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Abstract

Obsolescence of embedded parts is a serious concern for managers of complex systems where the *design life* of the system typically exceeds 20 years. Capital asset management teams have been exploring several strategies to mitigate risks associated with Diminishing Manufacturing Sources (DMS) and repeated life extensions of complex systems. Asset management cost and the performance of a system depend heavily on the obsolescence mitigation strategy chosen by the decision maker. We have developed mathematical models that can be used to calculate the impact of various obsolescence mitigation strategies on the Total Cost of Ownership (TCO) of a system. We have used classical multi-arm bandit (MAB), arm-acquiring bandit and restless bandit models to identify the best strategy for managing obsolescence in such instances wherein organizations have to deal with continuous technological evolution under uncertainty.

Keywords: Arm-acquiring bandit model, Gittins Index, Markov decision process, Multi-arm bandit (MAB) model, obsolescence, Restless bandits (RB), total cost of ownership.

1.0 Introduction

Technological innovation is no longer an option, but a necessity; indeed a vital survival strategy for many companies, especially the electronic parts manufacturers. The advantages of technological innovation are well documented in the literature (Capon *et al, 1992,* Gatignon *et al* 2002); Capon *et al* (1992), provide empirical evidence, based on a survey of 113 manufacturing firms selected from *Fortune 500* manufacturers, that the firms that invest heavily in innovation perform best financially. However, the main disadvantage of technological innovation is the reduction in the economic life of parts as availability of better technology renders the older technology less useful (Lee and Lee, 1988). Most complex systems, especially, defence systems such as fighter aircraft, battle ships, tanks etc have long life, much longer than many of their embedded parts. Managing such complex systems under continuous technological changes has become a major challenge to many defence services and defence original equipment manufacturers (OEMs). The main causes of part obsolescence are: (1) Diminishing Manufacturing Sources (DMS) triggered by rapid progress in technology and (2) Life extension of capital assets far beyond the life of its embedded parts, in particular, defence systems, and (3) Planned obsolescence by many consumer durable manufacturers. Defence systems usually have long design period (time spent on designing the product in the development life cycle), for example, the design period for F-22 aircraft was more than 10 years and many parts of F-22 became obsolete even before the production started (Hitt and Schmidt, 1998). Boeing's B-52 Bomber project began in 1946 and the first flight took off in 1952. It is predicted that B-52 would still be flying for the US air force in 2045; almost a century after its development began (Feltus, 2007). The projected electronic obsolescence budget for the F-22 fighter is in excess of one billion dollars (Tepp, 1999).

Defence Supply Centre Columbus (DSCC) deals with procurement and quality assurance of over 2.2 million spare parts and on an average 10,000 parts become obsolete every year because of discontinuance of production by the manufacturer (Johnson, 2002). It is reported that 84% of the discontinued items are electronic components and the rest are mechanical and passive devices (Tomczykowski, 2003). In 2005, more than 150,000 integrated circuits (ICs) were declared end of life (Short, 2006). Solomon *et al* (2000) claim that obsolescence of electronic parts is a major reason for the high life cycle cost of military systems. The problem has become a serious concern for product support managers of defence systems due to the rapid progress in electronic technology wherein a new generation of components replace the old generation components within months (Meyor *et al* 2004).

The impact of obsolescence, although severe on defence industry, has also affected several other industries. Szoch *et al* (1995) discusses policies to manage obsolescence issues in nuclear reactor protection systems. Hoorickx (2008) studied the impact of obsolescence on long-life medical instruments where support requirements for healthcare systems can extend beyond 10 years. He suggested several strategies to manage obsolescence of healthcare systems to reduce the total cost of ownership.

Several strategies have been developed by capital asset managers to mitigate the impact of obsolescence. Last-time-buy (also known as Life-time-buy), part replacement, aftermarket sources, emulation, re-engineering and design refresh are some of the strategies used by product support managers to reduce the impact of parts obsolescence (Porter, 1998). In this paper we focus on the following three strategies:

1. Last-time-buy (LTB): Under the LTB strategy, the users of the part are given one last chance to buy the spare items so that they can meet the demand for spare parts for entire remaining life of the item and maintain the availability at the system level.
2. Redesign strategy: Under redesign, the part and the system in which the part is embedded are redesigned to incorporate new technology.
3. Combination of LTB and redesign: Under the combination strategy, LTB strategy is used up to a certain period (up to j-1 periods out of N periods) and redesign is completed and implemented from period j onwards (till the remaining life of the system).

Mathematically, strategy 2 is a special case of strategy 3. It is assumed that the redesigned part will survive all remaining periods with probability Q_i, where redesign is performed during period i.

One key issue with LTB procurement is that it is very difficult to estimate the number of spare items required to support the system for the rest of its useful life, especially when the life of the system itself may be extended several times. The product support managers have to decide whether it is beneficial to redesign an existing subsystem or use LTB strategy or a combination of both. The use of LTB strategy involves carrying a huge quantity of inventory for a long period; on the other hand redesign programs for defence systems need to go through time and cost consuming system qualifications/certifications that make the entire process of redesign extremely expensive (Porter, 1998; Solomon *et al* 2000). Porter (1998) claims that the redesigning exercise may take up to 12 to 24 months if there are significant design changes to the LRU (line replaceable unit) in which the obsolete part is embedded.

The main focus of this paper is to develop mathematical models for choosing the best obsolescence strategy under the following three different scenarios:

1. The user receives information from the supplier that a part would be discontinued in the near future (deterministic scenario).
2. There is uncertainty about the availability of a part and the time to obsolescence is assumed to be a random variable. *A priori* distribution, F(X), for time to obsolescence is used to calculate the optimal time to redesign.
3. The time to obsolescence is assumed to be a random variable and the user is updated about the future availability of the part for procurement during each period of

operation (scenario 2 is a special case of scenario 3). In this scenario it is assumed that the probabilities get updated prior to every period.

The rest of the paper is organized as follows. In Section 2, we discuss the main drivers of obsolescence and literature on assent management models under obsolescence and technological change. In Section 3, we develop mathematical models for predicting the total cost of ownership under different obsolescence mitigation strategies and these models are further used to generate propositions on different strategies. The Bandit models for selection of optimal strategies are discussed in Section 4. An illustrative example of the Bandit process approach is demonstrated in Section 5. Conclusions and the course of future research are presented in Section 6.

2.0 Drivers of Obsolescence and Literature on Assent Management under Technological Change

The electronic parts market is driven by the commercial sector and the influence of defence on electronic parts manufacturers has decreased over a period of time. The market share of electronic parts for military systems is less than 0.3% (Hunt and Haug, 2001). Hamilton and Chin (2001) claim that the *Acquisition Reform* initiated by the US Department of Defence after the end of the Cold War probably resulted in the obsolescence of electronic parts within the defence industry. Acquisition Reform favoured the use of commercial off-the-shelf (COTS) parts in place of military specification (Mil-Spec) parts. Singh and Sandborn (2006) suggest that products that race to adapt to the latest technology are generally high volume consumer electronics goods such as mobile phones, laptops, iPods etc. In addition, many consumer durable manufacturers use planned obsolescence as a marketing strategy to introduce new products to compete in the market (Bulow, 1986; Grout and Park, 2005). Planned obsolescence is a marketing strategy used by manufacturers of consumer durable companies in which the manufacturer produces goods with uneconomically short useful life so that the customer will have to make repeat purchases. The defence industry has little or no control over the supply chain of electronic parts because of their low volume. Product sectors such as airplanes, ships and weapon systems find it difficult to adopt the leading edge technology. These product sectors often lag in adopting the latest technology because of the high costs and long time period associated with technology insertion (Singh and Sandborn, 2005).

Several debates on replacement decision models exist in literature (Oakford *et al* 1984, Banerjee and Kabadi 1994; Goldstein *et al* 1998; Hartman, 2001; Lai *et al* 2001; Regnier *et al* 2004) that address the issue of optimal replacement of old parts with new technology. The rationale behind most replacement models is the decreasing ownership cost of new technologies which justifies the replacement of an existing part with new technology. Nair and Hopp (1992) developed the model of equipment replacement due to technological obsolescence, using dynamic programming. However, the decision to replace an obsolete sub-system/part with one incorporating the latest technology is driven by many factors and requires careful analysis. For example, system integration issues may force a major redesigning of the LRU in which the obsolete part is embedded. In such cases, it may be convenient to choose the LTB strategy for the obsolete part for the time being; however, this may involve compromising on the performance and other design parameters such as reliability, maintainability and supportability when a more sophisticated technology is available. On the contrary, redesigning is a very time consuming process and there is no guarantee that the new technology will survive for a long time period or at least till the design life of the system.

Product support managers are updated frequently with information about parts that are likely to become obsolete in the near future, on the basis of which they have to choose a strategy that would enable them to maintain availability of the fleet at the least cost of ownership. Unavailability of fleet would result in heavy penalty and should be avoided. The following assumptions are used in developing the models presented in this paper:

1. Annual demand for parts is constant and the rate of demand is equal to its failure rate.
2. The time taken to redesign is known and is deterministic.
3. There is no shelf life for stocked parts. That is, the inventory is not subject to obsolescence.
4. The system life is not extended beyond the initial life of the system.

Notations:

F	Fleet size of the system
N	Remaining useful life of the system
λ_e	Failure rate per annum of the existing part
Q_e	$= \lceil \lambda_e \times F \times N \rceil$, the total number of parts required to support the fleet for N years under the LTB strategy
λ_d	Failure rate per annum of the redesigned part
O_e	Operating cost per annum for existing parts
M_e	Maintenance cost of the existing parts per maintenance activity
O_d	Operating cost per annum for the redesigned part
M_d	Maintenance cost of the redesigned part per maintenance activity
X	Random variable denoting the time to obsolescence of the existing part
r	Annual interest rate
$C_{p,e}$	Procurement cost of the existing parts per unit
$C_{h,e}$	Inventory holding cost per unit per annum for the existing parts
$C_{h,d}$	Inventory holding cost per unit per annum for the redesigned part
$C_{p,d}$	Procurement cost of redesigned part per unit
C_{RD}	Total cost associated with redesigning the part
U_p	Penalty cost associated with unavailability of the fleet due to non-availability of obsolete parts.

3.0 Total Cost of Ownership under LTB, Redesign and Combination Strategies .

Among the many strategies used to mitigate obsolescence, LTB and redesign have emerged as the most debated strategies. The LTB strategy is likely to involve procuring and storing huge inventory of spare parts and the use of relatively inferior technologies compared to other known technologies; whereas a redesign strategy may be costly due to stringent qualification and certification requirements of the defence and avionics systems. One approach traditionally used to decide between various obsolescence strategies is to check the impact of these choices on Life Cycle Cost (LCC) or Total Cost of Ownership (TCO). Singh and Sandborn (2005) claim that they have developed a mitigation of obsolescence cost analysis model (MOCA) which predicts the optimal time to refresh using the LCC of the system. Porter (1998) proposes a net present value (NPV) cost analysis technique which would help managers to decide between LTB and redesign. Porter also makes a conjecture that if the total non-recurring engineering cost of redesign is expensive and yields a ratio of 200 or greater, then the LTB would be the most cost effective strategy between the two. Regnier *et al* (2004) have developed replacement decision models under continuous technological progress when capital costs and operation and maintenance cost decrease at different rates; however, the focus of their work is on the use of cost effective new technologies rather than on the obsolescence of embedded parts within a system.

TCO calculation can be very complex depending on the operations strategy adopted by the user (Asiedu and Gu, 1998). In this paper we include cost elements that have significant impact on the selection of alternative obsolescence mitigation strategies. Using the framework for TCO discussed by Regnier *et al (2004)* and Dinesh Kumar *et al* (2007), the TCO for N years of normal operation (in absence of obsolescence) is given by:

$$TCO(N) = \overbrace{C_{p.e} \times \lambda_e \times F \times D_N}^{1} + \overbrace{\frac{\lambda_e \times F}{2} \times D_N \times C_{h,e}}^{2} + \overbrace{D_N \times F \times (\lambda_e \times M_e + O_e)}^{3} \quad (1)$$

Where,

$$D_N = \sum_{i=1}^{N} \frac{1}{(1+r)^i} = \frac{1-(1+r)^{-N}}{r} \quad (2)$$

In Equation (1), terms 1, 2 and 3 represent procurement, inventory holding and operation & maintenance costs respectively over N years discounted at the rate of r. For mathematical simplicity we have ignored the cost of disposal from the TCO expression in Equation (1). Equation (2) can be easily derived since it is a geometric series. Under the LTB strategy, the decision maker is forced to buy all the spare components that are required to support the system for the remaining useful life of the system. The TCO under LTB is given by:

$$TCO_{LTB}(N) = \overbrace{D_1 \times C_{p,e} \times \lambda_e \times F \times N}^{1} + \overbrace{\sum_{i=1}^{N} \left[Q_e - \frac{(2i-1) \times \lambda_e \times F}{2} \right] \times C_{h,e} \times \frac{1}{(1+r)^i}}^{2} \quad (3)$$
$$+ \overbrace{D_N \times F \times (\lambda_e \times M_e + O_e)}^{3}$$

The first term in Equation (3) is the total procurement cost associated with purchasing the LTB component. The second term calculates the inventory holding cost of the LTB component over N years. $Q_e - (i-1) \times \lambda_e \times F$ is the amount of inventory left at the beginning of year i and $Q_e - \frac{(2i-1) \times \lambda_e \times F}{2}$ is the average inventory of the LTB component during the ith year. The third expression accounts for the operation and maintenance cost of the LTB component over N years. TCO under redesign strategy is given by:

$$TCO_{RD}(N) = \overbrace{C_{RD} \times D_1}^{1} + \overbrace{C_{p.d} \times \lambda_d \times F \times D_N}^{2} + \overbrace{\frac{\lambda_d \times F}{2} \times D_N \times C_{h,d}}^{3} + \overbrace{D_N \times F \times (\lambda_d \times M_d + O_d)}^{4} \quad (4)$$

In Equation (4), term 1 accounts for the discounted redesign cost assuming that the redesigning is performed at the beginning of the remaining life. Terms 2, 3 and 4 in Equation (4) account for procurement, inventory and operation and maintenance cost of the redesigned part respectively. TCO under the combination strategy can be calculated as follows:

Let

$$\delta_j = \begin{cases} 1, & \text{If redesign is performed during j}^{\text{th}} \text{ year, j} \leq \text{N} \\ 0, & \text{Otherwise} \end{cases}$$

Then the cost of ownership under the combination strategy is given by:

$$TCO_C(j,N) = \overbrace{\sum_{j=1}^{N} TCO_{LTB}(j-1) \times \delta_j}^{1} + \overbrace{\sum_{j=1}^{N} TCO_{RD}(j,N) \times \delta_j}^{2} \quad (5)$$

Where $TCO_{LTB}(j-1)$ is the total cost of ownership for j-1 years under LTB and $TCO_{RD}(j,N)$ is the total cost of ownership for (N-j+1) years where redesign is performed on the j^th^ year. $TCO_{RD}(j,N)$ is given by:

$$TCO_{RD}(j,N) = \overbrace{\frac{C_{RD}}{(1+r)^j}}^{1} + \overbrace{C_{p.d} \times \lambda_d \times F \times D_{j,N}}^{2} + \overbrace{\frac{\lambda_d \times F}{2} \times D_{j,N} \times C_{h,d}}^{3} + \overbrace{D_{j,N} \times F \times (\lambda_d \times M_d + O_d)}^{4} \quad (6)$$

Where

$$D_{j,N} = \sum_{i=1}^{N} \frac{1}{(1+r)^i} - \sum_{k=1}^{j-1} \frac{1}{(1+r)^k} = \left(\frac{(1+r)^{-(j-1)} - (1+r)^{-N}}{r} \right) \quad (7)$$

Equation (7) is derived using Equation (2). A simple zero-one programming model can be used to find the optimal time at which the redesigning should be performed, and the corresponding optimization model is given by:

$$Min\, TCO_C(j,N) = Min\sum_{j=1}^{N}\left(TCO_{LTB}(j-1) + TCO_{RD}(j,N)\right)\times\delta_j \tag{8}$$

Subject to:

$$\sum_{j=1}^{N}\delta_j = 1,\ \text{where}\ \delta_j = 0\ \text{or}\ 1 \tag{9}$$

The following propositions are derived using the TCO model discussed above for different obsolescence mitigation strategies.

Proposition 1: If an embedded part becomes obsolete, then there exists an upper limit for the remaining life, beyond which, the redesign strategy is preferred over the LTB strategy.

Proof: The proposition can be easily proved by comparing the TCO under the LTB and redesign strategies. Let:

$$H_e = \overbrace{\sum_{i=1}^{N}\left[Q_e - \frac{(2i-1)\times\lambda_e\times F}{2}\right]\times C_{h,e}\times\frac{1}{(1+r)^i}}^{1} \tag{10}$$

The expression for TCO under LTB is given by:

$$TCO_{LTB}(N) = \overbrace{D_1\times C_{p,e}\times\lambda_e\times F\times N}^{1} + \overbrace{H_e}^{2} + \overbrace{\mathrm{D_N}\times F\times(\lambda_e\times M_e + O_e)}^{3} \tag{11}$$

Equation (4) can be rewritten as:

$$TCO_{RD}(N) = C_{RD}\times D_1 + D_N\times F\times\left(\lambda_d(C_{p,d} + \frac{C_{h,d}}{2} + M_d) + O_d\right) \tag{12}$$

The redesign strategy should be chosen if $TCO_{LBT}(N) - TCO_{RD}(N) \geq 0$. Using Equations (11) and (12) we get:

$$C_{p,e}\lambda_e FND_1 \geq C_{RD}D_1 + D_N F\left(\lambda_d(C_{p,d} + \frac{C_{h,d}}{2} + M_d) + O_d - \lambda_e \times M_e - O_e\right) - H_e$$

Or

$$N \geq \frac{1}{D_1 C_{p,e}\lambda_e F}\left[C_{RD}D_1 + D_N F\left(\lambda_d(C_{p,d} + \frac{C_{h,d}}{2} + M_d) + O_d - \lambda_e M_e - O_e\right) - H_e\right] \quad (13)$$

Figure 1 is obtained for a hypothetical example wherein for a given part all the cost and design parameters under both LTB and redesign are assumed to be the same. In this case it is observed that if the remaining life is more than 5 years then the redesign strategy is preferred over the LTB strategy.

We define: *Redesigning is cost effective when the sum of the procurement, operation and maintenance cost of the redesigned part is less than or equal to that of the existing part.*

Proposition 2: If an embedded part becomes obsolete and the redesign is cost effective and $C_{RD} < H_e - \frac{\lambda_d \times F \times D_N \times C_{h,d}}{2}$ **then it is optimal to choose the redesign strategy, provided that the redesigning is performed in period 1 of the remaining life.**

Proof: Assume that the redesign is cost effective. That is:

$$(\lambda_e \times M_e + O_e) \geq (\lambda_d \times M_d + O_d) \quad (14)$$

Let

$$C_{RD} < H_e - \frac{\lambda_d \times F \times D_N \times C_{h,d}}{2} \quad (15)$$

We know that:

$$C_{p.e} \times \lambda_e \times F \times N \times D_1 > C_{p,d} \times \lambda_d \times F \times D_N \quad (16)$$

Using Equations (14), (15) and (16) in Equations (3) and (4), we can show that $TCO_{LTB}(N) \geq TCO_{RD}(N)$. In proposition 1 and 2, the underlying assumption is that the part under consideration would certainly become obsolete (or the manufacturer has informed the user that he is discontinuing the part). In proposition 3, we assume that there is some uncertainty about the time at which the part may become obsolete. An important question which every decision maker responsible for obsolescence management poses is, 'What is the optimal time to redesign the obsolete part?' (Porter, 1998; Singh and Sandborn, 2006). In proposition 3, we assume that the time-to-obsolescence is uncertain and derive the optimal time to redesign.

Proposition 3: If the time to Obsolescence is a random variable with distribution F(x), then the optimal redesign period, M, can be derived using the following inequality:

$$P(X < M) \leq \frac{TCO_{RD}(M) - TCO(M)}{\left(TCO_{RD}(M) - TCO(M) + U_p \times D_M\right)} \qquad (17)$$

Proof:

Assume that the decision maker chooses to perform redesign during the M^{th} period and till then it will be used under normal conditions. Then, using the concept of *expected marginal benefit*, we can write that the optimal redesign period, M (< N), should be the maximum value of M, for which:

$$P(X \geq M) \times \left(TCO_{RD}(M) - TCO(M)\right) \geq P(X < M) \times U_P \times D_M \qquad (18)$$

Or

$$\left(1 - P(X < M)\right) \times \left(TCO_{RD}(M) - TCO(M)\right) \geq P(X < M) \times U_P \times D_M \qquad (19)$$

Note that in Equation (17), the penalty cost (U_p) incurred due to the non-availability of spare parts is discounted at the rate r since the penalty is incurred in year M. It is easy to show from Equation (17), that:

$$P(X < M) \leq \frac{TCO_{RD}(M) - TCO(M)}{\left(TCO_{RD}(M) - TCO(M)\right) + U_p \times D_M}$$

If the time to obsolescence follows an exponential distribution, with mean obsolescence time (μ), then equation (17) can be written as:

$$1-e^{-\frac{M}{\mu}} \le \frac{TCO_{RD}(M)-TCO(M)}{\left(TCO_{RD}(M)-TCO(M)\right)+U_p \times D_M} \qquad (20)$$

Rearranging equation (20), we get:

$$M \ge \mu \times \ln\left(\frac{1}{1-R}\right) \qquad (21)$$

Where,

$$R = \frac{TCO_{RD}(M)-TCO(M)}{\left(TCO_{RD}(M)-TCO(M)\right)+U_p \times D_M}$$

Figure 2 depicts the graphs of expected benefits and expected cost curves for a part for which the time-to-obsolescence is assumed to follow a normal distribution with mean 8 years and standard deviation 2 years. In this example, the optimal time to redesign is 7 years. When the remaining life of the system is very large and the technology obsolescence rate is high, it is likely that the redesigned part may become obsolete before the system is decommissioned. Here normal distribution is used for time-to-obsolescence for illustrative purpose only. In the next section, we use the Markov decision processes called the Bandit processes to model and find the optimal sequence of decisions.

4.0 Bandit Process Approach for Optimal Selection of Obsolescence Mitigation Strategies

Assume that at the beginning of each period the decision maker has to choose one of the many strategies available to her. Let S_{ij} be the strategy j used for period i and $R_{ij}(S_{ij})$ is the expected reward for choosing strategy j for period i. Let Π be the sequence of decisions made over N periods. That is:

$$\Pi = (S_{1j}, S_{2j}, \ldots, S_{Nj}) \quad \cdot \tag{22}$$

The corresponding expected total reward $R(\Pi)$ is given by:

$$R(\Pi) = \sum_{i=1}^{N} \sum_{j=1}^{M} \beta^i P_{ij}(S_{ij}) R_{ij}(S_{ij}) \tag{23}$$

Where $P_{ij}(S_{ij})$ is the probability of obtaining the reward $R_{ij}(S_{ij})$ when strategy S_{ij} is chosen. A sequence Π^* is called β-optimal (Mine and Osaki, 1970) if there exists a sequence of strategies Π^* such that $R(\Pi^*) \geq R(\Pi) \ \ \forall \Pi$, where $(0 \leq \beta < 1)$. The problem stated in (22) and (23) is a classical Multi-arm Bandit Problem (MAB) and the resulting process is called the Bandit process in which a decision maker has to take a sequence of decisions that maximizes her total expected discounted reward. The rewards are discounted since they are earned at different time periods. In this case we have assumed finite time horizon which is determined by the designed life of the systems, after that the system is condemned or decommissioned. It is possible that the designed life of the system may be extended, but in this paper we restrict our analysis to systems without life extensions beyond their initial design period.

A common example used in explaining a MAB is the sequential selection of projects to optimize the total reward over T (t = 1, 2, .., T) periods. Assume that there are N parallel projects indexed by k = 1, 2, ..., N. The project k can be in $n_t(k)$ states, where $n_t(k)$ is finite. At any instant of time t, only one project is chosen for implementation. If project k in state $n_t(k)$ is chosen for period t, then one receives an expected reward $R(n_t(k))$. The rewards are additive and discounted by a factor β, where $0 < \beta < 1$. The state $n_t(k)$ changes to $n_{t+1}(k)$, the state evolution is a Markov process in which change of state depends on k, but not on t. In the classical MAB, the states of projects that are not chosen remain same. The problem is to choose the projects sequentially to maximise the total discounted reward. Thus, MAB is basically an independent Markov decision process with aforementioned additional conditions.

Thompson (1933) introduced the two-arm bandit problem in the context of clinical trials, and since then they were used extensively by others in clinical drug trials, petroleum exploration and consumer product or service settings (e.g., Gittins (1989), Meyer and Shi (1995), Erdem and Keane (1996), Banks *et al* (1997), Anderson (2001)). Breakthrough research on Bandit problems were carried out by Gittins and Jones (1974), Whittle (1981, 1988), Berry and Fristedt (1985) and Glazebrook (1987, 1990). Gittins and Jones (1974) proved that a k-armed Bandit problem can be solved by solving k-one armed bandit problems. This theorem asserts that in any independent-armed Bandit problem with geometric discounting over an infinite horizon, it is possible to associate with each arm an index (dynamic allocation index), known as the Gittins Index, with the property that a strategy in the Bandit problem is an optimal strategy if and only if it involves playing an arm with the highest value of the Gittins Index at that point (such a strategy is called the Gittins Index strategy). Whittle (1981) introduced the concept of arm-acquiring bandits, in which new arms may be added to the problem at a later stage and proved that Gittins index policy of classical MAB is optimal to arm-acquiring bandits as well. The Gittins index (dynamic allocation index, Gittins (1979)) for the j^{th} arm with a discount rate of β is calculated as follows:

$$G^j = \sup_{N \geq 1} \sum_{i=1}^{N} \beta^i E(R_{ij}) / E \sum_{i=1}^{N} \beta^i \qquad \text{for} \quad j = 1, \ldots, N \tag{24}$$

The feature that gives this result especial potency is that the Gittins Index on an arm depends solely on the characteristics of that arm and on the rate of discounting, and not on any other feature of the problem under study (Sundaram, 2003).

Whittle (1988) generalized the classical MAB by allowing state evaluation of passive arms. The new model called "restless bandit" problems allowed rewards for arms under passive state and the decision maker can choose m out of n arms ($m < n$). Restless bandit (RB) problems are intractable and are proved to be PSPACE-hard (Papadimitriou and Tsitsiklis, 1999). Whittle (1988) proposed an index policy for RB problems which reduces to Gittins index for the classical MAB. However, Whittle's index policy is applicable to only certain class of RB problems that satisfy a certain indexability property, which may be hard to check. Nino-Mora (2001) derived sufficient conditions for indexability of Whittle's indices based on partial conservation laws.

Glazebrook (1987, 1990, and 1996) made several important contributions to the theory of MAB and RB problems and presented approaches to policy evaluation and sensitivity analysis in stochastic scheduling via index based sub-optimality bounds and procedures for the evaluation of Gittins Index strategies for resource allocation in a stochastic environment. Recently, Glazebrook *et al* (2005, 2006) used RB model to analyse machine maintenance problems, where bandits represent machines that evolve under the influence of maintenance actions. Glazebrook *et al* (2006) use Whittle's index policy to arrive at optimal decisions for machine maintenance problem.

The selection of optimal obsolescence strategies in its general form is an arm-acquiring restless bandit problem. New technologies would emerge from time to time and the decision maker has to decide on the optimal strategy based on all currently available technological choices. Also, the state evolution of arms depends on the time, that is, the rate at which a technology becomes obsolete would depend on the time (current age of the technology), and thus the problem is also a restless bandit problem. State evolution occur independent of whether an arm (in this case the strategy) is chosen or not. However, at any stage, the decision maker chooses only one arm, and there is no passive reward for the arms which are not chosen. Dayanik *et al* (2007) have proved that when the passive rewards are equal to zero, the Whittle's index converges to Gittin's index. In fact, for the optimal selection of obsolescence strategies, since only one strategy can be chosen at any period and passive arms carry no reward; there is no difference between Whittle's and Gittin's indices.

4.1 Two Armed Bandit Model For Selection Of Obsolescence Strategy

Every year, during the useful life of a system, the decision maker faces the issue of having to decide on the strategy she is going to use to deal with the obsolescence of parts. While the decision maker is informed about parts that are likely to become obsolete in the near future, she may ignore the warning and store only those parts required for that particular period to support the system, or go for the redesign strategy (if redesign strategy turns out to be an optimum strategy, the decision maker can always choose LTB till that period). That is, there is a state change in the form of availability of parts in the next period, irrespective of the fact whether an arm is chosen or not. A two armed restless bandit approach can be used to model this problem, where each arm represents a unique strategy as defined below. At any period only one arm is chosen and there is no passive reward.

Arm 1: The decision maker chooses to procure parts required for the current period (period i) only.

Arm 2: The decision maker chooses to redesign the part during period i.

Now we calculate the Gittins Indices for arm 1 and arm 2 and choose the arm which has the highest Gittins Index (please note that in this case both Whittle's and Gittin's indices are same). Since we are dealing with a two armed restless bandit problem, we continue the estimation of Gittins Indices until arm 2 is chosen, and this point becomes the stopping rule in our case. It is obvious that the decision maker would like to minimize the total cost of ownership of the system. At the same time however, if the decision maker is unable to maintain the availability of the system due to the chosen strategy, she is likely to incur a heavy penalty. The expected reward $E(R_{ij})$ corresponding to arm j during period i is defined as the difference between penalty cost and the total cost of ownership under the corresponding obsolescence strategy.

Calculation of Gittins Index for Arm 1:

The expected total cost of ownership when a decision maker chooses to store parts necessary for a particular period only (say period i), is given by:

$$E(TCO_{i,a1}) = \lambda_e \times F \times \left(C_{p,e} + \frac{C_{h,e}}{2} + M_e + \frac{O_e}{\lambda_e} \right) \times P_i \times \frac{1}{(1+r)^i} + (1 - P_i) \times U_p \quad \text{i} = 1,2,..., \text{N} \tag{25}$$

Where P_i is the probability that the part will be available for procurement during period i and U_p is the penalty cost associated with the risk of not being able to maintain fleet availability due to obsolescence (non availability of parts).

$$E(R_{i,a1}) = U_p - \sum_{k=1}^{i} E(TCO_{k,a1}), \quad \text{i} = 1, 2, ..., \text{N} \tag{26}$$

Note that the expected reward is defined as the difference between the penalty cost and the total cost of ownership up to period i. The objective here is to minimise the total cost of ownership.

The Gittins index for arm 1 up to period N is given by:

$$
\begin{aligned}
G_N^1 &= \sup_{N\geq 1} \sum_{i=1}^{N} \beta^i E(R_{i,a1}) / E \sum_{i=1}^{N} \beta^i \\
&= \sup\{ E(R_{1,a1}), \frac{\beta E(R_{1,a1}) + \beta^2 E(R_{2,a1})}{(\beta + \beta^2)}, \\
&\quad \frac{\beta E(R_{1,a1}) + \beta^2 E(R_{2,a1}) + \beta^3 E(R_{3,a1})}{(\beta + \beta^2 + \beta^3)}, \ldots\}
\end{aligned}
\tag{27}
$$

Calculation of Gittins Index for Arm 2:

The total cost of ownership for arm 2 is given by:

$$
E(TCO_{i,a2}) = TCO(i-1) + TCO_{RD,i} \times Q_i + (1-Q_i)U_p \quad \text{i}=1,2,\ldots,\text{N} \tag{28}
$$

where,

$$
TCO(i-1) = \overbrace{C_{p.e} \times \lambda_e \times F \times D_{i-1}}^{1} + \overbrace{\frac{\lambda_e \times F}{2} \times D_{i-1} \times C_{h,e}}^{2} + \overbrace{D_{i-1} \times F \times (\lambda_e \times M_e + O_e)}^{3}
$$

$$
TCO_{RD,i} = \frac{1}{(1+r)^i} \left(C_{RD} + C_{p.d} \times \lambda_d \times F + \frac{\lambda_d \times F}{2} \times C_{h,d} + F \times (\lambda_d \times M_d + O_d) \right)
$$

and Q_i is the probability that the redesigned item will survive the remaining (N-i) periods of the designed life. The expected reward for period i is given by:

$$
E(R_{i,a2}) = U_p - E(TCO_{i,a2}) \quad \text{i}=1,2,\ldots,\text{N} \tag{29}
$$

The Gittins Index up to period N for arm 2 is given by:

$$
\begin{aligned}
G_N^2 &= \sup_{N\geq 1} \sum_{i=1}^{N} \beta^i E(R_{i,a2}) / E \sum_{i=1}^{N} \beta^i \\
&= \sup\{E(R_{1,a2}), \frac{\beta E(R_{1,a2}) + \beta^2 E(R_{2,a2})}{(\beta + \beta^2)}, \\
&\quad \frac{\beta E(R_{1,a2}) + \beta^2 E(R_{2,a2}) + \beta^3 E(R_{3,a2})}{(\beta + \beta^2 + \beta^3)}, \ldots\}
\end{aligned}
\tag{30}
$$

Ultimately, the optimal strategy for period i is given by the arm whose Gittins Index value is the highest and substitution of this optimal strategy for each period in Equation (22) gives us the optimum sequence of decisions Π^* and the corresponding optimum expected reward $R(\Pi^*)$, over N periods can be obtained from Equation (23).

Proposition 4: If $\frac{TCO(i-1)}{TCO(i)} > P_i > Q_i$ then it is optimal to choose arm 1 for period i.

Proof:

$$\frac{TCO(i-1)}{TCO(i)} > P_i \Rightarrow TCO(i-1) > P_i TCO(i) \tag{31}$$

$$P_i > Q_i \Rightarrow (1-P_i)U_p < (1-Q_i)U_p \tag{32}$$

Equations (31) and (32) imply:

$$TCO(i-1) + TCO_{RD,i}Q_i + (1-Q_i)U_p > P_i TCO(i) + (1-P_i)U_p \tag{33}$$

$$\Rightarrow E(TCO_{i,a2}) > P_i TCO(i) + (1-P_i)U_p \tag{34}$$

$$\Rightarrow E(TCO_{i,a2}) > E(TCO_{i,a1})$$

(35)$\Rightarrow E(R_{i,a1}) > E(R_{i,a2})$, That is arm 1 is optimal for period i.

Proposition 5: If there exists a j such that, $\frac{TCO(j-1)}{TCO(j)} > P_j$ and $TCO_{RD,j}Q_i > TCO_{RD,j+1}Q_{i+1}$; then arm 2 is the optimal strategy and a stopping rule.

Proof:

If $G_j^2 > G_j^1$, then we have:

$$TCO(j-1) + TCO_{RD,j}Q_j + U_p(1-Q_j) < P_j TCO(j) + (1-P_j)U_p \tag{36}$$

From conditions stated in the proposition we have:

$$\frac{TCO(j-1)}{TCO(j)} > P_j \Rightarrow TCO_{RD,j} + U_p(1-Q_j) < (1-P_j)U_p \tag{37}$$

Since $TCO_{RD,j}Q_i > TCO_{RD,j+1}Q_{i+1}$ we get:

$$TCO_{RD,k}Q_k + U_p(1-Q_k) < (1-P_k)U_p \,.\ \forall\, k > j \tag{38}$$

Thus, if there exists a 'j' such that $G_j^2 > G_j^1$ and, $\frac{TCO(j-1)}{TCO(j)} > P_j$ and $TCO_{RD,j}Q_i > TCO_{RD,j+1}Q_{i+1}$ then arm 2 is the optimal strategy and a stopping rule.

4.2 Arm Acquiring MAB Model for Selection of Obsolescence Strategy

Consider a scenario similar to the one in section 4.1, except that more technologies can appear in the future periods, and the decision maker may have to choose strategies by considering all possible technologies available at any particular period. At any period only one arm is chosen and there is no passive reward. Arms are defined as follows:

Arm 1: The decision maker chooses to procure parts required for the current period (period i) only.

Arm j: The decision maker chooses to redesign the part using technology j (= 2, 3, ..., n) during period i.

Thus we have a arm-acquiring bandit problem and here again the Gittins index policy provides the optimal solution. The Gittins index for arm j can be calculated as follows:

The total cost of ownership for arm j, $E(TCO_{i,j})$ is given by:

$$E(TCO_{i,j}) = \left(TCO(i-1) + TCO_{RD,i} \times Q_i + (1-Q_i)U_p\right) \quad i = 1, 2, ..., N, \tag{39}$$

$$TCO(i-1) = \overbrace{C_{p.e} \times \lambda_e \times F \times D_{i-1}}^{1} + \overbrace{\frac{\lambda_e \times F}{2} \times D_{i-1} \times C_{h,e}}^{2} + \overbrace{D_{i-1} \times F \times (\lambda_e \times M_e + O_e)}^{3}$$

$$TCO_{RD,i} = \frac{1}{(1+r)^i}\left(C_{RD} + C_{p.d} \times \lambda_d \times F + \frac{\lambda_d \times F}{2} \times C_{h,d} + F \times (\lambda_d \times M_d + O_d)\right)$$

and Q_i is the probability that the redesigned item will survive the remaining (N-i) periods of the designed life. The expected reward for arm j period i is given by:

$$E(R_{i,j}) = \delta_{i,j}\left(U_p - E(TCO_{i,j})\right) \quad \mathrm{i} = 1, 2, ..., \mathrm{N}; \mathrm{j} = 2, 3, ..., \mathrm{n} \tag{40}$$

where,

$$\delta_{i,j} = \begin{cases} 1, \text{ if arm j is available during period i} \\ 0, \text{otherwise} \end{cases}$$

5.0 Calculation of Gittins Index – Illustrative Example

In this section, we use a hypothetical example to illustrate the Bandit process models discussed in section 4. The values of the parameters are shown in Table 1. Table 1 contains three sets of values, the first set of values correspond to the existing part which has become obsolete (arm 1), the second set of values correspond to the redesign option using technology 1 (arm 2) and third set of values correspond to the redesign option using technology 2 which will be available at the beginning of year 3 (arm 3). Using the data defined in Table 1 we have calculated Gittins indices for the following two scenarios.

Scenario 1:

The decision maker chooses the optimal strategy for management of obsolescence by considering the technological options available at the beginning of the decision making period. That is, a two-armed bandit model is used to calculate the Gittins Index values. The Gittins index values for arms 1 and 2 are shown in Table 2. From the values of the Gittins Index, the optimal strategy is arm 1 for the first six periods and arm 2 from period 7 onwards which is a stopping rule. The optimal strategy for the hypothetical problem is to redesign the part during period 7 and have normal operation for the first six periods. From the data in Table 2, one may notice that there is a 30% chance that the part may not be available in the market in period 6.

Scenario 2:

In scenario 2, we assume that the part may be redesigned using an alternative technology which will be available from period 3 onwards. This scenario is modelled using arm-acquiring bandit model. This is a three-armed bandit problem in which the third arm is available from period 3 onwards. The optimal solution to this problem is to use arm 1 for first four periods and arm 3 in period 5. Figure 3 shows Gittins index values for 3 arms. In both scenarios we have used decreasing probabilities for survival of the redesigned part since the technology used to redesign may also become obsolete before the designed life of the system.

6. Conclusions and Future Research

The life span of a capital asset is a critical period because the asset manager has to make several important decisions to ensure that availability of the system is maintained at the least cost of ownership. During the designed life of the system, some embedded parts may either become obsolete or become technologically inferior. In this paper, we have developed a few mathematical models that would assist a decision maker in choosing the best obsolescence mitigation strategy from a set of available strategies. In the first set of mathematical models, we have assumed that the decision maker either receives information about the part obsolescence or has knowledge about the prior distribution of time to obsolescence of a part embedded within the system or LRU. Using techniques like zero-one programming and expected marginal benefit, the model identifies the optimal time to redesign in each case.

The aforementioned myopic models may not be suitable when the remaining life of the system is large or if the rate of technological obsolescence is very high. In such situations, the decision maker has to use a sequence of decisions that is optimal. During each period, the decision maker gains some new information about the parts and is in a better position to judge between various options. We have modelled this problem using the restless multi-armed Bandit approach. As an illustrative example, a restless two-armed Bandit framework is used to show how the optimal strategy can be chosen in case of two strategic choices. The main advantage of the Bandit process approach is that the model allows the decision maker to update the model parameters when she moves from one period to the next. We have also illustrated how to calculate the Gittins Indices in the case of two-armed Bandit problems, which can be extended to cases of multi-armed bandit problems also. Another important aspect of obsolescence management problem is that more technologies may become available that can be used to redesign the obsolete part and the decision maker has to include all available technology to choose the best strategy. This scenario is modelled using arm acquiring bandit models.

In this paper, we have assumed that the arms are independent; however, this need not be true always. Consequently, there is scope for future research to develop mathematical models for choosing optimal obsolescence strategies under dependent arms. In the current paper, we have assumed that the system's life is not extended beyond the design life of the system, however, for many systems, life extensions are common practice and future research should consider these cases.

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Figures

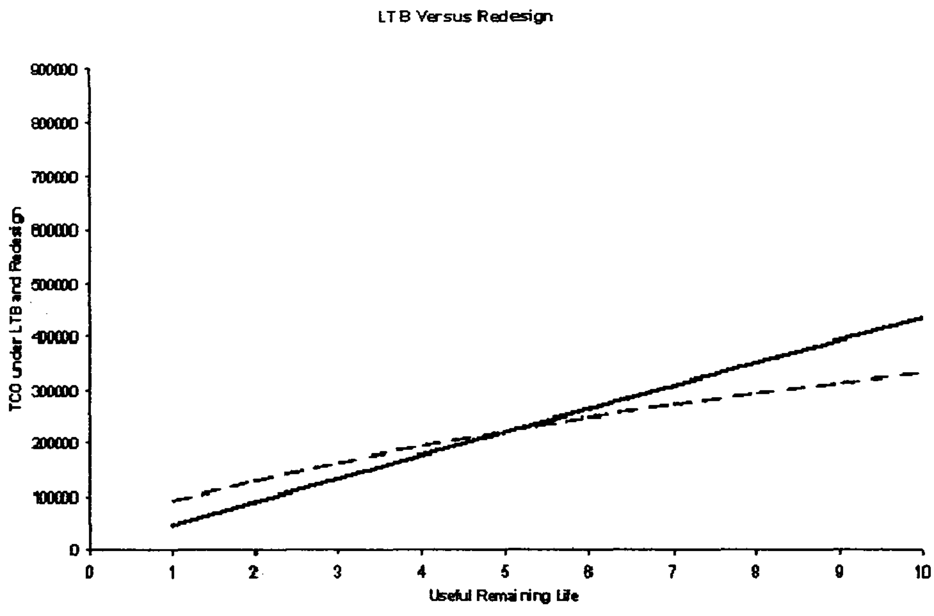

Figure 1. Comparison of TCO under LTB and Redesign strategies

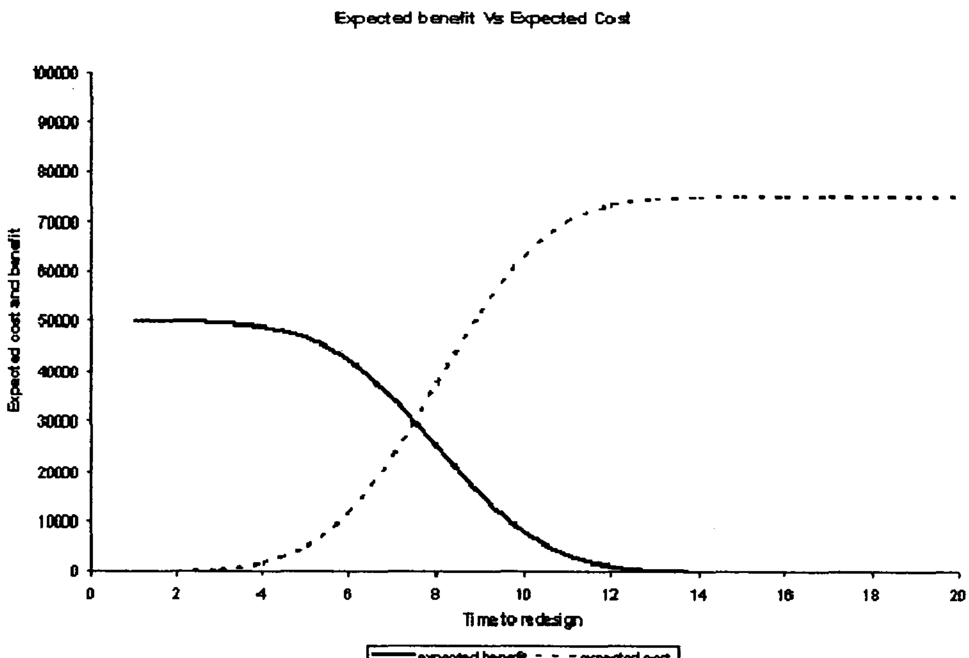

Figure 2. Cost benefit analysis of time-to-redesign

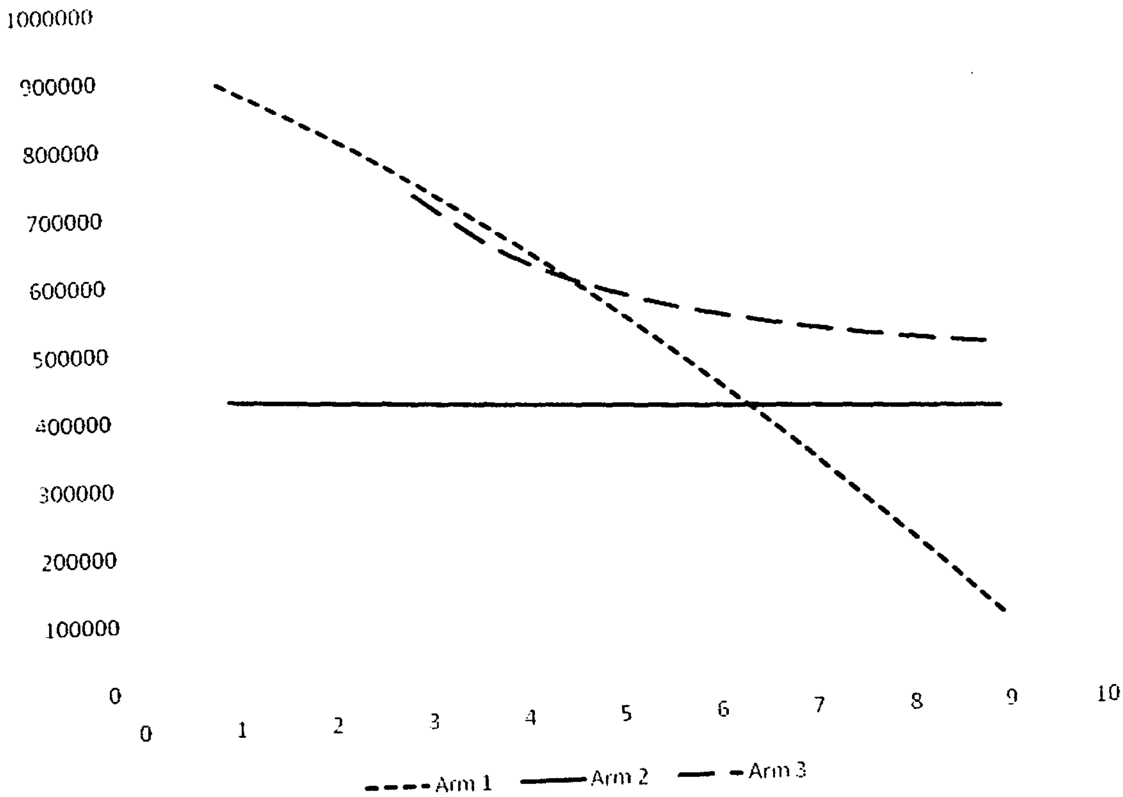

Figure 3. Gittins index values for arms 1, 2 and 3 (arm acquiring bandit model)