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A Scheduling Model Incorporating Information on Daily Availability of Personnel

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Abstract: The process of scheduling involves allocation of off days and assignment of shifts on working days for each employee in a manner such that the demand, service rules and work preferences are met at a minimum cost. This paper presents a scheduling model for known demand and staff size that is fixed for the planning horizon but can vary from day to day due to planned leaves taken by the personnel. The model integrates prescribed service rules and information on daily availability of personnel during the scheduling period. The problem is formulated as a 0/1 goal-programming model where the objective is to minimize the violations of requested leaves in the planning period. The paper does not address the larger problem of workforce planning and budgeting. The model is tested using real data from a hospital and solutions are obtained in reasonable time.

Keywords: Personnel scheduling, integer goal programming, nurse scheduling.

Introduction

Personnel scheduling in hospitals, including the scheduling of nurses, has been an area of considerable research interest. Good schedules can help improve quality of service by providing better service levels with existing staff resources. The problem has received extensive attention as generating desirable schedules such that demands are met, service rules relating to off-days, work load and work stretches are satisfied and individual nurse preferences are taken into consideration lends it a great deal of complexity. Ernst et. al. (2004) gives an excellent bibliography of literature related to personnel scheduling and rostering including scheduling of nurses. Earlier literature surveys on nurse scheduling are given by Cheang (2003), Bradley and Martin (1991), and Sitompul and Randhawa (1990).

Most of the work on scheduling of nurses takes into consideration the demand to be met and the various service rules under which the hospital might operate. At times individual preferences of the staff are also incorporated. However, while generating schedules it is assumed that the daily availability of staff over the planning horizon is fixed though this is bound to change every planning period. Given that each person is allowed a number of leaves in a year, it becomes necessary that duty assignments every planning period take dynamic information on leave requests into consideration. This assumes greater importance when the number of persons to be scheduled is fixed. At the same time in such instances it is possible to elicit this information ahead of time and incorporate it in the scheduling process. If done manually it is bound to not only be a time consuming activity. Manual generation of schedules cannot evaluate all permutations and combinations of possible assignments and this often results in schedules that violate some of the constraints even though it might actually be possible to satisfy them. In addition, the workloads allocated may not be equitable. An automated procedure is likely to minimize such sub-optimal use of available resources. The objective of this paper is to develop a mathematical model for nurse scheduling that incorporates information on staff unavailability due to leave requests during the planning period. Möz and Vazpato (2004) is the only paper, to the author's knowledge, that addresses a related though different re-rostering problem.

The process of scheduling involves allocation of off days and assignment of shifts on working days for each employee in a manner such that the demand, service rules and work preferences are met at a minimum cost. At times only cyclic schedules for days of work are determined (Megeath (1989) and Rosenbloom and Goertzen (1987)) and assignment of shifts on these days is left to the user. When both days off and shift scheduling are handled the solution technique can be based on splitting the schedule into days off scheduling and shift scheduling phases as in Abdennadher and Schlenker (1999), Ahuja and Sheppard (1975), Smith and Wiggins (1977) and Smith et al. (1979). This approach is typically used when scheduling is cyclical implying that each person follows a work pattern that is repeated every 'n' periods. Mathematical formulations (Bailey (1985)) may be solved using heuristics or otherwise to generate off day patterns and purely heuristic approaches are generally used for allocating shifts on the assigned work days. Arthur and Ravindran (1981) present a goal programming approach to generate days of work and a heuristic to assign shifts. Berrada et al. (1996), Hurang (1999), Musa and Saxena (1984) and Ozkarahan and Bailey (1988) develop goal programming models that allow trade-offs between desirability and schedule requirements and staff preferences. Bell et al. (1986) and Randhawa and Sitompul (1993) present heuristics that combine days off and shift scheduling and Ozkarahan (1991) presents a set covering IP and heuristics. Jaumard et al. (1998) use a column generation approach to find good solutions for an integer programming formulation. Warner

(1972) formulate the problem as a mixed integer quadratic program. Millar and Kiragu (1998) use network programming to generate schedules. Aickelin (1999), Aickelin and Dowsland (2000) use a genetic algorithm approach. A Tabu search approach is used by Dowsland (1998) and Dowsland and Thompson (2000).

This paper integrates days off and shift scheduling with availability information during the planning period. We take into consideration the prescribed service rules, seniority level requirements and desirability criteria and also incorporate information on availability of personnel in any given planning period. Leave requests by nurses are taken to imply that the nurse is unavailable for that many days. The scheduling problem is formulated as a 0-1 integer-program where all service rules and desirability constraints are taken to be hard constraints except the availability constraints. The objective is to minimize the violations of requested leaves. ILOG CPLEX 8.0 is used to solve the problem. The larger problem of workforce planning and budgeting is not addressed in this paper.

1. Scheduling Environment

Currently staffing and scheduling of nurses in Indian hospitals is done manually to our knowledge. We use a typical hospital as an example to develop a model and generate schedules that meet the required demands and service rules. The service rules across hospitals are likely to be quite similar and the approach could be used with modified service rules if necessary. Preparing a schedule manually for the 600-bed hospital's entire nursing staff is difficult and unnecessary. Duty scheduling is, therefore, done on a ward basis and left to the head nurse of the ward with a given number of nurses assigned to the ward. This number can possibly change from time to time but is fixed in general. Manual scheduling is done on a monthly basis in this case and is undesirable in many respects. Some pertinent issues are:

- Preparing a manual schedule consumes a considerable amount of the head nurse's time and the process needs to be started much ahead of the month.
- Dynamic inputs such as availability of nurses need to be known much ahead of the planning period. Nurses are required to specify their desired leave days, if any, for a given month by the middle of the previous month. This leads to a considerable amount of inflexibility and is undesirable from the nurses' point of view.
- The resulting schedules are often sub-optimal and do not satisfy some of the duty and service requirements.
- The workloads generated are not equitable.

The duty requirements and restrictions considered for the scheduling model are:

- At least four nurses are required daily in each of the morning, evening and night shifts. A morning or an evening shift is of six hours duration while a night shift is of twelve hours duration. The shifts are taken as non-overlapping for scheduling purposes.
- Sixteen nurses are assigned to each ward and six of them, taken to be the first six listed, are senior nurses.
- At least one senior nurse is required in each shift each day.
- A nurse can be allocated at most one shift a day provided she has not requested for leave that day.
- An off day is to be necessarily given after six consecutive working days.

- For equitable distribution of work no nurse should be assigned more than 192 hours of work in a month.
- A nurse cannot be assigned more than eight night shifts in a month.
- A sequence of night shift assignments is to be followed by an off day.

The following desirable aspects for schedules are also taken into consideration:

- A night shift is not assigned before a leave day.
- A nurse should work for at least two consecutive days in a particular shift.

2. Model Description

There are sixteen nurses assigned to the ward who are to be scheduled as per the requirements listed above. Some objective functions that are typically considered in the literature are to minimize the number of nurses or minimize the number of over assignments. Alternatively in a goal programming approach violations of constraints that are specified as goals are minimized.

In this case the first objective does not serve much purpose as a quick calculation indicates that the number of nurses is just sufficient to satisfy the requirement of scheduling four nurses in each shift taking into cognizance the given constraints and the slack necessary for requested leaves. Minimizing the number of over assignments can be a possibility provided all other requirements can be met. However, there is no guarantee that such a schedule will always be possible since information pertaining to leave requests changes from month to month. Therefore, the constraint that there is no assignment of any shift to a nurse on the day that she requests for leave is taken to be a goal constraint and the objective function minimizes the sum of resulting violations, if any.

The decision variables defined for the model are:

x_{ijk} = 1 if a nurse 'i' is assigned on day 'j' to shift 'k' and 0 otherwise. Where i = index for nurses: 1,2, ..., number of nurses; j = index for days: 1,2, ..., number of days in the scheduling period and k = index for shifts 1,2,3.

o_{ijk} = 1, if a nurse 'i' is assigned for duty on day 'j' to shift 'k' when she has requested for a leave on that particular day and 0 otherwise.

Parameters used in the model are:

a_{ij} = 1 if a nurse 'i' is available for duty on day 'j' and 0 otherwise.

The complete formulation is given in Appendix 1a and 1b.

3. Solution Procedure

The model is tested using data obtained from the hospital pertaining to leave requests for ten months. Table 1 gives a sample of leave requests during a month. A scheduling period of one month gives rise to 1440 binary variables. In order to reduce the CPU time taken to solve the problem with this many variables we divide the problem into two fifteen-day periods. In the first step a solution is obtained for the first 15 days of each month. The schedule obtained is taken into account and a solution for the latter fifteen days of the month is obtained. In order to maintain continuity of the schedule over the month necessary information is passed on to the second fifteen day model. The parameters defined for this purpose are:

p_{idk} = 1 if a nurse 'i' was assigned for duty on day 'd' to shift 'k' in the previous 15 day period and 0 otherwise. Where d = index for previous 15 days: 1,2, ...15.

n_i = 1 if a nurse 'i' is available for duty on the first day in the forthcoming scheduling period and 0 otherwise.

The time taken to obtain schedules in this manner is under 5 minutes (CPU time) in most cases. However in two cases the constraint that the maximum number of nights should be eight is found to be restrictive due to unavailability of nurses. The model is, therefore, modified to allow this constraint to be violated by introducing the variable:

$onite_{ijk}$ = 1 if a nurse 'i' is assigned for a night shift on day 'j' in excess of the maximum eight night shifts in a month and 0 otherwise.

A penalty is imposed in the objective function to minimize such violations. This penalty is taken to be larger than the penalty for violating leave requests as fulfilling this constraint is considered to be more important. The modified model is given in Appendix 1c.

4. Results

The model is solved for 10 months with non-availabilities varying from 4 to 31. Table 2 lists the details regarding non-availabilities, CPU time taken and objective function value obtained. The model runs in reasonable time and finds schedules satisfying all hard constraints. The solution obtained is declared to be optimal in most cases except when a solution limit is given as indicated in the table. Even in these cases the solution is likely to be optimal but has been terminated early to reduce the time taken. Leave requests are met in most of the months. In May, August and December only one request is not met. The November schedule has four unmet leave requests and one nurse is assigned nine nights instead of eight. This can be handled by adjusting the number of nights for that nurse in the subsequent month, which in any case is the general procedure followed at the hospital. The schedules obtained are superior to those used at the hospital where several constraints are violated, most important being the desired staffing level in each shift. Violation of this constraint implies that service levels being provided at the hospital are lower than desired even though the solutions obtained here indicate that there is no necessity to do so. The desired staffing levels can be provided without increasing staff strength.

A sample schedule generated is given in Table 3. This corresponds to the non-availability data given in Table 1. The table includes the number of morning, evening and night assignments for each nurse and the total number of hours assigned to the nurse. It also lists the number of nurses scheduled in each shift. It is to be noted that the first six nurses listed are taken to be senior nurses. As can be seen from the schedule no further processing is required. Any emergency leave requests can be handled as and when they come which is normal practice.

5. Conclusions

The model can be used to generate desirable schedules incorporating the information on nurse availability during the planning period. The time taken for scheduling is a few minutes rather than several hours spread over a few days when done manually. The service levels provided are as desired. Nurses can give their leave information for a given 15-day period, instead of having to provide it for a month much ahead of time. This is far more feasible as well as practical and lends greater flexibility.

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Table 1: Nurse Availability Data for a Sample Month

		DAY																													
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	2	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
N	**7**	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U	**8**	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
R	**9**	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
S	**10**	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
E	**11**	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
	14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Total Non-Availabilities = 22

Table 2: Results for the 10 months with varying non-availability information

Month	CPU Time	ObjectiveFunction Value	Number of Non-availabilities	Remarks
Mar1	50 secs	0	12	
Mar2	3 min 29 secs	0	7	
Apr1	1 min 43 secs	0	15	
Apr2	2 min	0	7	
May1	1 min 41 secs	0	20	
May2	2 min 34 secs	1	11	
Jun1	1 min 33 secs	0	15	
Jun2	3 min 4 secs	0	5	
Jul1	1 min 14 secs	0	11	
Jul2	4 min 28 secs	0	8	
Aug1	12 secs	0	6	
Aug2	4 min 14 secs	1	10	Solutionlim = 4
Sep1	2 min 33 secs	0	15	
Sep2	4 min 1 secs	0	5	
Oct1	57 secs	0	3	
Oct2	2 min 5 secs	0	1	
Nov1	1 min 41 secs	0	9	
Nov2	4 min 57 secs	6	17	Solutionlim = 3
Dec1	9 secs	0	0	
Dec2	3 min 47 secs	1	14	Solutionlim = 3

Table 3: Schedule for data in Table 1

		DAYS																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	M	E	N	Hrs
	1	M	M				E	E	M	M	M	M			E	E	6	4	0	60
	2			E	E	E	M	M				M	M	M	M	M	7	3	0	60
	3			E	E	E		E	E	E	E		M	M	N	N	2	7	2	78
	4	E	E	E		M	M	N	N	N	N		N	N			2	3	6	102
	5	N	N	N	N	N	N		M	M	E	E	E	E			2	4	6	108
	6			M	M	E	E	M	M		N	N	N	N	N	N	4	2	6	108
N	7	N	N	N	N		E	E	E	E	M	M					2	4	4	84
U	8	E	E	M	M	M	M		N	N		N	N	N	N		4	2	6	108
R	9	M	M	M	M		N	N		N	N	N		E	E	E	4	3	5	102
S	10	E	E	M	M	M		E	E	M	M		E	E	E	E	5	8	0	78
E	11								E	E		E	E	N	N	N	0	4	3	60
S	12	N	N	N		M	M	M	N	N	N		E	E	M	M	5	2	6	114
	13	M	M		E	E	E	N	N		E	E	M	M	E	E	4	7	2	90
	14	N	N		N	N	N	N		E	E	M	M		M	M	4	2	6	108
	15	E	E	N	N	N		M	M	E	E	E		M	M	M	5	5	3	96
	16	M	M	E	E	N	N		M	M	M	N	N		N	N	5	2	6	114
	Nurses																			
	M	4	4	4	4	4	4	4	5	4	4	4	4	4	4	4				
	E	4	4	4	4	4	4	4	4	5	5	4	4	4	4	4				
	N	4	4	4	4	4	4	4	4	4	4	4	4	4	5	4				
	Total	12	12	12	12	12	12	12	13	13	13	12	12	12	13	12				

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	M	E	N	Hrs	Total N	Total Hrs
	1	N	N	N	N		M	M	M	M	M		N	N	N	N	5	0	8	126	8	186
	2	M		N	N		E	E	N	N		N	N	N			1	2	7	102	7	162
	3			E	E	N	N	N	N		E	E	N	N			0	4	6	96	8	174
	4	M	M	N	N		M	M	E	E		M	M	E	E	E	6	5	2	90	8	192
	5	E	E		M	M	N	N		M	M	E	E	M	M		6	4	2	84	8	192
	6		M	M	E	E		E	E	E	N	N		M	M	M	5	5	2	84	8	192
N	7	E	E	E	M	M		M	M	N	N	N		M	M	M	7	3	3	96	7	180
U	8	E	E	E	N	N		E	E		M	M	M		E	E	3	7	2	84	8	192
R	9	E	M	M		N	N		M	M	E	E	M	M			6	3	2	78	7	180
S	10	N	N		E	E	E			N	N		N	N	N	N	0	3	8	114	8	192
E	11	N		M	M	N	N		E	E	E	E	E		N	N	2	5	5	102	8	162
S	12	N	N		E	E	E			E	E	M	M		M	M	4	5	2	78	8	192
	13						N	N	N	N	N		E	E	E	E	0	4	5	84	7	174
	14	M	E	E		E	E	E	M	M			E	E	N	N	3	7	2	84	8	192
	15	M	N	N		M	M	N	N		M	M		M	M	M	8	0	4	96	7	192
	16		M	M	M	M	M	M			N	N		E	E	E	6	3	2	78	8	192
	Nurses																					
	M	4	4	4	4	4	4	4	4	4	4	4	4	5	5	4						
	E	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4						
	N	4	4	4	4	4	5	4	4	4	5	4	4	4	4	4						
	Total	12	12	12	12	12	13	12	12	12	13	12	12	13	13	12						

M: Morning **E: Evening** **N: Night**

Appendix 1a

Period-1 (Model for the first 15 days of a month):

Objective function: Minimize the assignment of nurses on days they are not available.

$$\text{Min } Z = \sum_{ijk} o_{ijk}$$

Subject to: -

(1) A nurse is allocated to utmost one shift, a day.

$$\sum_{k} x_{ijk} \leq 1 \qquad (\forall i, j)$$

(2) There should be at least one senior nurse in each shift everyday.

$$\sum_{i=1,6} x_{i,j,k} >= 1 \qquad (\forall j,k)$$

(3) Morning or evening shift is not to be assigned on any day, if the previous day was a night duty.

$$x_{i,j-1,3} + x_{i,j,1} \leq 1 \qquad (\forall i, j)$$

$$x_{i,j-1,3} + x_{i,j,2} \leq 1 \qquad (\forall i, j)$$

(4) Nurse to be granted off after 6 consecutive days of work.

$$\sum_{k} \sum_{j-6}^{j} x_{ijk} \leq 6 \qquad (\forall i, j > 6)$$

(5) Nurse is to be assigned a maximum of 6 night shifts in this period.

$$\sum_{j} x_{ij3} \leq 6 \qquad (\forall i)$$

(6) Minimum of four nurses to be assigned in a shift on a given day.

$$\sum_{i} x_{ijk} >= 4 \qquad (\forall j,k)$$

(7) Nurse to be assigned for at least 2 consecutive days in a particular shift.

$$x_{i,2,k} >= x_{i,1,k} \qquad (\forall i,k)$$

$$x_{i,j-1,k} + x_{i,j+1,k} >= x_{i,1,k} \qquad (\forall i, j = 2..14, k)$$

$$x_{i,14,k} >= x_{i,15,k} \qquad (\forall i, k)$$

(8) Assign a nurse only on days she is available (violation allowed).

$$x_{i,j,k} = a_{i,j} + o_{i,j,k} - u_{i,j,k} \qquad (\forall i, j, k)$$

(9) Nurse not to be assigned for night duty before a casual day off.

$$x_{i,j,3} <= a_{i,j+1} + \sum_k x_{i,j+1,k} \qquad (\forall i, j < 15)$$

$$x_{i,15,3} <= n_i \qquad (\forall i)$$

Appendix 1b

Period-II (Model for the second 15 days of a month):

The following constraints are required for assignments over a month.

(1) Nurse to work for a Maximum of 192 hours per month.

$$6*(\sum_{d=1}^{15} p_{i,d,1} + \sum_j x_{i,j,1}) + 6*(\sum_{d=1}^{15} p_{i,d,2} + \sum_j x_{i,j,2}) + 6*(\sum_{d=1}^{15} p_{i,d,2} + \sum_j x_{i,j,2}) <= 192 \qquad (\forall i)$$

(2) The total night shifts assigned in the month cannot exceed eight. This constraint replaces constraint number (5) for period 1.

$$\sum_{d=1}^{15} p_{i,d,3} + \sum_j x_{i,j,3} = 8 \qquad (\forall i)$$

To establish connectivity with the second 15-day period the following constraints are also introduced.

(3) Morning or Evening shift is not to be assigned on the first day, if the previous day was a night duty

$$p_{i,15,3} + x_{i,1,1} \le 1 \qquad (\forall i)$$

$$p_{i,15,3} + x_{i,1,2} \le 1 \qquad (\forall i)$$

Appendix 1c

Modified Model: Relaxation in constraint for maximum of eight night shifts in a month.

Objective function: Minimize the weighted sum of over assignments of nurses on days they are not available and the number of night shifts assigned to a nurse over a total of eight.

$$\text{Min } Z = \sum_{ijk} (o_{ijk} + 2 * onite_{ijk})$$

Modified constraint (2) for period 2:

$$\sum_{d=1}^{15} p_{i,d,3} + \sum_{j} x_{i,j,3} = 8 + \sum_{j} onite_{i,j,3} - \sum_{j} unite_{i,j,3} \qquad (\forall i)$$