In this paper actuarial justification is explored in details for equal or unequal sharing of premiums and benefits between policyholders in a product involving joint lives. The analysis reveals a fundamental difference between endowment and assurance type of products in this regard. In assurance plans, there is a clear basis for differential structure in terms of sharing premium payment that is illustrated with examples. In pure endowment plans, the default system of equal premium for equal benefit may be more justified although implication of other alternatives are also considered. A justification is derived for such an alternative through an appropriate discount figures as compared to the individual live policies. An alternative actuarial principle is also suggested to deal with joint endowment plan and solutions have been worked out under this framework.

Keywords: Actuarial, assurance, discount, endowment, EPV, independent, sharing.

1 Introduction

Joint Life insurance policies are becoming increasingly popular because of their wider applicability to cover risk from different sources. The traditional policies covering the lives of couples continue to be important, but the concern of sharing premiums and benefit is relatively unimportant in such cases. However, the relative importance of the joint policies involving business partners and family members from different generations has increased over the years and in these circumstances, the issue of sharing the premium and benefit among the policy members can be critical. The fairness in equal sharing is critically examined in this work.

Consider, for example, a joint life policy for two business partners, one at age 30 and the other at age 55. Is it appropriate to require that the two partners share the premium(s) or benefit equally? After all, they bring in different quantum of risk to the table! A naïve view might be to dismiss such a question at the outset on the ground that the policyholders having agreed to the contract of joint lives do not have any right to individualistic concerns. We do not subscribe to that view; instead we inspect the fairness from individual perspective also. Towards that goal we study different kinds of joint-life policies, mainly the endowment as well assurance plans.

In assurance plans involving joint lives, there is a natural way sharing the premium on the basis of their expected benefit or payoff from this joint assurance policy. The details is shown in Section 2 along with illustrative tables for different pairs of ages for the case with two policyholders.

It is fairly obvious intuitively, as well as from an actuarial perspective, that the fair price from individual and joint consideration cannot be consistent. In (pure) endowment type policies, it is quite natural to talk about possible differential share of premiums and benefits by the individuals insured. Differential benefit sharing is also considered as a balancing tool to take into account of the difference in risk contribution of the individiduals and these various possibilities are worked out in Section 3.1. It is however shown that irrespective of whether the sharing is equal or not, one can obtain an interesting perspective by comparing the joint and the individual plan. In particular, the premium obtained by the fundamental actuarial principle from the joint consideration is a discounted amount from the same from individual policy. The fairness and interpretation of this discount is discussed in details in Section 3.2, keeping a focus on the relative risk factors brought in by the different policyholders. The work has natural extension for joint endowment plans involving multiple lives and this is taken up in Section 3.3. The above treatment takes place under the paradigm of fundamental actuarial principle whereby expected presented value of benefits to either individuals (and as a group) are taken as a basic cost or premium of the policy. However, specially in pure endowment policies that seems to bypass the basic question that we have attempted to address in this work. Consequently we also work under an alternative framework or actuarial principle and draw a comparison in Section 3.4.

Finally in Section 4 we deal with miscellaneous issues starting with discussion of joint endowment assurance type products in Section 4.1. We briefly touch upon the impact of departures from the critical assumptions made for this study (notably in the form of different mortality tables and the case with dependent lives) in Section 4.2. Finally the concluding Section 4.3 presents a summary of this work along with possibilities of carrying this research forward in various directions.

Assumptions and Notations:

In cost based pricing of insurance products, the expected present value of future benefits is usually taken to be the leading component in determining the premium of a life insurance product. To simply the scenario, the considerations of profit, and all other costs like administrative costs, defaulters etc. have been excluded in this work. Thus, cost (total premium to be paid) and the expected present value (EPV) of benefits from the plan are identical and hence used interchangeably in this manuscript.

We have also assumed that the concerned lives are independent and governed by the same mortality table. We also assume that the payments will be made immediately at the moment of death for assurance products. The impacts of the variations from these assumptions vary and have been briefly discussed in the concluding section.

Let P be the total premium to be paid. The objective of this research is to study how this amount P should be shared among two policyholders. For the ease of understanding, we would focus on the case with policies with two individuals X and Y, respectively at ages x and y, and then extend to the case with multiple lives. Let us denote these premium amounts by P_x and P_y respectively; naturally

$$P = P_x + P_y. \tag{1}$$

Let us denote the future lives of the two individuals by T_x and T_y respectively, with the future life of the joint status being denoted by $T_{xy} = \min(T_x, T_y)$. Unless otherwise stated, we will take the sum assured of the policy to be unit. We also assume a constant rate of interest δ, and denote v^n as the present value of unit amount payable at time n. For continuous compounded interest, v^n would be equal to $\exp(-\delta n)$. In the case of annual compounding $v = (1+\delta)^{-1}$, which is more prevalent in Indian markets; however, for numerical illustration, we have taken $v = \exp(-\delta)$.

2 Assurance plans for Joint life

Consider a (whole life) joint assurance plan of unit amount for the two individuals with the amount being payable immediately at the failure of the joint status. The expected present value of the benefit from the plan is given by: (see for example: [1])

$$\bar{A}_{xy} = \boldsymbol{E}(v^{T_{xy}}) = \int_0^\infty v^t \cdot {}_tp_{xy} \cdot \mu_{x+t:y+t}\, dt, \tag{2}$$

where $\mu_{x:y}$ is the force of mortality operating on the joint life status (at ages x and y). Under the assumption of independence of the two lives,

$$\mu_{x:y} = \mu_x + \mu_y. \tag{3}$$

Ignoring other costs and assuming no-profit situation as in the framework of this work, the worth of the policy is given by (2), and this can be computed by numerical integration. An approximate form of $\bar{A}_{xy}$, via Euler-MacLaurin formula, is given by:

$$1 - \delta(0.5 + \sum_{t=1}^{n} v^t \cdot {}_tp_{xy}). \tag{4}$$

Note that X receives the sum assured if and only if X survives longer than Y, i.e. $T_x > T_y$. Thus, the expected present value of the benefit to X is given by:

$$\bar{A}_{x\overset{1}{y}} = \boldsymbol{E}\left[v^{T_y}\mathbb{I}_{(T_x \geq T_y)}\right] = \int_0^\infty v^t \cdot {}_tp_{xy} \cdot \mu_{y+t}\, dt. \tag{5}$$

The derivation of the last equality in the above equation is fairly standard and is available in standard texts (viz. [1]). In the literature (5) is also referred to as *contingency assured.* The expected present value of amount receivable by Y is similarly given by

$$\bar{A}_{\overset{1}{x}y} = \int_0^\infty v^t \cdot {}_tp_{xy} \cdot \mu_{x+t}\, dt.$$

Using (3), it is easy to see that

$$\bar{A}_{xy} = \bar{A}_{x\overset{1}{y}} + \bar{A}_{\overset{1}{x}y}.$$

From an individual perspective, it seems only fair that the cost of the joint assurance policy should be shared among the two policyholders X and Y as per the present values of their individual returns or payoff from the policy, i.e. we propose

$$P_x = \bar{A}_{x\overset{1}{y}} \quad \text{and} \quad P_y = \bar{A}_{\overset{1}{x}y}. \tag{6}$$

In practice, the other costs (ignoring in the current framework) and profit contributions may be equally or proportionately shared. The Tables 1 and 2 provide the respective contributions or expected present values of the two policyholders at different ages (assuming a 5% interest rate) for a joint assurance plan of unit amount. Table 1 is valid when the younger member is at age 20, while Table 2 is for the same at age 30. The calculation is based on the Published Mortality Tables within the meaning of Regulation 4 of IRDA (Assets, Liabilities and Solvency margin of insurers) Regulations, 2000 (ultimate mortality table published by Life Insurance Corporation of India), for illustration. An EXCEL macro is used for the relevant computation and this can be used in similar situation with alterations in the problem parameter. The table should be interpreted as follows. Consider a joint (whole-life) assurance policy with sum assured of one lakh concerning two lives, one at age 20 and the other at age 25. Then, the present value of benefits to the two lives are respectively, 7560 and 5106; thus the total cost of the policy (ignoring other considerations, as usual) is 12666. Entries in the last column (e.g., 12637 in the above example) represent the corresponding total premium, as obtained by the Euler-MacLaurin approximation (4).

Table 1: Cost of joint assurance with 1 lakh sum assured

Age of the younger member = 20; interest rate = 5%

	Cost to			
Age-old	Old	Young	Total	Total_EM
20	5726	5726	11452	11424
25	5106	7560	12666	12637
35	3827	12849	16676	16647
45	2720	20888	23608	23579
60	1554	37985	39539	39516
75	816	61557	62373	62391

Table 2: Cost of joint assurance with 1 lakh sum assured

Age of the younger member = 30; interest rate = 5%

	Cost to			
Age-old	Old	Young	Total	Total_EM
30	8264	8264	16528	16496
35	7226	11235	18461	18429
45	5102	19571	24673	24642
60	2615	37281	39896	39871
75	1118	61329	62447	62465

The following observations can be made from the Tables 1 and 2; these are also intuitively obvious from an actuarial angle.

- If the age of the younger partner is held constant and that of the older partner keeps on increasing (move down the second column in either table), the value to the older partner decreases, while the value to the younger partner increases. This is because the younger policyholder becomes progressively more likely to draw the benefit from the plan.

- The (total present) value of the assurance product increases with increase in age of either of the partners because the policy payout draws progressively nearer to the issue of the policy.

- The Euler Maclaurin approximation seems to be working reasonably well, on the basis of the computations we have performed. However, this seems to consistently underestimating the actual value, except when one of the partners are very old (look at the entries corresponding to old age 75). It must be pointed out though that the calculation in other columns are also approximate values since they are obtained on the basis of numerical integration technique. Also, the force of mortality at the integral ages have been estimated from the life table as

$$\mu(x) = \frac{d_x + d_{x-1}}{2l_x} \tag{7}$$

Assurance plan with multiple Lives

The above logic and treatment has a natural extension when the joint assurance policy involve multiple (more than two) lives. For example, if we consider a third individual Z with age z, then the EPV of the total benefit from the plan (cost of the policy) is given by

$$\bar{A}_{xyz} = E(v^{T_{xyz}}) = \int_0^\infty v^t \cdot {}_tp_{xyz} \cdot \mu_{x+t:y+t:z+t}\, dt \tag{8}$$

As a natural extension to (3), one gets

$$\mu_{x:y:z} = \mu_x + \mu_y + \mu_z,$$

which implies that

$$\bar{A}_{xyz} = \bar{A}_{\overset{1}{x}yz} + \bar{A}_{x\overset{1}{y}z} + \bar{A}_{xy\overset{1}{z}}, \tag{9}$$

where

$$\begin{aligned}
\bar{A}_{\overset{1}{x}yz} &= E(v^{T_{xyz}} | T_x = T_{xyz}) = \int_0^\infty v^t \cdot {}_tp_{xyz} \cdot \mu_{x+t}\, dt, \\
\bar{A}_{x\overset{1}{y}z} &= E(v^{T_{xyz}} | T_y = T_{xyz}) = \int_0^\infty v^t \cdot {}_tp_{xyz} \cdot \mu_{y+t}\, dt \\
\bar{A}_{xy\overset{1}{z}} &= E(v^{T_{xyz}} | T_z = T_{xyz}) = \int_0^\infty v^t \cdot {}_tp_{xyz} \cdot \mu_{z+t}\, dt
\end{aligned}$$

are the expected present values of benefits respectively, when X,Y and Z are the first to die. Now assuming that the benefits are shared equally among the surviving members, it is clear that the EPV of benefit to X is given by $0.5(\bar{A}_{x\overset{1}{y}z} + \bar{A}_{xy\overset{1}{z}})$, which should in turn, determine the share of premium to be paid by X. Thus, the premium for the joint assurance of three lives may be shared as:

$$
\begin{aligned}
P_x &= \frac{\bar{A}_{x\overset{1}{y}z} + \bar{A}_{xy\overset{1}{z}}}{2} = \frac{\bar{A}_{xyz} - \bar{A}_{\overset{1}{x}yz}}{2} \\
P_y &= \frac{\bar{A}_{\overset{1}{x}yz} + \bar{A}_{xy\overset{1}{z}}}{2} = \frac{\bar{A}_{xyz} - \bar{A}_{x\overset{1}{y}z}}{2} \\
P_z &= \frac{\bar{A}_{\overset{1}{x}yz} + \bar{A}_{x\overset{1}{y}z}}{2} = \frac{\bar{A}_{xyz} - \bar{A}_{xy\overset{1}{z}}}{2},
\end{aligned}
$$

where the last identities follow from (9) and very amenable to extension for policies involving more than three lives.

Let us now take up an illustration for the case with 3 lives. Consider the whole-life joint assurance policy (sum assured = INR one lakh) for 3 business partners at ages $x = 25$, $y = 45$ and $z = 60$ respectively. Then computation similar to the illustration earlier (via a generalization of the earlier program routine), we get

$$\bar{A}_{\overset{1}{x}yz} = 1743, \quad \bar{A}_{x\overset{1}{y}z} = 8339, \quad \text{and} \quad \bar{A}_{xy\overset{1}{z}} = 32854.$$

These, in turn, should determine the share of premiums to be paid, (ignoring fraction of a INR) by the three partners, viz.

$$P_x = \frac{8339 + 32854}{2} = 20597, \quad P_y = \frac{1743 + 32854}{2} = 17298, \quad P_z = \frac{1743 + 8339}{2} = 5041.$$

Joint Term Assurance plans:

Along the same line, one can compute the relevant expected present values and ensuing share of premiums for a n-year term assurance plan where an insured is eligible to receive the sum assured only if the first death among the policyholders occurs within the next n years. The adjustments in the computation would be in the forms of replacing the upper limit (namely ∞) of the integrals in (2) and (5) by n. Appendix B contains selected tables in connection with this.

3 Pure Endowment policy for Joint life

Let us next consider the pure endowment policy for the joint life for a period of n years. For simplicity, we will consider the endowment to be of unit amount, unless otherwise specified. Let the amount to be received upon joint survival be B_x and B_y respectively, for X and Y; thus

$$B_x + B_y = 1. \tag{10}$$

Equating the EPV in the n-year joint endowment plan to the premium, we get

$$P = {}_np_{xy} \cdot v^n, \tag{11}$$

where ${}_np_{xy}$ stands for the joint survival probability of the two lives for another n years. Note that ${}_np_{xy} = {}_np_x \cdot {}_np_y$ under the assumption of independence of the two lives, and this is the probability at force for either X to Y to receive the endowment benefit. Obviously, the contribution of X and Y to this survival probability is not necessarily the same (and, accordingly, the benefit or the premium may be unequally shared). We will refer this principle (of equating present value of individual benefit to premium on the basis of aggregate risk) as **fundamental actuarial principle**. Later on the next section, we consider working under an alternative actuarial principle for dealing with joint endowment policy.

3.1 Fundamental Actuarial Principle

With reference to the joint pure endowment plan, we would refer to the principle as the 'Fundamental actuarial Principle for pure Endowment Plan involving Joint lives' or FPEJ. Applying it to the individual benefits, we get

$$P_x = B_x \cdot {}_np_{xy} \cdot v^n, \qquad P_y = B_y \cdot {}_np_{xy} \cdot v^n. \tag{12}$$

The left-hand sides of the two equations in (12) represent the present values of the amounts to be received by X and Y.

The default and the most popular choice of sharing the endowment would be that of equal sharing, i.e.

$$B_x = B_y = 0.5; \tag{13}$$

the above condition to be abbreviated as ES.

An alternative proposal is to divide the endowment in inverse proportion to the risk they bring in. Taking the survival probabilities, namely ${}_np_x$ and ${}_np_y$, as the driving force for risk, we may enforce

$$B_x : B_y = {}_np_x : {}_np_y; \tag{14}$$

this condition will be referred to as 'proportion to survival probability' or PSP, in short.

The condition of ES or PSP or any other alternative form of sharing the unit endowment amount can be expressed under the uniform platform by introduction the *ratio index of benefit sharing*. Let us define the ratio index of benefit sharing $b_{x:y}$ as

$$b_{x:y} = \frac{B_x}{B_y}. \tag{15}$$

Thus, under ES, $b_{x:y}$ is equal to 1, while for PSP, the ratio index is

$$\frac{{}_np_x}{{}_np_y}.$$

Yet another possibility is to enforce inverse ratio to death probability (IRDP), i.e.

$$b_{x:y} = \frac{{}_nq_y}{{}_nq_x}. \tag{16}$$

Note that by (10), the individual benefit shares are uniquely defined from the ratio index of benefit sharing $b_{x:y}$; explicitly, this means:

$$B_x = \frac{b_{x:y}}{1 + b_{x:y}}, \qquad B_y = \frac{1}{1 + b_{x:y}}. \tag{17}$$

Under the fundamental actuarial principle, we get (12), which implies $\frac{P_x}{P_y} = b_{x:y}$ and consequently the result follows trivially.

Result 1 *If the benefits for a n-year pure endowment of joint lives (one at age x and the other at y) are to be shared at a decided ratio of $b_{x:y} = b$, then the fair contribution of the premiums from the two lives are given by:*

$$P_x = \frac{b}{1+b} \cdot {}_np_{xy} \cdot v^n, \qquad P_y = \frac{1}{1+b} \cdot {}_np_{xy} \cdot v^n. \tag{18}$$

Corollary 1 *Under ES (equal sharing of pure endowment benefits), the premiums should be shared equally too, i.e.*

$$P_x = P_y = 0.5 \cdot {}_np_{xy} \cdot v^n. \tag{19}$$

Corollary 2 *Under PSP (proportional benefit to survival probability), the share of premiums is given by*

$$P_x = \frac{({}_np_x)^2 {}_np_y}{{}_np_x + {}_np_y} \cdot v^n, \qquad P_y = \frac{{}_np_x ({}_np_y)^2}{{}_np_x + {}_np_y} \cdot v^n. \tag{20}$$

Corollary 3 *Under IRDP (inversely proportional benefit to death probability), the share of premiums is given by*

$$P_x = \frac{{}_np_x \; {}_np_y \; {}_nq_y}{{}_nq_x + {}_nq_y} \cdot v^n, \qquad P_y = \frac{{}_np_x \; {}_nq_x \; {}_np_y}{{}_nq_x + {}_nq_y} \cdot v^n. \tag{21}$$

As an illustration, we enclose below sharing of premiums under different benefit sharing for a pure joint endowment of INR 1 lakh for a period of 10 years when the first life (x) is at age 30 (${}_np_x = 0.9859$) (and the second varies from 20 to 75) considering a 5% rate of interest.

Table 3: Sharing Premiums under FPEJ

10 year pure endowment for Joint life: first life at age 30,

Sum assured = INR 1 lakh; Rate of interest = 5%

y=age2	${}_{10}p_y$	ES	PSP		IRDP	
25	0.988193	29547	29514	29581	26979	32115
30	0.985945	29480	29480	29480	29480	29480
35	0.979982	29302	29391	29213	34430	24173
40	0.968961	28972	29224	28721	39885	18060
45	0.949392	28387	28923	27851	44434	12340
50	0.919292	27487	28449	26525	46821	8153
55	0.875687	26183	27734	24633	47048	5319
60	0.807262	24137	26543	21732	44994	3281
65	0.692213	20697	24320	17075	39587	1808
70	0.526794	15751	20532	10970	30594	909
75	0.337158	10081	15024	5138	19744	419

3.2 Fairness in Share: An Interpretation through *Discount* in Premium

In search for fairness of premium and the benefit to be shared by the policyholders, we look from an individualistic perspective. For an individual endowment policy of n years, the premiums they would have to pay (to cover the risk for their respective lives life) are given by:

$$P_x^0 = v^n \cdot B_x \cdot {}_np_x, \qquad \text{and} \qquad P_y^0 = v^n \cdot B_y \cdot {}_np_y, \tag{22}$$

which are smaller in magnitude than the premiums that they are required to pay for the joint endowment policy, namely P_x and P_y, as given by (12). Of course, the difference in premiums (between the joint policy and the individual) is justified considering the relative risk factors. However it may be instructive to focus on the *discount* that one would receive by going for a joint policy, as compared to the individual plan and this discount turns out to be fair on considerations of the relative risk.

To elaborate, the 'discount' X receives in going for the joint policy is given by

$$D_x = 1 - \frac{P_x}{P_x^0} = 1 - \frac{{}_np_{xy}}{{}_np_x} \overset{*}{=} 1 - {}_np_y = {}_nq_y,$$

where the equality with $*$ follows from independence of the two lives. Similarly the discount for Y is given by:

$$D_y = 1 - {}_np_x = {}_nq_x.$$

Thus, if $x < y$, we have (for matured X and Y) ${}_np_x > {}_np_y \Leftrightarrow {}_nq_x < {}_nq_y$, and thus the discount for the younger policy holder is proportionately higher.

3.3 Endowment Policies Involving Multiple Independent Lives

The above concept of discount and associated explanation can now easily be extended to an endowment policy involving m independent lives currently at ages $x_1, \ldots, x_m$. Extending the introduced notations to denote the benefit by B_i and share of premium by P_i, for the i-th person, with $i = 1, \ldots, m$, we have

$$P_i = v^n B_i \, {}_np_{\mathbf{x}}, \quad i = 1, \ldots, m.$$

Whereas the fair premium (with corresponding endowment) for the individual life policy of i-member is

$$P_i^0 = v^n B_i \, {}_np_{x_i}, \quad i = 1, \ldots, m,$$

and hence the discount factor for the i-th person is given by

$$D_i = 1 - \frac{P_i}{P_i^0} = 1 - \frac{{}_np_{\mathbf{x}}}{{}_np_{x_i}} = 1 - \prod_{j \neq i} {}_np_{x_j},$$

with the last identity following from the independence of the lives concerned. Note that, as in the case of joint life two individuals at matured ages, the 'discount' decreases with age, i.e.

$$x_i > x_j \Leftrightarrow {}_np_{x_i} < {}_np_{x_j} \Leftrightarrow D_i < D_j.$$

3.4 Alternate Actuarial Principle for Pure Endowment Plans involving Joint Lives (APEJ)

In some sense, the argument presented in Section 3.2 and 3.3 does not fully address the individual concern and bypass the question which led to inspecting the sharing issue. (12) implies that unless Partner 1 is prepared to sacrifice his benefit amount, in spite of being younger (assuming $x < y$), he would not get to pay any less premium as compared to Partner 2, although he may be bringing a lot less risk in the endowment plan on account of being younger. This is a direct fall out of the FPEJ. A possibility is to violate (12) by modifying it to

$$P_x = B_x \cdot R_x \cdot v^n, \qquad \text{and} \qquad P_y = B_y \cdot R_y \cdot v^n, \tag{23}$$

where R_x and R_y represent the risk factors brought in by X and Y respectively. Since probability of surviving the period n is inversely related to the risk contribution, one natural alternative is to require:

$$\frac{R_x}{R_y} = \frac{{}_np_y}{{}_np_x}. \tag{24}$$

However, note that, the fairness of costing the policy (ignoring other costs and profit considerations), demands that

$$P_x + P_y = {}_np_x \cdot {}_np_y \cdot v^n, \tag{25}$$

which is a restatement of (11) (and satisfied automatically under FPEJ following (12)).

Solving (23), (24) and (25) simultaneously, we get

$$P_x = \frac{B_x \, {}_np_x({}_np_y)^2}{{}_np_x B_y + \, {}_np_y B_x} \cdot v^n, \qquad P_y = \frac{B_y({}_np_x)^2 \, {}_np_y}{{}_np_x B_y + \, {}_np_y B_x} \cdot v^n. \tag{26}$$

Note the similarity of the above solution with (20). Indeed, if the benefits are to be equally shared then APEJ demands exactly the reverse sharing of premiums as determined by FPEJ with proportional sharing of benefits. This follows algebraically as well from a intuitive stand point. The Table 4 below shows an example of sharing of premiums under APEJ. For the sake of comparing with the scenario under FPEJ, again we consider a 10 year endowment with the first life being at age 30. Note that under APEJ, the premiums will be equally shared among the policyholders only with PSP.

Table 4: Sharing Premiums under APEJ

10 year pure endowment for Joint life: first life at age 30,

Sum assured = INR 1 lakh; Rate of interest = 5%

y=age2	ES		PSP	IRDP	
	P_x	P_y	$P_x = P_y$	P_x	P_y
25	29581	29514	29547	27013	32082
30	29480	29480	29480	29480	29480
35	29213	29391	29302	34344	24259
40	28721	29224	28972	39668	18276
45	27851	28923	28387	44065	12709
50	26525	28449	27487	46323	8652
55	24633	27734	26183	46453	5913
60	21732	26543	24137	44327	3948
65	17075	24320	20697	38867	2528
70	10970	20532	15751	29844	1659
75	5138	15024	10081	18985	1177

4 Concluding Remarks and Summary

4.1 Endowment Assurance plan

In endowment assurance plans, as it is customary, the share of the premiums should be separately calculated for the pure endowment part and added to the same for the joint assurance plan. Of course, under standard of FPEJ and equal sharing of benefits, the premiums will be equal for the pure endowment part and the differences will be entirely due to the possible claim in case of death within the policy period due to joint assurance part. We will illustrate it with a 10 year endowment assurance plan for two lives one at age 30, the other at age 45 with the sum assured (payable to the living individual when the other dies within the next 10 years) being 10 lakh, while the endowment amount being Rs. 1 lakh which would be paid if both the policyholder survive the next 10 years. We assume a 5% rate of interest and equal sharing of endowment. The following table summarizes the calculation:

age	endowment part		assurance part	total	
	FPEJ	APEJ		FPEJ	APEJ
30	28387	27851	31602	59989	59453
45	28387	28923	9794	38181	38717

4.2 Impact of relaxing Assumptions

The assumption of the lives being governed by *identical mortality table* is hardly critical. All the results will continue to be valid except that the formulae would appear to be more cumbersome otherwise.

The assumption of the lives being *independent* is however fairly critical. We plan to carry forward this work in case of dependent lives, possibly governed by the common hazard model.

In case of assurance plans, if the payment is made out at the end of the year of death as opposed to the assumed case of instantaneous payment, the analysis presented here can be extended on standard route.

4.3 Summary

In analyzing the sharing premium policy of joint insurance products, we found that the fairness in equal sharing depends critically on the type of product. In assurance plans, for either the whole-lives or for a limited duration, there seems to be a strong justification for unequal sharing. However, for endowment plans, equal sharing seemed to be more justified under traditional actuarial setup. In that case, a comparison with premium for individual policies reveals a rational discount in terms of premium to be paid. However, there may be some rationale behind individualistic thinking and for that an alternative actuarial principle is formulated. The applicability of this work depends on growing popularity of joint live products involving business partners or distant relations, where the default choice of equal sharing may be called into question. The work may be extended through relaxing assumptions as mentioned in the previous subsection.

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A Selected Tables for EPV of Individual Benefits in (Whole Life) Joint Life Assurance plans

Table No A1: Individual EPV in Whole Life Joint Assurance Plan

Sum assured = 1; Rate of Interest = 5%

Age of the	Age of the primary life						
Second life	20	25	30	35	45	60	75
20	0.05726	0.05106	0.04458	0.03827	0.02720	0.01554	0.00816
25	0.07560	0.06859	0.06059	0.05220	0.03659	0.01949	0.00934
30	0.09889	0.09167	0.08264	0.07226	0.05102	0.02615	0.01118
35	0.12849	0.12164	0.11235	0.10064	0.07343	0.03798	0.01541
45	0.20888	0.20359	0.19571	0.18427	0.14954	0.08549	0.03665
60	0.37985	0.37724	0.37281	0.36534	0.33698	0.24632	0.11981
75	0.61557	0.61470	0.61329	0.61011	0.59461	0.53467	0.36566

Table No A2: Individual EPV in Whole Life Joint Assurance Plan

Sum assured = 1; Rate of Interest = 7%

Age of the	Age of the primary life						
Second life	20	25	30	35	45	60	75
20	0.03024	0.02817	0.02586	0.02344	0.01861	0.01235	0.00719
25	0.03984	0.03738	0.03443	0.03113	0.02427	0.01510	0.00817
30	0.05304	0.05036	0.04687	0.04264	0.03313	0.01974	0.00962
35	0.07172	0.06900	0.06521	0.06021	0.04764	0.02836	0.01306
45	0.13027	0.12781	0.12411	0.11859	0.10100	0.06471	0.03108
60	0.27801	0.27650	0.27390	0.26939	0.25187	0.19363	0.10270
75	0.52320	0.52256	0.52156	0.51925	0.50761	0.46214	0.32723

B Selected Tables for EPV of Individual Benefits in Fixed Term Joint Life Assurance plans

Table No A3: Period for Joint Term assurance = 10 years

Sum assured = 1; Rate of Interest = 5%

Age of the	Age of the primary life						
Second life	20	25	30	35	45	60	75
20	0.00819	0.00819	0.00819	0.00817	0.00810	0.00775	0.00630
25	0.00895	0.00894	0.00894	0.00893	0.00885	0.00848	0.00694
30	0.00991	0.00991	0.00990	0.00989	0.00979	0.00937	0.00758
35	0.01306	0.01305	0.01305	0.01303	0.01289	0.01229	0.00976
45	0.03164	0.03162	0.03160	0.03155	0.03120	0.02967	0.02325
60	0.10467	0.10461	0.10457	0.10440	0.10333	0.09861	0.07880
75	0.39648	0.39626	0.39611	0.39556	0.39182	0.37512	0.30477

Table No A4: Period for Joint Term assurance = 10 years

Sum assured = 1; Rate of Interest = 7%

Age of the	Age of the primary life						
Second life	20	25	30	35	45	60	75
20	0.00746	0.00745	0.00745	0.00744	0.00737	0.00707	0.00580
25	0.00817	0.00816	0.00816	0.00815	0.00808	0.00776	0.00640
30	0.00901	0.00901	0.00901	0.00899	0.00891	0.00854	0.00697
35	0.01179	0.01179	0.01178	0.01176	0.01164	0.01112	0.00892
45	0.02843	0.02841	0.02840	0.02835	0.02805	0.02672	0.02113
60	0.09472	0.09466	0.09463	0.09449	0.09355	0.08944	0.07215
75	0.36090	0.36070	0.36058	0.36010	0.35683	0.34216	0.28039

Table No A5: Period for Joint Term assurance = 20 years

Sum assured = 1; Rate of Interest = 5%

Age of the Second life	Age of the primary life 20	25	30	35	45	60	75
20	0.01407	0.01406	0.01402	0.01395	0.01357	0.01213	0.00792
25	0.01669	0.01666	0.01662	0.01652	0.01603	0.01416	0.00897
30	0.02147	0.02144	0.02137	0.02123	0.02052	0.01782	0.01057
35	0.03170	0.03164	0.03154	0.03130	0.03015	0.02583	0.01447
45	0.07497	0.07485	0.07462	0.07410	0.07150	0.06174	0.03492
60	0.24333	0.24293	0.24218	0.24050	0.23212	0.20064	0.11538
75	0.58606	0.58547	0.58456	0.58228	0.56985	0.52150	0.36406

Table No A6: Period for Joint Term assurance = 20 years

Sum assured = 1; Rate of Interest = 7%

Age of the Second life	Age of the primary life 20	25	30	35	45	60	75
20	0.01184	0.01182	0.01180	0.01174	0.01146	0.01035	0.00704
25	0.01389	0.01387	0.01384	0.01377	0.01340	0.01198	0.00794
30	0.01754	0.01752	0.01747	0.01737	0.01683	0.01482	0.00924
35	0.02550	0.02546	0.02539	0.02522	0.02436	0.02115	0.01247
45	0.06058	0.06049	0.06033	0.05994	0.05800	0.05067	0.03000
60	0.19733	0.19703	0.19648	0.19525	0.18900	0.16544	0.09988
75	0.50462	0.50415	0.50346	0.50171	0.49197	0.45374	0.32620

Table No A7: Period for Joint Term assurance = 30 years

Sum assured = 1; Rate of Interest = 5%

Age of the	Age of the primary life						
Second life	20	25	30	35	45	60	75
20	0.02088	0.02078	0.02060	0.02027	0.01906	0.01462	0.00815
25	0.02766	0.02750	0.02720	0.02670	0.02485	0.01810	0.00932
30	0.03926	0.03901	0.03854	0.03775	0.03486	0.02430	0.01116
35	0.05707	0.05672	0.05606	0.05492	0.05074	0.03537	0.01538
45	0.12570	0.12496	0.12357	0.12118	0.11235	0.08015	0.03658
60	0.34902	0.34748	0.34461	0.33933	0.31901	0.24294	0.11976
75	0.61467	0.61383	0.61245	0.60933	0.59403	0.53454	0.36566

Table No A8: Period for Joint Term assurance = 30 years

Sum assured = 1; Rate of Interest = 7%

Age of the	Age of the primary life						
Second life	20	25	30	35	45	60	75
20	0.01595	0.01589	0.01577	0.01557	0.01479	0.01188	0.00718
25	0.02050	0.02040	0.02022	0.01990	0.01873	0.01440	0.00816
30	0.02829	0.02813	0.02785	0.02735	0.02552	0.01880	0.00961
35	0.04092	0.04071	0.04030	0.03958	0.03692	0.02705	0.01304
45	0.09120	0.09074	0.08989	0.08839	0.08274	0.06197	0.03105
60	0.26235	0.26137	0.25955	0.25613	0.24263	0.19186	0.10267
75	0.52273	0.52210	0.52112	0.51884	0.50730	0.46207	0.32723

C Program Codes for Excel Macro to compute Contingent assurance value

```
Sub Calculate()

'Taking input values
Dim age1 As Integer, age2 As Integer, age3 As Integer, delta As Double
Dim ul As Integer, px(120) As Double, py(120) As Double, pz(120) As Double
Dim mx(120) As Double, my(120) As Double, mz(120) As Double
Dim x As Integer, y As Integer, z As Integer, t As Integer, i As Integer
Dim Fn1(120) As Double, Fn2(120) As Double, Fn3(120) As Double
Dim fsum1 As Double, fsum2 As Double, fsum3 As Double
Dim k As Integer, val1 As Double, val2 As Double, val3 As Double
Dim respon As String, fmess As Variant, dummy As Variant
Dim ir1 As Integer, ir2 As Integer, ir3 As Integer, ir4 As Integer
'start row for input values
ir4 = 9
ir1 = 3
ir2 = 5
ir3 = 7

Worksheets("Simpson").Activate

dummy = checkval()
' assigning values to x = min and y = max

x = Application.WorksheetFunction.Min(Cells(ir1, 3), Cells(ir2, 3), Cells(ir3, 3))
y = Application.WorksheetFunction.Max(Cells(ir1, 3), Cells(ir2, 3), Cells(ir3, 3))

age1 = Cells(ir1, 3)
age2 = Cells(ir2, 3)
age3 = Cells(ir3, 3)
z = age1 + age2 + age3 - x - y
'*************************************************
delta = Cells(ir4, 3)

ul = ulimit(y)
Worksheets("table").Activate
Cells(2, 7) = ul
Fn1(0) = mu(age1)
fsum1 = Fn1(0)
Fn2(0) = mu(age2)
fsum2 = Fn2(0)
Fn3(0) = mu(age3)
fsum3 = Fn3(0)
'*************************************************
' first loop from lower integral limit to upper limit
For t = 1 To ul Step 1

val1 = 1
val2 = 1
val3 = 1
' second loop for computing pxt pyt
   For i = 1 To t Step 1
     val1 = val1 * (1 - tabval(age1 + i - 1))
```

```
     val2 = val2 * (1 - tabval(age2 + i - 1))
     val3 = val3 * (1 - tabval(age3 + i - 1))
     Next
px(t) = val1
py(t) = val2
pz(t) = val3
'*********************************************
mx(t) = mu(age1 + t)
my(t) = mu(age2 + t)
mz(t) = mu(age3 + t)
k = findk(t, ul)

Fn1(t) = Exp(-delta * t) * px(t) * py(t) * pz(t) * mx(t) * k
Fn2(t) = Exp(-delta * t) * px(t) * py(t) * pz(t) * my(t) * k
Fn3(t) = Exp(-delta * t) * px(t) * py(t) * pz(t) * mz(t) * k

fsum1 = fsum1 + Fn1(t)
fsum2 = fsum2 + Fn2(t)
fsum3 = fsum3 + Fn3(t)
Next

fsum1 = fsum1 / 3
fsum2 = fsum2 / 3
fsum3 = fsum3 / 3
Worksheets("Simpson").Activate
Cells(12, 3) = fsum1
Cells(13, 3) = fsum2
Cells(11, 3) = fsum3

Range(Cells(21, 11), Cells(5000, 14)).ClearContents
For t = 1 To ul Step 1
Cells((20 + t), 11) = t
Cells((20 + t), 12) = px(t)
Cells((20 + t), 13) = py(t)
Cells((20 + t), 14) = pz(t)
Next
End Sub

Function ulimit(m As Integer) As Integer
Dim val As Integer
val = 118 - m
If (iseven(val) = 1) Then
ulimit = val
Else
ulimit = val - 1
End If
End Function

Function tabval(pt As Integer) As Double
Worksheets("table").Activate
tabval = Cells(pt - 18, 2)
End Function
Function mu(pt As Integer) As Double

Worksheets("table").Activate
mu = Cells(pt - 18, 5)
```

```
End Function

Function iseven(val As Integer) As Integer
If ((val Mod 2) = 0) Then
iseven = 1
Else
iseven = 0
End If
End Function

Function findk(val As Integer, mval As Integer) As Integer
If (val = mval) Then
findk = 1
ElseIf ((val Mod 2) = 0) Then
findk = 2
Else
findk = 4
End If
End Function

Function checkval() As Variant
Dim age1 As Integer, age2 As Integer, rat As Double
Dim mess1 As Variant, ratio As Variant
Dim ir1 As Integer, ir2 As Integer, ir3 As Integer
'start row for input values
ir1 = 5
ir2 = 7
ir4 = 3
ir3 = 9
Worksheets("Simpson").Activate
If (Cells(ir1, 3) = Empty) Then
age1 = InputBox("Please enter Age of Partner-1 ", "Enter Age in Whole Years", 20)
Cells(ir1, 3) = age1
Else
age1 = Cells(ir1, 3)
End If
If (Cells(ir2, 3) = Empty) Then
age2 = InputBox("Please enter Age of Partner-2 ", "Enter Age in Whole Years", 20)
Cells(ir2, 3) = age2
Else
age2 = Cells(ir2, 3)
End If
If (Cells(ir3, 3) = Empty) Then
age2 = InputBox("Please enter Age of Partner-3 ", "Enter Age in Whole Years", 20)
Cells(ir3, 3) = age3
Else
age3 = Cells(ir3, 3)
End If
ratio = Cells(ir4, 3)
If (IsEmpty(ratio) = True) Then
rat = InputBox("Please enter Interest rate in %age ", "Enter Interest rate", 4)
Cells(ir4, 3) = rat / 100
Else
rat = Cells(ir4, 3)
End If
```

Code for generating matrix of contingency assurances for 2 lives

```
Sub matrixcalc()
Dim siz As Integer, agex(100) As Integer, agey(100) As Integer, simp(100, 100) As Double
Dim rat As Double, i As Integer, j As Integer, ageval(100) As Integer, prmess(100) As String
Dim ir1 As Integer, ir2 As Integer, ir3 As Integer, dummy As Variant
' row numbers that conatin ages and interest rates
ir1 = 5
ir2 = 7
ir3 = 9

Worksheets("Simpson").Activate
siz = InputBox("Please enter the size of Matrix ", "No of Entries to be made (for Age)", 5)

For i = 1 To siz Step 1
prmess(i) = "Please enter the Age for person no:" + CStr(i)
ageval(i) = InputBox(prmess(i), "No of Entries to be made (for Age)", 20)
Next

rat = InputBox("Please enter Interest rate in %age ", "Enter Interest rate", 4)

'*************************** RUN cycle for each value *****************
For i = 1 To siz Step 1
Worksheets("Simpson").Activate
Cells(ir3, 3) = rat / 100
agex(i) = ageval(i)
Cells(ir1, 3) = agex(i)
   For j = 1 To siz Step 1
    Worksheets("Simpson").Activate
    agey(j) = ageval(j)
    Cells(ir2, 3) = agey(j)
    simp(i, j) = calc()
   Next
Next
'*************************** Writing the values ***************
Worksheets("Matrix").Activate
Range(Cells(4, 4), Cells(100, 100)).ClearFormats
Range(Cells(4, 5), Cells(100, 100)).ClearContents
Range(Cells(5, 4), Cells(100, 4)).ClearContents
For i = 1 To siz Step 1
   Cells(4 + i, 4) = agex(i)
   For j = 1 To siz Step 1
    Cells(4, 4 + j) = agey(j)
    Cells(4 + i, 4 + j) = simp(i, j)
   Next
Next
dummy = form(4, (siz + 4), 4, (4 + siz))

End Sub
Function form(sro As Integer, enr As Integer, scol As Integer, encol As Integer) As Variant
Dim horhead As Range, verhead As Range, valran As Range
Worksheets("Matrix").Activate
```

```
Set horhead = Range(Cells(sro, scol), Cells(sro, encol))
Set verhead = Range(Cells(sro, scol), Cells(enr, scol))
Set valran = Range(Cells(sro, scol), Cells(enr, encol))

horhead.Select
   Selection.Interior.ColorIndex = 9
   Selection.Font.Bold = True
   Selection.Font.ColorIndex = 2
verhead.Select
   Selection.Interior.ColorIndex = 9
   Selection.Font.Bold = True
   Selection.Font.ColorIndex = 2
'******************** Bordering *****************
    valran.Select
   Selection.Borders(xlDiagonalDown).LineStyle = xlNone
   Selection.Borders(xlDiagonalUp).LineStyle = xlNone
   With Selection.Borders(xlEdgeLeft)
      .LineStyle = xlContinuous
      .Weight = xlThin
      .ColorIndex = xlAutomatic
   End With
   With Selection.Borders(xlEdgeTop)
      .LineStyle = xlContinuous
      .Weight = xlThin
      .ColorIndex = xlAutomatic
   End With
   With Selection.Borders(xlEdgeBottom)
      .LineStyle = xlContinuous
      .Weight = xlThin
      .ColorIndex = xlAutomatic
   End With
   With Selection.Borders(xlEdgeRight)
      .LineStyle = xlContinuous
      .Weight = xlThin
      .ColorIndex = xlAutomatic
   End With
   With Selection.Borders(xlInsideVertical)
      .LineStyle = xlContinuous
      .Weight = xlThin
      .ColorIndex = xlAutomatic
   End With
   With Selection.Borders(xlInsideHorizontal)
      .LineStyle = xlContinuous
      .Weight = xlThin
      .ColorIndex = xlAutomatic
   End With
   Range("K19").Select

   End Function
```