# Estimating Healthcare Demand for an Aging Population: A flexible and robust Bayesian Joint Model

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**Abstract**

We propose a joint model to combine models for hospital visits and out-of-pocket medical expenditures. It allows for the presence of non-linear effects of covariates using splines to capture the effects of aging on healthcare demand. Sample heterogeneity is modeled robustly with the random effects following Dirichlet process priors with explicit cross-part correlation. We validate our model using a simulation study. We apply this model to Health and Retirement Survey data and show that healthcare varies with age and gender and exhibits significant cross-part correlation that provides a richer understanding of how aging affects healthcare demand.

**Keywords:** Bayesian Methods, Joint Model, Healthcare Demand, Aging, Splines

**JEL Code:** C11, C14, I10

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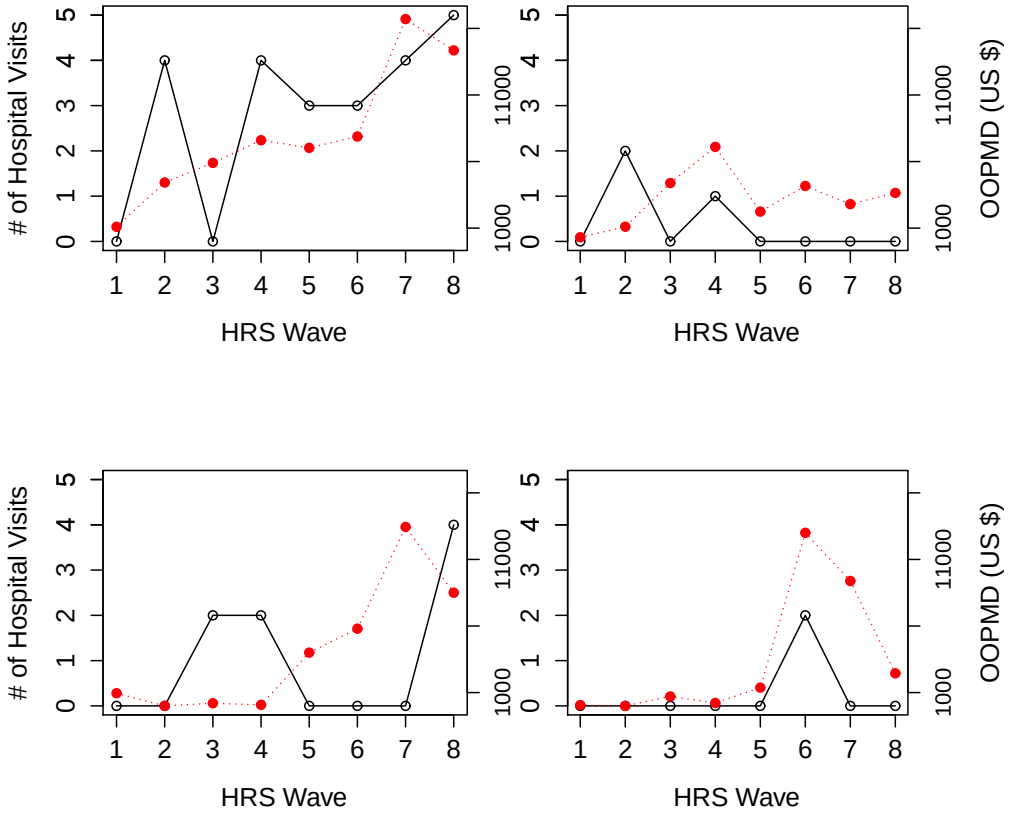
# 1 Introduction

The world population is aging. According to a joint report by the U.S. Department of State and the National Institute on Aging (NIA), almost 500 million people worldwide were 65 and older in 2006 (Dobriansky et al. 2007). That number is expected to increase to 1 billion, 1 in every 8 of the earths inhabitants, by the year of 2030. In the United States, life expectancy has increased from 49 years for Americans born in 1900 to 78 years for those born in 2006 (Arias 2010). Rapid demographic change is expected to lead to an increase in healthcare spending by 25% by 2030 (Strunk et al. 2006; Dobriansky et al. 2007). While global aging represents a triumph of medical, social, and economic advances, it also poses tremendous challenges for health systems. It is well understood that aging will change the mix of diseases in favor of chronic conditions for inpatient care and this alone is likely to increase demand for healthcare (Strunk et al. 2006; Hartman et al. 2008). With limited long-term benefits under Medicare, such increases in demand will lead to large out-of-pocket medical expenses for the elderly (Wei et al. 2004; Hartman et al. 2008). Thus, reliable estimates for the demand of healthcare has never been more important than now with aging becoming a worldwide challenge (Dobriansky et al. 2007).

Health economics has always focussed on healthcare demand and Duan et al. (1982)'s seminal work on healthcare demand explored different strategies to estimate medical expenditure to address data concerns specific to healthcare cost data. Another metric that is also frequently used to measure healthcare demand is the rate for hospital admission (Atella and Deb 2008). Clearly, out-of-pocket medical expenditure by an individual is closely related to the number of hospitalizations an individual experiences in a year (see Figure 1). In addition, the probability of needing healthcare increases with age, particularly with the onset of chronic conditions, as does the probability of out-of-pocket expenditures. Thus, managing healthcare demand better will require a better understanding of hospitalizations as well as medical expenditures. In this research we try to understand the key factors affecting both

Figure 1: Co-movement of Hospital Visits and OOP-Medical Expenditure




*Note*: On each x-axis we plot the survey wave in which the case was observed. On the left y-axis we plot the count of hospital visits made and on the right y-axis we plot the amount of out-of-pocket medical expenditure (OOPMD) in US dollars. The dotted line captures the OOPMD incurred at each wave while the straight line captures the count of hospital visits.

these endpoints by developing a novel joint modeling framework which allows us to reliably study healthcare demand and the correlation between these two endpoints.

Modeling these longitudinal events requires the full consideration of a number of complications specific to healthcare data. First, both hospitalization and out-of-pocket expenditure at the individual level usually have a considerable amount of zero observations, which cannot be adequately described by a simple distribution (e.g., Poisson or lognormal). For example,

in our data 90% of the sample have no hospital visits and 17% report zero out-of-pocket expenditures in wave 1; these numbers are 40% and 4% respectively by wave 8 (see Table 1). Spurious overdispersion occurs due to the presence of these extra zeros. Recently, Naya et al. (2008) compared model fits of a Poisson model and a zero-inflated Poisson (ZIP) model to zero-inflated data and found that a ZIP model gave estimates closer to the true values. Thus, we need to modify parametric distributions to incorporate excess zeros in the distribution of the hospitalizations and out-of-pocket expenditure. Recent literature (Deb and Trivedi 1997, Winkelmann 2004 and Atella and Deb 2008 and references therein) has developed zero-inflated distributions for modeling the count of hospital visits and medical costs; however, these are modeled independently. Second, the two main responses, hospital visits and medical costs, are likely to be correlated with each other over time for the same individual. Accounting for this correlation will result in borrowing information, which can lead to a better understanding of the healthcare demand. Third, some important characteristics of an individual, such as age, may have complex nonlinear effects. In addition, the potential nonlinear effects of this variable could vary with other demographic covariates, such as gender, resulting in an interaction effect that influences healthcare demand in a nonlinear fashion. Fourth, both the count of hospital visits and medical costs are known to be skewed (Liu et al. 2010). Although, some authors have argued for log transformation for dealing with skewness, this is problematic. Re-transformation presents no problem when errors achieve linearity, normality and homoscedasticity assumptions (Jones 2000). When any one of these does not hold, re-transformation bias arises when we try to revert back to the original scale. Since the log-transformed model results in geometric means rather than arithmetic means, log scale predictions will in general provide biased estimates of the impact of any explanatory variable on the arithmetic mean (Yu et al. 2011).

In this paper, we develop a joint model for describing count of hospital visits and out-of-pocket medical expenditure in an integrated framework to accommodate the aforementioned

complications as follows. We model the count of hospital visits by an individual using a Poisson hurdle model (Mullahy 1986) and we model the out-of-pocket medical expenditure using a semicontinuous model (Liu et al. 2010). The Poisson hurdle model (semicontinuous model) consists of two components: a Bernoulli component that models the probability of hospitalization (any positive expense) and a truncated Poisson component (log-normally distributed component) that models the number of repeat hospital visits (amount of money spent) among users. Together, these components accommodate both the high proportion of zeros and the right-skewness of the nonzero events. In addition, we explicitly account for interdependencies of these events by modeling the correlation of these two processes. While the literature on healthcare demand discusses "multi-part" models such as in the original work of Duan et al. (1982) or the more recent work of Liu et al. (2008), these are different from our model in a number of ways. These models look at a single outcome and the multi-part model allows for flexibility in model parameters across sub-groups with different demands for health care. For example, Duan et al. (1982) were interested in how the parameters vary by non-spenders, ambulatory spenders, and inpatient spenders; more recently, Liu et al. (2008) are interested in the differences between non-spenders, out-patients spenders and inpatient spenders. Our model provides a richer specification of healthcare demand that not only captures healthcare costs but also hospital visits within the same joint model with explicitly modeled random effects. In addition, our sample is a predominantly aging population where the effects of "age" on hospital visits and medical costs is poorly understood.

We thus adopt a semi-parametric approach using spline models to flexibly capture the possibly nonlinear effects of age. This approach not only protects the model from the possible misspecifications of age effects but also explores if this nonlinear effect varies across gender. For the distribution of the latent random effects terms of the joint model a standard assumption is to use a parametric distribution, such as the multivariate normal. The importance of

Table 1: Distribution of Outcomes

| | Count of Hospital Visits | | | | Out-of-Pocket Medical Expense | | | |
|---|---|---|---|---|---|---|---|---|
| HRS | % | Non-zeros | | | % | Non-zeros | | |
| Wave | Zeros | Mean | Min | Max | Zeros | Mean | Min | Max |
| 1 | 91.33 | 1.50 | 1 | 3 | 16.33 | 1,108 | 2 | 18,494 |
| 2 | 81.00 | 1.72 | 1 | 8 | 12.33 | 1,175 | 9 | 26,629 |
| 3 | 76.33 | 1.63 | 1 | 6 | 11.67 | 1,767 | 10 | 58,250 |
| 4 | 75.33 | 1.99 | 1 | 10 | 8.00 | 1,325 | 6 | 39,800 |
| 5 | 68.33 | 1.80 | 1 | 8 | 8.00 | 1,522 | 15 | 24,800 |
| 6 | 58.33 | 2.82 | 1 | 60 | 9.00 | 3,710 | 35 | 232,400 |
| 7 | 51.00 | 1.64 | 1 | 5 | 7.00 | 3,422 | 10 | 301,000 |
| 8 | 40.33 | 2.47 | 1 | 25 | 4.00 | 2,516 | 5 | 45,200 |

such a choice has received a lot of attention in the joint modeling literature. Particularly, it has been also shown that a restrictive parametric assumption for this distribution could influence the results (Tsonaka et al. 2009 and Naskar and Das 2006). Thus, in order to protect the derived inferences against potential misspecification effects, we opt for a semiparametric approach based on a Dirichlet Process prior. A similar approach to modeling random effects has been proposed by Jochmann and Leon-Gonzalez (2004), although they have considered it with a single endpoint and without splines.

The rest of the paper is organized as follows: Section 2 discusses the notation and presents the four part model with cross-equation and cross-part correlation as well as details for Bayesian inference. Section 3 presents simulation results that investigate the advantage of this class of models. Section 4 discusses the data we use, Section 5 presents our results, and Section 6 closes with a discussion of our proposed model.

# 2 A 4 Part Robust Semi-parametric Joint Model

Our joint model consists of three components: a semiparametric Poisson hurdle mixed effects model for the number of hospitalizations, a semiparametric semicontinuous model for out-of-pocket medical expenses, and a Dirichlet process for the joint distribution of the latent

random effects from the Poisson hurdle model and the semi-continuous models.

## 2.1 Poisson Hurdle Model for the Count of Hospital Visits

The hurdle Poisson model is a two-component mixture model consisting of a point mass at zero followed by a truncated Poisson for the nonzero observations (Mullahy 1986). For independent and identically distributed (i.i.d.) responses, the hurdle model is given by

$$\begin{aligned} \Pr(Y_i = 0) &= 1 - p, \quad 0 \leq p \leq 1 \\ \Pr(Y_i = k) &= p\frac{\mu^k e^{-\mu}}{k!(1 - \mathrm{e}^{-\mu})}, \quad k = 1, \ldots, \infty,\ 0 < \mu < \infty, \end{aligned} \tag{1}$$

where $Y_i$ denotes the response for subject $i = 1, \ldots, n$, and $\mu$ is the mean for an untruncated Poisson distribution. As the zeros and nonzero counts are modeled uniquely, the hurdle model accommodates both an excess number of zeros and a right-skewed distribution for the positive counts. By comparison, a standard Poisson regression would have to compromise between these two competing goals, since excess zeros would tend to lower the Poisson mean while large nonzero values would tend to increase it. The expected count under the Poisson hurdle model is given by $\mathrm{E}(Y) = p\mu/\left(1 - \mathrm{e}^{-\mu}\right)$.

In health services research, $p$ is known as the *usage probability*—i.e., the probability of using services at least once. When $(1 - p) > \mathrm{e}^{-\mu}$, the data are zero inflated relative to an ordinary Poisson; when $(1-p) < e^{-\mu}$ there is zero deflation (i.e., fewer than expected zeros). In the extremes, $p = 0$ or 1. When $p = 1$, there are no zero counts and the model reduces to a truncated Poisson, and when $p = 0$, there are no users (i.e., all counts equal zero), and the model is degenerate at zero. Typically, one assumes that $p$ is strictly between 0 and 1, so that all subjects have a nonzero probability of usage and are therefore considered "potential" users even if they do not actually use services during the study period.A special case of (1) is the zero-inflated Poisson model (Lambert 1992), which consists of a degenerate distribution

at zero mixed with an un-truncated Poisson distribution:

$$
\begin{aligned}
P(Y_i = 0) &= (1-p) + p\mathrm{e}^{-\mu}, \quad 0 < p < 1 \qquad (2)\\
P(Y_i = k) &= p\frac{\mu^k \mathrm{e}^{-\mu}}{k!}, \quad k = 1, \ldots, \infty,\ 0 < \mu < \infty. \qquad (3)
\end{aligned}
$$

Note that the zero-inflated Poisson model can be rewritten as a hurdle model with mixing probability $\theta = p(1 - \mathrm{e}^{-\mu})$. Unlike the hurdle model, which accommodates zero deflation as well as zero inflation, the ZIP allows only for zero inflation and thus allows for greater flexibility (Neelon et al. 2010). Let $Y_{ij}^H$ be the count of number of hospital stays reported by the $i$th subject in the $j$th wave, $i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, n$, where $m$ represents the number of subjects in the study, and $n$ is the total number of waves over which the individual is surveyed. Depending on the fact whether a subject is hospitalized or not, a large number of zeros is observed in $Y_{ij}^H$. Also, let $X_{ijk}$ be the $k^{th}$ covariate for subject $i$ at time $j$; such covariates include baseline and time-varying variables.

Each subject's total count of hospital visits is determined simultaneously by needing some healthcare ($p_{ij}$) as well as the level of care needed given that the person needs care $\lambda_{ij}$. Given that these are jointly determined, and that the determinants of either may or may not be relevant for the other, we consider simultaneous modeling of both $\lambda_{ij}$ and $p_{ij}$. The hurdle model can be extended to accommodate covariates and random effects as follows:

$$
\begin{aligned}
p(y_{ij}^H|\boldsymbol{\phi}_i) &= (1 - p_{ij}^H)1_{(y_{ij}^H=0)} + p_{ij}^H \mathrm{Tpois}(y_{ij}^H; \mu_{ij}^H)1_{(y_{ij}^H>0)}\\
\mathrm{logit}(p_{ij}^H) &= \mathbf{X}_{ij1}^T\boldsymbol{\beta}_1^p + \mathbf{Z}_{ij1}^T\boldsymbol{b}_{i1} + f^p(W_{ij})\\
\log(\mu_{ij}^H) &= \mathbf{X}_{ij2}^T\boldsymbol{\beta}_1^\lambda + \mathbf{Z}_{ij2}^T\boldsymbol{b}_{i2} + f^\lambda(W_{ij}) \qquad (4)
\end{aligned}
$$

where, $\mathbf{X}_{ij1}, \mathbf{X}_{ij2}$ are the vector of covariates corresponding to fixed effects and $\mathbf{Z}_{ij1}, \mathbf{Z}_{ij2}$ are the vector of covariates corresponding to the random effects. Note that the zero-state and

the Poisson state do not need to have the same set of covariates. The $b_{i1}$ and $b_{i2}$ are the random subject effects on $p_{ij}$ and $\lambda_{ij}$, respectively. We will discuss the distribution of the random subject effects later. In many situations, such as ours, the effect of some covariates, viz., $W_{ij}$ on $p_{ij}^H$ and $\mu_{ij}^H$ may not be linear. Thus, effects of those covariates can be modeled by unspecified nonparametric functions $f^p(W_{ij})$ and $f^\lambda(W_{ij})$. These unknown smooth functions reflect the nonlinear effects of the covariate. However, these functions represent only the population averages for a single population.

We now consider a modified model for multiple factors/populations. Instead of fitting one nonparametric smoothing spline for a single population, we can include multiple nonparametric smoothing splines for multiple populations in one model. We consider:

$$\begin{aligned}
\text{logit}(p_{ij}^H) &= \mathbf{X}_{ij1}^T\boldsymbol{\beta}_1^p + \mathbf{Z}_{ij1}^T\boldsymbol{b}_{i1} \\
&+ f_1^p(W_{ij})d_{ij1}^p + f_2^p(W_{ij})d_{ij2}^p + \cdots + f_L^p(W_{ij})(1 - d_{ij1}^p - d_{ij2}^p - \cdots - d_{ij(L-1)}^p) \quad (5) \\
\log(\mu_{ij}^H) &= \mathbf{X}_{ij2}^T\boldsymbol{\beta}_1^\lambda + \mathbf{Z}_{ij2}^T\boldsymbol{b}_{i2} \\
&+ f_1^\lambda(W_{ij})d_{ij1}^\lambda + f_2^\lambda(W_{ij})d_{ij2}^\lambda + \cdots + f_L^\lambda(W_{ij})(1 - d_{ij1}^\lambda - d_{ij2}^\lambda - \cdots - d_{ij(L-1)}^\lambda) \quad (6)
\end{aligned}$$

where, $d_{ijk}$; $k = 1, 2, \cdots, L$ are indicator variables for multiple populations. With $L$ populations, the first group is indicated by $(d_{ij1} = 1,\ d_{ij2} = 0, \cdots, d_{ij(L-1)} = 0)$, the second group is indicated by $(d_{ij1} = 0,\ d_{ij2} = 1, \cdots, d_{ij(L-1)} = 0)$ and the last group is indicated by $(d_{ij1} = 0,\ d_{ij2} = 0, \cdots, d_{ij(L-1)} = 0)$. The $f_1, f_2, \cdots, f_L$ are their respective nonparametric smoothing splines.

We approximate the spline function $f(W_{ij})$ (suppressing the subscripts) by a piecewise polynomial of degree $\tau$. Let the knots $\tilde{w} = (\tilde{w_1}, \tilde{w_2}, \cdots, \tilde{w_m})$ are placed within the range of $W_{ij}$, such that $\min(W_{ij}) < \tilde{w_1} < \tilde{w_2} < \cdots < \tilde{w_m} < \max(W_{ij})$. Then $f(W_{ij})$ is approximated

by

$$f(W_{ij}) = \nu_1 W_{ij} + \nu_2 W_{ij}^2 + \cdots + \nu_\tau W_{ij}^\tau + \sum_{c=1}^{C} u_c \gamma_c (W_{ij} - \tilde{w}_c)_+^\tau$$

where $X_+ = x$ if $x > 0$, and 0 otherwise, $\nu = (\nu_1, \cdots, \nu_\tau)$, $\tilde{w}$ are the vectors for regression coefficients in the polynomial regression spline. Note that there is no intercept in the polynomial regression to avoid the identifiability. We assume $u_c \sim^{iid} N(0, \sigma_u^2); i = 1, \ldots, C$. In the above formulation one of the important issue is the choice of how many knot point and where to locate them. If there are too few knots or they are poorly located, estimated curve may be biased, while too many knots will inflate the local variance. Thus, following Smith and Kohn (1996) we incorporate selector indices, $\gamma_c$, that allow the spline coefficients to be included or excluded and that are defined for each knot. The $\gamma_c$ are then drawn independently from a Bernoulli prior, viz., $\gamma_c \sim$ Bernoulli(0.5). By introducing this, we can select a subset of well supported knots from a larger space. For each knot point $u_c$ the $\gamma_c$ will weight the importance of a particular knot point.

## 2.2 Semicontinuous Model for Out-of-Pocket Medical Costs

In this section a semi-continuous model for longitudinal data on out-of-pocket medical cost data is introduced. Since in some years the subject may not have any medical cost, this kind of data has a mix of many zeros and positive continuous observations. To formulate the model, let $y_{ij}^M$ be the medical cost for subject $i$ at year $j$. Let $R_{ij}$ be a random variable denoting the yearly medical cost where,

$$R_{ij} = \begin{cases} 0, & \text{if } y_{ij}^M = 0 \\ 1, & \text{if } y_{ij}^M > 0, \end{cases} \tag{7}$$

with conditional probabilities

$$\Pr(R_{ij} = r_{ij}) = \begin{cases} 1 - p_{ij}^M, & \text{if } r_{ij} = 0 \\ p_{ij}^M, & \text{if } r_{ij} = 1, \end{cases} \tag{8}$$

For this semicontinuous data, we introduce an analogous semi-continuous model consisting of a degenerate distribution at zero and a positive continuous distribution, such as as a lognormal (LN), for the nonzero values:

$$\begin{aligned} f(y_{ij}^M|\mathbf{p}_i^M) &= (1-p_{ij}^M)^{1-r_{ij}} \left\{p_{ij}^M \times \text{LN}(y_{ij}^M; \mu_{ij}^M, \sigma^2)\right\}^{r_{ij}} \\ \text{logit}(p_{ij}^M) &= \mathbf{X}_{ij}^T \boldsymbol{\beta}_1^{Mp} + \mathbf{Z}_{ij1}^T \boldsymbol{b}_{i3} \\ &+ h_1^p(W_{ij}) e_{ij1}^p + h_2^p(W_{ij}) e_{ij2}^p + \cdots \\ &+ h_L^p(W_{ij})(1 - e_{ij1}^p - e_{ij2}^p - \cdots - e_{ij(L-1)}^p) \qquad (9) \\ \log(\mu_{ij}^M) &= \mathbf{X}_{ij}^T \boldsymbol{\beta}_1^{M\lambda} + \mathbf{Z}_{ij2}^T \boldsymbol{b}_{i4} \\ &+ h_1^\lambda(W_{ij}) e_{ij1}^\lambda + h_2^\lambda(W_{ij}) e_{ij2}^\lambda \cdots \\ &+ h_L^\lambda(W_{ij})(1 - e_{ij1}^\lambda - e_{ij2}^\lambda - \cdots - e_{ij(L-1)}^\lambda) \qquad (10) \end{aligned}$$

where, $r_{ij}$ is an indicator defined above, $\mu_{ij}^M$ and $\sigma^2$ are the mean and variance of $\log(y_{ij}^M)$. The interpretation of $e_{ijk}$ is same as $d_{ijk}$ in the ZIP model and the nonparametric spline functions $h(.)$ is also defined in a similar fashion. Model (9-10) is a semiparametric counterpart of the correlated two-part model proposed by Olsen and Schafer (2001); a gamma or log-skew-normal may also be used to model the nonzero values.

## 2.3 Latent Random Effects Distribution: Dirichlet Process Prior

Without loss of generality, we assume that all $\mathbf{b}_{ik}$ in (5,6,9,10) are $r \times 1$ unobserved vectors. Let $\mathbf{b} = (\mathbf{b}_1, \ldots, \mathbf{b}_m)$ denote the random effects for all the $m$ subjects, where $\mathbf{b}_i = (\mathbf{b}_{i1}^\top, \mathbf{b}_{i2}^\top, \mathbf{b}_{i3}^\top, \mathbf{b}_{i4}^\top)^\top \in \mathbb{R}^{4r}$, $i = 1, \ldots, m$, is a $4r \times 1$ vector representing the random effects

for the $i$th subject. To allow for correlation structure between repeated observations from the same subject taken over different years and also to account for uncertainty in probability distributions of the random effects, we assume the unknown distribution $G$ of random effects for different subjects, $\mathbf{b}_i$, $i = 1, \ldots, m$, to be a Dirichlet process (DP), which is a popular choice of a random probability measure served as a prior distribution over the space of probability measures discussed in Ferguson (1973). That is,

$$\begin{aligned} \mathbf{b}_i|G &\overset{\text{iid}}{\sim} G,\ i = 1, \ldots, m, \\ G|a, G_0 &\sim \texttt{DP}(aG_0), \qquad \text{with } G_0 = \texttt{N}_{4r}(\mathbf{0}, \boldsymbol{\Sigma}), \end{aligned} \tag{11}$$

where random effects for different subjects are exchangeable, and they are random vectors distributed as $G$, which is a random probability measure from the DP characterized by a total mass $a > 0$ and a base probability measure $G_0$ as a $4r$-variate normal distribution with a zero mean vector and a $4r \times 4r$ variance-covariance matrix $\boldsymbol{\Sigma} = (\sigma_{ij})_{4r \times 4r}$.

A priori the unknown $G$ is expected to be the same as $G_0$, while $a$ is a precision parameter that measures the strength of this prior belief. Following Blackwell and MacQueen (1973), given $a, G_0$, the joint (marginal) distribution of the exchangeable sequence of unobserved random effects $\mathbf{b}_1, \ldots, \mathbf{b}_m$ is summarized by the Pólya urn distribution. That is,

$$\mathbf{b}_1 \sim G_0, \text{ and } \mathbf{b}_i|\mathbf{b}_1, \ldots, \mathbf{b}_{i-1} \sim \frac{a}{a+i-1} G_0 + \sum_{k=1}^{i-1} \frac{1}{a+i-1} \delta_{\mathbf{b}_k},\ i = 2, \ldots, m.$$

This implies that $G$ is an almost surely discrete random probability measure which assigns positive probability to ties/duplicates among $\mathbf{b}$. Let $\mathbf{B} = (\mathbf{B}_1, \ldots, \mathbf{B}_{|\mathbf{c}|})$ denote the unique vectors among $\mathbf{b}$, where $\mathbf{c} = \{c_1, \ldots, c_m\}$, with $c_i = 1, \ldots, m$, is a classification vector induced by the relationship that $c_i = j$ if and only if $\mathbf{b}_i = \mathbf{B}_j$, and $|\mathbf{c}| \leq m$ records the number of unique values (or equivalently, the largest value) in the vector $\mathbf{c}$. An alternative description

of the joint distribution of all the random effects $\mathbf{b}_1, \ldots, \mathbf{b}_m$ follows from Antoniak (1974) in terms of a classification vector $\mathbf{c}$ and the unique vectors among $\mathbf{b}$, which gives the joint density of the random effects as

$$\mathbf{m}(\mathbf{b}) = \Pr(\mathbf{c}|a) \prod_{k=1}^{|\mathbf{c}|} \phi_{4r}(\mathbf{B}_k; \mathbf{\Sigma}), \tag{12}$$

where

$$\Pr(\mathbf{c}|a) = \frac{a^{|\mathbf{c}|} \prod_{k=1}^{|\mathbf{c}|}(n_k - 1)}{\prod_{i=1}^{m}(a + i - 1)}$$

with $n_k$ being the total number of $\mathbf{b}_i$, $i = 1, \ldots, m$, identical to $\mathbf{B}_k$, or equivalently, the number of $c_i$ equal to $k$, and $\phi_p(\cdot; \mathbf{\Sigma})$ represents the probability density function of a $p$-variate normal vector with a zero mean vector and a variance-covariance matrix $\mathbf{\Sigma}$. That is, given the vector $\mathbf{c}$, one can summarize the random effects $\mathbf{b}$ for all $m$ subjects by $\mathbf{B}_1, \ldots, \mathbf{B}_{|\mathbf{c}|}$. This idea, or analogous idea in terms of partitions, plays a key role in not only characterizing the posterior distribution but also designing most efficient and popular numerical algorithms for inference in Bayesian hierarchical models involving DP.

## 2.4 Bayesian Inference

Under the joint model described in (5,6,9,10), the likelihood of the observed data for the $i$th subject, denoted by $\mathbf{Y}_{i1}, \ldots, \mathbf{Y}_{in}$, with $\mathbf{Y}_{ij} = (y_{ij}^H, y_{ij}^M)^\top$ for $j = 1, \ldots, n$, based on the parameters set $\Omega$ and the random effects $\mathbf{b}_i$ is proportional to

$$\begin{aligned} L_i(\mathbf{Y}_{i1}, \ldots, \mathbf{Y}_{in}|\Omega, \mathbf{b}_i) &= \prod_{j=1}^{n} \left[(1 - p_{ij}^H)\right]^{I_{[y_{ij}^H = 0]}} \times \left[\frac{p_{ij}^H \mu_{ij}^{H\, y_{ij}^H} e^{-\mu_{ij}^H}}{y_{ij}^H!(1 - e^{-\mu_{ij}^H})}\right]^{1 - I_{[y_{ij}^H = 0]}} \\ &\times (1 - p_{ij}^M)^{1 - r_{ij}} \left\{p_{ij}^M \times \mathrm{LN}(y_{ij}^M; \mu_{ij}^M, \sigma^2)\right\}^{r_{ij}} \end{aligned} \tag{13}$$

Assuming independence between observations from different subjects, the resulting likelihood for all the observations from the $m$ subjects is the product of these individual likelihood values. Then, marginalizing out all the random effects which are modeled by a DP as given

in (11) with a fixed $a > 0$ yields that the likelihood of all the observed data is proportional to

$$L(\Omega|\text{data}) = \int_{\mathbb{R}^r} \cdots \int_{\mathbb{R}^r} \prod_{i=1}^{m} L_i(\Omega, \mathbf{b}_i|\mathbf{Y}_{i1}, \ldots, \mathbf{Y}_{in})\mathbf{m}(\mathbf{b})d\mathbf{b}_1 \cdots d\mathbf{b}_m,$$

which is an $m$-folded integral. To complete the Bayesian specification of the model, we assign priors to the unknown parameters in the above likelihood function. Thus, the set of parameters from the model may be listed as:

$$\begin{aligned}\Omega \;=\; & \Big(\beta_{11}^{Hp}, \beta_{21}^{H_\lambda}, \beta_{31}^{Mp}, \beta_{41}^{M_\lambda}, \ldots, \beta_{19}^{Hp}, \beta_{29}^{H_\lambda}, \beta_{39}^{Mp}, \beta_{49}^{M_\lambda}, \nu_1^{Hp}, \ldots, \nu_\tau^{Hp}, \sigma_{H_p}^2, \\ & \nu_1^{H_\lambda}, \ldots, \nu_\tau^{H_\lambda}, \sigma_{H_\lambda}^2, \nu_1^{Mp}, \ldots, \nu_\tau^{Mp}, \sigma_{M_p}^2, \nu_1^{M_\lambda}, \ldots, \nu_\tau^{M_\lambda}, \sigma_{M_\lambda}^2, \Sigma, a\Big) \end{aligned} \tag{14}$$

For each parameter in $\Omega$ we next specify a prior: for each model specific regression coefficient ($\beta_{ij}^{\theta}$) and each spline specific regression coefficient ($\nu_i^{\theta}$) we assume a normal density prior; for each variance parameter ($\sigma_\theta^2$) we assume an inverse-gamma (IG) prior and finally for the cross-part variance covariance matrix ($\Sigma$) we assume an inverse Wishart prior. Further, for the total mass, $a$ we assume a uniform distribution (Ohlseen et. al. 2007).

An IG prior with shapre parameter $c$ and scale parameter $d$ is denoted by $x \sim IG(c, d)$ and its density is given by $f(x) \propto x^{-c}e^{(d/2x^2)}$. Additionally, we assume a Wishart distribution for the inverse of a variance covariance matrix where $W_G(\rho, s)$ is a G-dimensional Wishart distribution with $\rho$ degrees of freedom and a mean of $\rho s^{-1}$. Thus, we specify the following

priors on the model parameters:

$$\begin{aligned}
\pi(\underset{\sim}{\beta}) &= \left(\beta_{11}^{Hp}, \ldots, \beta_{49}^{M_\lambda}, \right) \sim N(\underset{\sim}{\mu_\beta}, \Sigma_\beta) \\
\pi(\underset{\sim}{\nu}^{Hp}) &= \left(\nu_1^{Hp}, \ldots, \nu_\tau^{Hp}\right) \sim N(\mu_\nu^{Hp}, \Sigma_\nu^{Hp}) \\
\pi(\underset{\sim}{\nu}^{H_\lambda}) &= \left(\nu_1^{H_\lambda}, \ldots, \nu_\tau^{H_\lambda}\right) \sim N(\mu_\nu^{H_\lambda}, \Sigma_\nu^{H_\lambda}) \\
\pi(\underset{\sim}{\nu}^{Mp}) &= \left(\nu_1^{Mp}, \ldots, \nu_\tau^{Mp}\right) \sim N(\mu_\nu^{Mp}, \Sigma_\nu^{Hp}) \\
\pi(\underset{\sim}{\nu}^{M_\lambda}) &= \left(\nu_1^{M_\lambda}, \ldots, \nu_\tau^{M_\lambda}\right) \sim N(\mu_\nu^{M_\lambda}, \Sigma_\nu^{M_\lambda})
\end{aligned}$$

For the remaining variance parameters, the variance covariance matrix and $a$ we assume:

$$\begin{aligned}
\pi(\sigma_{Hp}^2) &\sim IG(c_{Hp}, d_{Hp}) \\
\pi(\sigma_{H_\lambda}^2) &\sim IG(c_{H_\lambda}, d_{H_\lambda}) \\
\pi(\sigma_{Mp}^2) &\sim IG(c_{Mp}, d_{Mp}) \\
\pi(\sigma_{M_\lambda}^2) &\sim IG(c_{M_\lambda}, d_{M_\lambda}) \\
\pi(\Sigma^{-1}) &= Wishart(\rho, s) \\
\pi(a) &= Uniform(e, f)
\end{aligned}$$

The joint posterior distribution of the parameters of the models conditional on the data are obtained by combining the likelihood and the prior densities using Bayes Theorem:

$$\begin{aligned}
Post(\Omega, \mathbf{b}|\mathbf{Y}) \propto & \int_{\mathbb{R}^r} \cdots \int_{\mathbb{R}^r} \prod_{i=1}^{m} L_i(\mathbf{Y}_{i1}, \ldots, \mathbf{Y}_{in}|\Omega, \mathbf{b}_i)\mathbf{m}(\mathbf{b})\pi(\underset{\sim}{\beta})\pi(\underset{\sim}{\nu}^{Hp})\pi(\underset{\sim}{\nu}^{H_\lambda}) \\
& \pi(\underset{\sim}{\nu}^{Mp})\pi(\underset{\sim}{\nu}^{M_\lambda})\pi(\sigma_{Hp}^2)\pi(\sigma_{H_\lambda}^2)\pi(\sigma_{Mp}^2)\pi(\Sigma^{-1})\pi(a)d\mathbf{b}_1 \cdots d\mathbf{b}_m \qquad (15)
\end{aligned}$$

The posterior distributions are analytically intractable. However, models described previously can be fit using Markov chain Monte Carlo (MCMC) methods such as the Gibbs sampler (Gelfand, Dey and Chang 1992). Since the full conditional distributions are not

standard, a straightforward implementation of the Gibbs sampler using standard sampling techniques may not be possible. However, sampling methods can be performed using adaptive rejection sampling (ARS; Gilks and Wild 1992). In this research, we follow their procedure, which first uses a data augmentation step to sample the values of the latent variables based on the current value of the parameters, and then samples the parameters using the ARS method given the latent variables. Samples were directly obtained from the joint posterior distribution of the parameters as well as the latent variables. The samples from the posterior obtained from the MCMC will allow us to achieve summary measures of the parameter estimates and to obtain credible intervals (CIs) of the parameters of interest.

# 3 Simulation Study

In this section, we present two simulation exercises to justify the relative complexity of the proposed model. This will also verify the performance of the model fitting procedure over more conventional models. The complexity of the proposed model arises from two aspects: 1) using a DP for the skewed distributed random effects $\boldsymbol{b_i}$ and (2) spline-based modeling of nonlinear time effects. The purpose of our simulation study is to verify the performance of our proposed model in comparison to simpler and parsimonious but parametric models.

## 3.1 Using DP model for skewed distributed random effects $b_i$

This simulation evaluates the performance of our method when the random effects are from skewed distribution. For this simulation we consider the following models:

$$\begin{aligned}
\text{logit}(p_{ij}^H) &= \beta_{11} + \beta_{12}t_{ij} + \beta_{13}X_i + \beta_{14}Z_{ij} + b_{i1} \\
\log(\lambda_{ij}) &= \beta_{21} + \beta_{22}t_{ij} + \beta_{23}X_i + \beta_{24}Z_{ij} + b_{i2} \\
\text{logit}(p_{ij}^M) &= \beta_{31} + \beta_{32}t_{ij} + \beta_{33}X_i + \beta_{34}Z_{ij} + b_{i3} \\
\log(s_{ij}+1) &= \beta_{41} + \beta_{42}t_{ij} + \beta_{43}X_i + \beta_{44}Z_{ij} + b_{i4} + e_{ij} \qquad (16)
\end{aligned}$$

In this model, we consider a subject-specific baseline covariate $X_i$, random intercepts $b_i =$

$(b_{i1}, b_{i2}, b_{i3}, b_{i4})'$, and a time-varying covariate $Z_{ij}$, where $i = 1, 2, \ldots, 100$ and $j = 1, 2, \ldots, 16$. Data are generated from Equation (16) to mimic the real data presented in the article. The data is generated using following steps:

1. $X_i$'s are assumed to be continuous and generated from a univariate normal distribution with mean $\mu_X = 0$ and $\sigma_X = 0.5$.

2. Time dependent covariates ($Z_{ij}$) for 16 time-points are generated using $MVN(\boldsymbol{\mu_Z}, \boldsymbol{\Sigma_Z})$. In order to maintain a correlation between the $Z$ values in adjacent time-points within one subject $\boldsymbol{\Sigma_Z}$ was assumed an AR(1) variance-covariance structure.

3. Random intercepts $b_{i1}$ are generated from a skewed bimodal distribution (a balanced mixture of the N(-1, 2.25) and log normal (2.30, 0.48) distributions). In order to create correlated random effects $b_{il}$ was generated as linear combination of $b_{i1}, b_{i2}, \ldots b_{il-1}$ and skewed bimodal distribution described as above ($l = 2, 3, 4$).

4. The $e_{ij}$'s of the two part model are generated from normal distribution.

5. Finally, we generated $Y_{i1}$ from a hurdle Poission distribution($\boldsymbol{p^H}$, $\boldsymbol{\lambda}$) and $Y_{i2}$ from TP($\boldsymbol{p^M}$, $\boldsymbol{s}$). Parameter values used in the simulation are chosen to produce data that are similar to the real data. In particular, we take $\beta_{11} = 6.23$, $\beta_{12} = 1.41$, $\beta_{13} = 0.81$, $\beta_{14} = -0.21$, $\beta_{21} = -3.32$, $\beta_{22} = -0.32$, $\beta_{23} = -0.94$, $\beta_{24} = 0.08$, $\beta_{31} = -0.10$, $\beta_{32} = -0.34$, $\beta_{33} = 0.02$, $\beta_{34} = -1.65$, $\beta_{41} = -3.70$, $\beta_{42} = 0.30$, $\beta_{43} = 0.49$ and $\beta_{44} = -0.23$.

6. One thousand simulated data sets are used in the simulation study.

Using generated data described above, we fit our proposed model with normal random effects and DP random effects. Model performance is evaluated for both the normal and DP model for random effects $b_i$. Our results are presented in Table 2. We have computed the bias, mean square error (MSE) and coverage probability (CP). The numbers in parentheses in

column 1 of Table 2 are the true population values of the parameters. Our simulation results show that the DP model produces better estimates of the model parameters with minimal bias and better coverage probabilities compare to normal model.

Table 2: Results for Normal and DP models in the presence of skewed random effects

| | Normal Model | | | | DP Model | | | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Mean | Bias | MSE | CP | Mean | Bias | MSE | CP |
| *Logit-$p^H$* | | | | | | | | |
| $\beta_{11}^{H_p}$(6.23) | 4.33 | -1.91 | 2.444 | 0.85 | 7.02 | 0.79 | 1.087 | 0.93 |
| $\beta_{12}^{H_p}$(1.41) | 0.62 | -0.79 | 0.457 | 0.82 | 1.77 | 0.15 | 0.202 | 0.95 |
| $\beta_{13}^{H_p}$(0.81) | 0.87 | 0.06 | 0.313 | 0.90 | 0.85 | 0.04 | 0.133 | 0.97 |
| $\beta_{14}^{H_p}$(-0.21) | -0.13 | 0.08 | 0.271 | 0.89 | -0.25 | -0.03 | 0.274 | 0.90 |
| *Log-linear-$\mu^H$* | | | | | | | | |
| $\beta_{11}^{H_\lambda}$(-3.32) | -5.25 | -1.93 | 1.476 | 0.86 | -3.93 | -0.61 | 1.003 | 0.91 |
| $\beta_{12}^{H_\lambda}$(-0.32) | -0.23 | 0.09 | 0.333 | 0.89 | - 0.29 | 0.03 | 0.091 | 0.94 |
| $\beta_{13}^{H_\lambda}$(-0.94) | -0.81 | 0.13 | 0.452 | 0.90 | -0.91 | -0.03 | 0.512 | 0.91 |
| $\beta_{14}^{H_\lambda}$(0.08) | -0.01 | -0.07 | 0.122 | 0.84 | 0.05 | -0.03 | 0.006 | 0.95 |
| *Logit-$p^M$* | | | | | | | | |
| $\beta_{11}^{M_p}$(-0.10) | -0.08 | 0.02 | 0.022 | 0.93 | -0.11 | -0.01 | 0.062 | 0.94 |
| $\beta_{12}^{M_p}$(-0.34) | 0.12 | 0.46 | 0.013 | 0.76 | -0.36 | -0.02 | 0.014 | 0.96 |
| $\beta_{13}^{M_p}$(0.02) | 0.005 | -0.01 | 0.070 | 0.83 | 0.02 | 0.00 | 0.086 | 0.98 |
| $\beta_{14}^{M_p}$(-1.65) | -0.47 | 1.18 | 0.097 | 0.85 | -0.99 | 0.66 | 0.035 | 0.91 |
| *Log-$\mu^M$* | | | | | | | | |
| $\beta_{11}^{M_\lambda}$(-3.70) | -7.12 | -3.42 | 1.303 | 0.79 | -4.17 | -0.47 | 1.406 | 0.94 |
| $\beta_{12}^{M_\lambda}$( 0.30) | 0.09 | -0.21 | 0.082 | 0.81 | 0.33 | -0.03 | 0.047 | 0.92 |
| $\beta_{13}^{M_\lambda}$(0.49) | 0.48 | -0.01 | 0.013 | 0.96 | 0.48 | -0.01 | 0.029 | 0.96 |
| $\beta_{14}^{M_\lambda}$(-0.23) | -0.11 | 0.12 | 0.107 | 0.90 | -0.35 | -0.12 | 0.103 | 0.95 |

*Note:* Number in parenthesis next to each parameter indicates its true population value.

## 3.2 Spline-based modeling of nonlinear time effects

This simulation study illustrates the performance of our proposed model under complexity of nonlinear time effect. For this simulation we have considered the following model:

$$\begin{aligned}
\text{logit}(p_{ij}^H) &= \beta_{11} + \beta_{12}t_{ij} + \beta_{13}X_i + \beta_{14}Z_{ij} + b_{i1} + f^p(t_{ij}) \\
\log(\lambda_{ij}) &= \beta_{21} + \beta_{22}t_{ij} + \beta_{23}X_i + \beta_{24}Z_{ij} + b_{i2} + f^\lambda(t_{ij}) \\
\text{logit}(p_{ij}^M) &= \beta_{31} + \beta_{32}t_{ij} + \beta_{33}X_i + \beta_{34}Z_{ij} + b_{i3} + f^M(t_{ij}) \\
\log(s_{ij}+1) &= \beta_{41} + \beta_{42}t_{ij} + \beta_{43}X_i + \beta_{44}Z_{ij} + b_{i4} + f^s(t_{ij}) + e_{ij} \qquad (17)
\end{aligned}$$

Table 3: Results for Parametric and Spline models in the presence of nonlinear time effects

| | Linear Model | | | | Model with Spline | | | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Mean | Bias | MSE | CP | Mean | Bias | MSE | CP |
| *Logit-$p^H$* | | | | | | | | |
| $\beta_{11}^{H_p}$(6.23) | 5.33 | 0.90 | 1.044 | 0.90 | 6.02 | 0.21 | 1.087 | 0.93 |
| $\beta_{12}^{H_p}$(1.41) | 0.92 | -0.59 | 0.457 | 0.89 | 1.57 | 0.16 | 0.202 | 0.95 |
| $\beta_{13}^{H_p}$(0.81) | 0.87 | 0.06 | 0.313 | 0.90 | 0.85 | 0.04 | 0.133 | 0.97 |
| $\beta_{14}^{H_p}$(-0.21) | 0.13 | 0.44 | 0.471 | 0.83 | -0.25 | -0.03 | 0.274 | 0.90 |
| *Log-linear-$\mu^H$* | | | | | | | | |
| $\beta_{11}^{H_\lambda}$(-3.32) | -5.25 | -1.93 | 1.476 | 0.90 | -3.93 | -0.61 | 1.003 | 0.92 |
| $\beta_{12}^{H_\lambda}$(-0.32) | -0.23 | 0.09 | 0.333 | 0.89 | - 0.29 | 0.03 | 0.091 | 0.93 |
| $\beta_{13}^{H_\lambda}$(-0.94) | -0.81 | 0.13 | 0.452 | 0.90 | -0.91 | -0.03 | 0.512 | 0.91 |
| $\beta_{14}^{H_\lambda}$(0.08) | -0.01 | -0.07 | 0.122 | 0.84 | 0.05 | -0.03 | 0.006 | 0.95 |
| *Logit-$p^M$* | | | | | | | | |
| $\beta_{11}^{M_p}$(-0.10) | -0.21 | -0.11 | 0.070 | 0.90 | -0.08 | 0.02 | 0.074 | 0.92 |
| $\beta_{12}^{M_p}$(-0.34) | -0.89 | -0.55 | 0.116 | 0.89 | -0.16 | 0.18 | 0.131 | 0.90 |
| $\beta_{13}^{M_p}$(0.02) | 0.05 | 0.03 | 0.162 | 0.95 | 0.03 | 0.01 | 0.057 | 0.97 |
| $\beta_{14}^{M_p}$(-1.65) | -2.78 | -1.23 | 0.172 | 0.89 | -1.41 | 0.24 | 0.078 | 0.95 |
| *Log-$\mu^H$* | | | | | | | | |
| $\beta_{11}^{M_\lambda}$(-3.70) | -3.51 | 0.19 | 0.109 | 0.90 | -3.56 | 0.14 | 0.112 | 0.93 |
| $\beta_{12}^{M_\lambda}$(0.30) | 0.66 | 0.36 | 0.082 | 0.88 | 0.54 | 0.24 | 0.068 | 0.90 |
| $\beta_{13}^{M_\lambda}$(0.49) | 0.82 | 0.39 | 0.207 | 0.90 | 0.33 | -0.16 | 0.058 | 0.91 |
| $\beta_{14}^{M_\lambda}$(-0.23) | -0.84 | -0.61 | 0.066 | 0.87 | -0.11 | 0.12 | 0.042 | 0.93 |

*Note:* Number in parenthesis next to each parameter indicates its true population value.

In this model, $f^p$, $f^\lambda$, $f^M$ and $f^s$ are nonlinear time effects for 16 time-points, while the remaining variables have the same interpretation as Equation 16. Data for the simulation is generated from 17 using following steps:

1. $X_i$, $Z_{ij}$ and $e_{ij}$ are generate same way as described in Step 1, 2 and 4 in Simulation 1.

2. Random effects $b_i$'s are generated from multivariate normal distribution.

3. Nonlinear time effects are generated using the nonlinear functions $f^p(t) = 1/9\cos^2((t+9)/17)$, $f^\lambda(t) = -0.9 + 0.005\exp((12+t)/12)$, $f^M(t) = 1/2\cos((t+12)/12)\sin(t/19)$ and $f^s(t) = -1.7 + 0.005\exp(t/2)I_{\{t\geq 8\}}$.

4. $Y_{i1}$ and $Y_{i2}$ are generated from a hurdle model and semi-continuous distribution respectively as described in Step 5 of Simulation 1 using $\boldsymbol{p^H}$, $\boldsymbol{\lambda}$, $\boldsymbol{p^M}$ and $\boldsymbol{s}$ from 17.

5. One thousand data sets are generated for illustrating model performance.

Spline model and linear time effect model are fitted with normal random effect. Results of the simulation is presented in Table 3. We have computed the bias, mean square error (MSE) and coverage probability (CP).In the presence of nonlinear time effects, linear time effect model often produce higher bias and substantially lower CP in the time-varying covariates, although estimates of other covariates appears comparable. Based on both simulations, we conclude that the models used in the analysis have good performance in the modeling aging data. Despite the increased complexity, the new analysis provides a safeguard against potential effects of misspecification of the time effects, thus preventing the occurrence of large biases in the estimation of time-varying effects.

# 4 Data Analysis

## 4.1 Motivating Data Description

We use data from the University of Michigan's Health and Retirement Study (HRS) to estimate the above model. The HRS is a longitudinal survey of Americans over the age of 50 with a follow-up frequency of two years and is designed to provide multi-disciplinary data to understand challenges of aging. In this paper we use data from the 1931-41 cohort - the HRS cohort. Baseline observations for the HRS cohort begins in 1992 when individuals were between 52-62 years of age and were near retirement.[1]For our outcome measures we use the number of hospital trips made in the past year and the total out-of-pocket medical expenses (OOPMD) that excludes all costs that were reimbursed or paid through insurance.[2]

Figure 1 presents plots of hospital visits and associated out-of-pocket medical expenditure (OOPMD) over time for four randomly chosen individuals. A number of things stand out - first, individuals vary widely in the number of hospital visits that they make. Not only do

[1]The data we use is maintained by RAND's Center for Study of Aging and has been comprehensively cleaned and documented (St.Clair et al. 2009).

[2]In practice we also restrict the HRS cohort further to include only those who did not drop-out of the study in the first 5 of the eight waves of the study to allow for sufficient length in the panel.

individuals have different intensities of hospital visits but the associated OOPMD for the same number of hospital visits varies; additionally, these graphs present preliminary evidence to show that the number of hospital visits and OOPMD are correlated - as the number of hospital visits increases (or decreases) so does the OOPMD. This co-movement in these outcomes suggest that modeling them jointly is important as OOPMD depends on the count of hospital visits made and vice-e-versa.

Table 1 presents summary statistics of both outcome measures for a randomly selected set of cases for whom we have non-missing observations for the first 4 waves. The count of hospital visits exhibits increasing frequency of missing observations in later waves; additionally, there is a high but declining fraction of the sample in each wave with zero hospital visits. This change in demand over time is also captured through narrower ranges of outcomes in earlier waves than in later waves, re-emphasizing the important effects of aging. Thus, while in wave 1 we have over 90% of the sample did not visit a hospital, by the last wave fraction has declined to 40%. This high frequency of zeros suggests support for the use of a Poisson hurdle model for hospital visits. Similarly, OOPMD also shows significant zero-inflation suggesting that treating it as a continuous variable would be problematic. As people age, the frequency of hospital visits rises, and so does OOPMD. We see this in Table 1 as the frequency of zeros declines the average OOPMD rises from USD 1,108 to USD 2,516. Descriptive statistics for the baseline and time varying covariates are presented in Table 4.

A key aspect of aging is a loss of functional abilities (muscular strength, ventilatory capacity, incontinence, or cardiovascular output); however the rate of this decay varies with lifestyle, and environmental factors (Wei et al. 2004). Many of these, such as gender, education, occupational status or functional independence (*rnodiffdress*) are observed in the HRS; additionally, data on each individual's self-reported health status, in the current and past wave is used as it is known to be predictive of health status (McGee et al 1998). The HRS also collects each respondent's expectation of being alive for the next ten years or more on 0

Table 4: Summary Statistics of Response and Predictors

| **Variables** | **Mean** | **SD** | **Min** | **Max** |
|---|---|---|---|---|
| *Time Invariant* | | | | |
| Is Female? ($female_i$) | 54.30% | 49.89% | 0 | 1 |
| Eduction: GED or Higher? ($gedplus_i$) | 71.67% | 45.14% | 0 | 1 |
| *Time Varying* | | | | |
| Count of Hospital Visits | 0.44 | 1.72 | 0 | 60 |
| OOPMD | 2563.26 | 9707.71 | 0 | 301000 |
| Do Health problems limit work? ($rhlthlm_{ij}$) | 0.26 | 0.44 | 0 | 1 |
| Has no difficulty in dressing ($rnodiffdress_{ij}$) | 0.95 | 0.22 | 0 | 1 |
| Self-reported expectation of living 10+ years ($rlive_{ij}$) | 61.31 | 29.60 | 0 | 100 |
| Change of health at current wave ($coh_{ij}$) | 0.09 | 0.86 | -2 | 2 |
| Change of health at previous wave ($coh_{i,j-1}$) | -0.01 | 1.43 | -4 | 4 |

to 100 scale (*rlive*); Hurd and McGarry (2002) have shown its importance to understanding mortality. A last covariate we use is the age of the respondent; in almost any aging study the age of the respondent is an important predictor of health outcomes (Strunk et al. 2006; and Wei et al. 2004), as it is believed that the age of the respondent is predictive of his or her healthcare demand and associated costs.

## 4.2 Model Specifications and Empirical Results

Before discussing our result we first compare our model with some other candidate models to test the quality of model fit that our model shows. To compare candidate models, we computed $P(Y_i|Y_{-i})$ which is the posterior predictive distribution of $Y_i$ conditional on the observed data with a single data point deleted. This value is known as the conditional predictive ordinate (CPO) and has been widely used for model diagnostic and assessment (Gelfand et al 1992). For the $i^{th}$ subject the CPO statistics under model $M_l : 1 \leq l \leq L$ is defined as:

$$CPO_i = P(Y_i|Y_{-i}) = E_{\underset{\sim}{\theta} l}\left[P(Y_i|\underset{\sim}{\theta} l)|Y_{-i}\right] \tag{18}$$

where $-i$ denotes the exclusion of the data from subject $i$. The $\underset{\sim}{\theta_l}$ is the set of parameters of the $M_l$ and $P(Y_i|\underset{\sim}{\theta_l})$ is the sampling density of the model evaluated at the $i^{th}$ observation. The preceding expectation is taken with respect to the posterior distribution of the model

parameter $\underset{\sim}{\theta_l}$ given the cross-validated data, $Y_{-i}$. For subject $i$ the $CPO_i$ can be obtained from the MCMC samples by computing the following weighted average:

$$C\hat{P}O_i = \left( \frac{1}{M} \sum_{m=1}^{M} \frac{1}{f(Y_i|\theta_l^{(m)})} \right)^{-1} \tag{19}$$

where $M$ is the number of simulations, $\theta_l^{(m)}$ denotes the parameter samples at the $m^{th}$ iteration. A large CPO value indicates a better fit. A useful summary statistic of the $CPO_i$ is the logarithm of the Psuedo-marginal Likelihood (LPML) defined as:

$$LPML = \sum_{i=1}^{n} log(C\hat{P}O_i) \tag{20}$$

Models with greater $LPML$ values represent a better fit. The LPML is well defined under the posterior predictive density it is computationally stable. We compared the following models using LPML:

**Model 1** A 4PM model used in the analysis and whose results we discuss below

**Model 2** A 4PM model where each part is modeled independently without Random Effects

**Model 3** A 4PM model with correlated random effects in a multivariate normal distribution

**Model 4** The 4PM model with robust Random Effects but no age splines or interaction.

The LPML values for models 1-4 were $-5405.7$, $-7198.4$, $-6201.8$ and $-61332.4$ respectively. The proposed model has the highest LPML values suggesting that it had the best fit amongst the candidate models. The large difference in the LPML values of our proposed model and other model indicated the presence of a nonlinear age effect and the need for DP in our analysis.

We, thus, formulate an empirical version of the 4-part model discussed above for our aging

data. Equations 21 and 22 present the zero-inflated semi-continuous component of the model that seeks to explain hospital visits. The same set of covariates are allowed to differentially impact the propensity for visiting a hospital (in Equation 21) and the count of such visits made (in Equation 22).

$$\begin{aligned}
\text{logit}(p_{ij}^H) &= \beta_{11}^p + \beta_{12}^p t_{ij} + \beta_{13}^p \text{gedplus}_i + \beta_{14}^p \text{female}_i + \beta_{15}^p \text{rhlthlm}_{ij} + \beta_{16}^p \text{rnodiff}_{ij} \\
&+ \beta_{17}^p \text{rlive}_{ij} + \beta_{18}^p \text{coh}_{ij} + \beta_{19}^p \text{coh}_{i,j-1} + f_1^p(age_{ij}) d_{ij1}^p + f_2^p(age_{ij})(1 - d_{ij1}^p) + b_{i1} \quad (21) \\
\log(\mu_{ij}^H) &= \beta_{11}^\lambda + \beta_{12}^\lambda t_{ij} + \beta_{13}^\lambda \text{gedplus}_i + \beta_{14}^\lambda \text{female}_i + \beta_{15}^\lambda \text{rhlthlm}_{ij} + \beta_{16}^\lambda \text{rnodiff}_{ij} \\
&+ \beta_{17}^\lambda \text{rlive}_{ij} + \beta_{18}^\lambda \text{coh}_{ij} + \beta_{19}^\lambda \text{coh}_{i,j-1} + f_1^\lambda(age_{ij}) d_{ij1}^\lambda + f_2^\lambda(age_{ij})(1 - d_{ij1}^\lambda) + b_{i2} \quad (22)
\end{aligned}$$

Similarly, Equations 23 and 24 are the two components of the semi-continuous hurdle model for out-of-pocket medical expenses incurred. For both, the Poisson hurdle model and the semicontinuous model, age is allowed to flexibly affect both the propensity and level of healthcare demand through a smoothing spline that is allowed to vary by gender.

$$\begin{aligned}
\text{logit}(p_{ij}^M) &= \beta_{11}^{M_p} + \beta_{12}^{M_p} t_{ij} + \beta_{13}^{M_p} \text{gedplus}_i + \beta_{14}^{M_p} \text{female}_i + \beta_{15}^{M_p} \text{rhlthlm}_{ij} + \beta_{16}^{M_p} \text{rnodiff}_{ij} \\
&+ \beta_{17}^{M_p} \text{rlive}_{ij} + \beta_{18}^{M_p} \text{coh}_{ij} + \beta_{19}^{M_p} \text{coh}_{i,j-1} + h_1^p(age_{ij}) e_{ij1}^p \\
&+ h_2^p(age_{ij})(1 - e_{ij1}^p) + b_{i3} \quad (23) \\
\log(\mu_{ij}^M) &= \beta_{11}^{M_\lambda} + \beta_{12}^{M_\lambda} t_{ij} + \beta_{13}^{M_\lambda} \text{gedplus}_i + \beta_{14}^{M_\lambda} \text{female}_i + \beta_{15}^{M_\lambda} \text{rhlthlm}_{ij} + \beta_{16}^{M_\lambda} \text{rnodiff}_{ij} \\
&+ \beta_{17}^{M_\lambda} \text{rlive}_{ij} + \beta_{18}^{M_\lambda} \text{coh}_{ij} + \beta_{19}^{M_\lambda} \text{coh}_{i,j-1} + h_1^{M_\lambda}(age_{ij}) e_{ij1}^{M_\lambda} \\
&+ h_2^{M_\lambda}(age_{ij})(1 - e_{ij1}^{M_\lambda}) + b_{i2} \quad (24)
\end{aligned}$$

Finally, in Equations 21, 22, 23 and 24, the random effects $\mathbf{b_i} = (b_{i1}, b_{i2}, b_{i3}, b_{i4})$ are jointly modeled as a DP ($aG_0 \equiv N_4(0, \Sigma)$ and $a \sim Uniform(0.4, 10)$. To fully specify the Bayesian model we assign weakly informative conjugate priors for the parameters. For each aggregate-level coefficient, we assume a normal density prior of $\text{N}(0, 100)$. For the variance parameters

we assume inverse-Gamma (IG) priors of IG$(2.01, 1.01)$, giving rise to a prior mean of 1 and a prior variance of 100. Lastly, we take an inverse-Wishart prior for the variance-covariance matrix by assuming $\Sigma^{-1} \sim \text{Wishart}(4, 0.1\text{I}_4)$, where $\text{I}_4$ is the $4 \times 4$ identity matrix. Each of this multi-part joint model with robust random effects captures important aspects of healthcare demand.

Table 5: Poisson Hurdle Model for Hospital Visits

| | parameter | mean | 95% Credible Interval |
|---|---|---|---|
| **Logit:** $p^H$ | | | |
| Intercept | $\beta_{11}^{H_p}$ | 6.32 | [ 0.46, 12.26] |
| Wave | $\beta_{12}^{H_p}$ | 1.40 | [ 0.75, 2.35] |
| Education: GED or Higher? | $\beta_{13}^{H_p}$ | 0.81 | [-0.99, 2.69] |
| Is Female? | $\beta_{14}^{H_p}$ | 0.76 | [ 0.45, 2.24] |
| Does health limit work? | $\beta_{15}^{H_p}$ | 0.58 | [ 0.07, 2.05] |
| Has no difficulty in dressing | $\beta_{16}^{H_p}$ | -1.09 | [-3.78,- 0.63] |
| Self-reported expectation of living 10+ years | $\beta_{17}^{H_p}$ | -0.17 | [-0.24,- 0.08] |
| Self-reported health: $\Delta$ in current wave | $\beta_{18}$ | 0.21 | [ 0.57, 1.04] |
| Self-reported health: $\Delta$ in previous wave | $\beta_{19}$ | 0.59 | [ 0.1, 1.41] |
| **Log:** $\mu^H$ | | | |
| Intercept | $\beta_{11}^{H_\lambda}$ | -3.32 | [-4.52,- 0.91] |
| Wave | $\beta_{12}^{H_\lambda}$ | -0.32 | [-1.04, 0.06] |
| Education: GED or Higher? | $\beta_{13}^{H_\lambda}$ | -0.94 | [-3.22, 0.37] |
| Is Female? | $\beta_{14}^{H_\lambda}$ | -1.02 | [-2.83, 0.03] |
| Does health limit work? | $\beta_{15}^{H_\lambda}$ | 0.56 | [ 0.03, 0.85] |
| Has no difficulty in dressing | $\beta_{16}^{H_\lambda}$ | -0.11 | [-1.4,- 0.08] |
| Self-reported expectation of living 10+ years | $\beta_{17}^{H_\lambda}$ | 0.08 | [ 0.04, 0.15] |
| Self-reported health: $\Delta$ in current wave | $\beta_{18}^{H_\lambda}$ | -0.19 | [-0.45, 0.001 ] |
| Self-reported health: $\Delta$ in previous wave | $\beta_{19}^{H_\lambda}$ | -0.06 | [-0.15, 0.002 ] |

Estimates for the two part Poisson hurdle model from Equations 21 and 22 are reported in Table 5. The top panel reports the determinants of the propensity for visiting a hospital while the bottom panel looks at the determinants of the count of hospital visits conditional on visits. Quite clearly, flexibility to differentially affect the logit and log portions are important with almost each variable behaving differentially in the two components. Two exceptions to this are if the respondent finds his health condition limits his ability to work and if he has any difficulty in dressing. If health condition limits work, or there is difficulty in dressing then they both raise the propensity to visit a hospital as well as the number of

hospital visits conditional on there being any visits at all. From the top panel of Table 5 it is evident that, holding all other effects constant, with time (wave) the propensity to visit the hospital increases. Here males are less likely to visit a hospital, on average, than women, while respondents with higher self-reported expectations of being alive for the next ten years have a lower propensity to visit the hospital. Changes in self-reported health, in this case, worsening health, is also associated with increases in the propensity for hospital visits.

Table 6: Two Part Model for Out-of-Pocket Medical Expenses

| | parameter | mean | 95% Credible Interval |
|---|---|---|---|
| **Logit:** $p^M$ | | | |
| Intercept | $\beta_{11}^{M_p}$ | -0.09 | [-3.06,2.86] |
| Wave | $\beta_{12}^{M_p}$ | -0.35 | [-3.41,-1] |
| Education: GED or Higher? | $\beta_{13}^{M_p}$ | 0.02 | [-2.74,3.08] |
| Is Female? | $\beta_{14}^{M_p}$ | -0.01 | [-3.04,2.84] |
| Does health limit work? | $\beta_{15}^{M_p}$ | -0.07 | [-3.08,2.85] |
| Has no difficulty in dressing | $\beta_{16}^{M_p}$ | -0.13 | [-3.32,-0.07] |
| Self-reported expectation of living 10+ years | $\beta_{17}^{M_p}$ | -1.65 | [-4.32,-0.46] |
| Self-reported health: $\Delta$ in current wave | $\beta_{18}^{M_p}$ | 0.05 | [-2.85,2.7] |
| Self-reported health: $\Delta$ in previous wave | $\beta_{19}^{M_p}$ | -0.04 | [-3.25,2.8] |
| **log:** $\mu^M$ | | | |
| Intercept | $\beta_{11}^{M_\lambda}$ | -3.69 | [-4.56,-2.95] |
| time | $\beta_{12}^{M_\lambda}$ | 0.31 | [0.08,0.42] |
| Education: GED or Higher? | $\beta_{13}^{M_\lambda}$ | 0.49 | [-0.24,1.29] |
| Is Female? | $\beta_{14}^{M_\lambda}$ | 0.45 | [0.14,1.11] |
| Does health limit work? | $\beta_{15}^{M_\lambda}$ | -0.01 | [-0.26,0.23] |
| Has no difficulty in dressing | $\beta_{16}^{M_\lambda}$ | 0.11 | [-0.02,-0.5] |
| Self-reported expectation of living 10+ years | $\beta_{17}^{M_\lambda}$ | -0.23 | [-0.38,-0.01] |
| Self-reported health: $\Delta$ in current wave | $\beta_{18}^{M_\lambda}$ | 0.01 | [-0.1,0.12] |
| Self-reported health: $\Delta$ in previous wave | $\beta_{19}^{M_\lambda}$ | 0.03 | [-0.06,0.13] |

Table 6 reports estimates from the semicontinuous model for out-of-pocket medical expenditure (OOPMD). A number of interesting differences with the Poisson hurdle model are noted. First, the propensity for any OOPMD is unaffected by education levels, gender, health conditions that may affect work, or self-perceived changes in health status. Thus, holding other things constant, with later waves, with no difficulty in dressing themselves, and with a higher self-reported expectation of being alive for the next ten years, respondents have a lower propensity for any OOPMD. However, once we condition on any OOPMD we

find that, holding other things constant, subsequent waves have higher OOPMD, women experience higher costs than men, and interestingly, higher self-reported probabilities of being alive for the next ten years are associated with lower OOPMD.

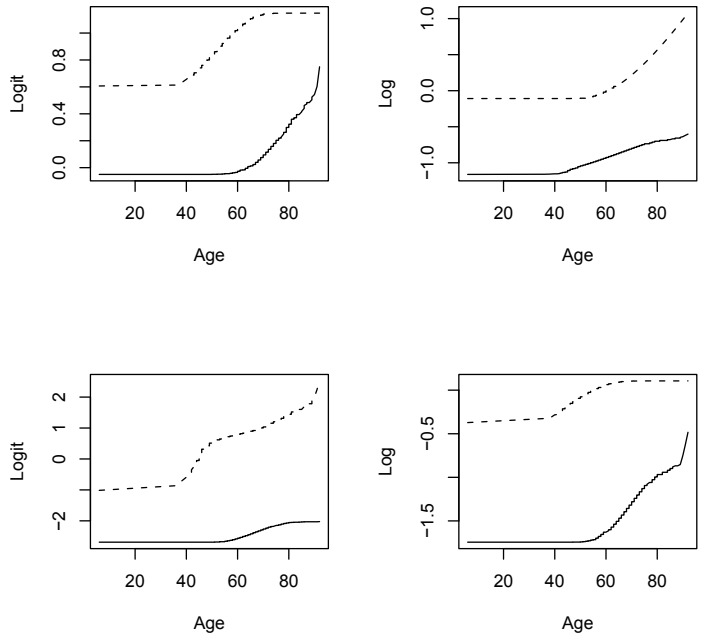
With the self-reported expectation of being alive for 10+ years variable and the difficulty in dressing variable being statistically significant in each of the four components of the 4 part model, it is only natural to expect significant correlation across the random effects from each of the components. Table 7 presents estimates for the correlation coefficients across the 4 components of the model. The two correlation coefficients between the random effects that are non-zero are the correlation between the random effects of the logit and log components of the Poisson hurdle sub-model and that between the random effects of the log portion of the Poisson hurdle model and the log portion of the semi-continuous hurdle model. The first is negative and suggests that individuals with larger unobserved effects on the propensity of hospitalization tend to have lower unobserved effects on the conditional count of hospital visits. While statistically significant, the correlation coefficient is much smaller (0.20) than the correlation seen between the random effects from the conditional count of hospital visits from the Poisson hurdle model and the random effects from the conditional OOPMD component of the semi-continuous model (0.66). The high correlation between the unobserved components of the conditional count of hospital visits and conditional OOPMD is expected as unobserved factors that determine hospital visits are closely related to unobserved factors that explain OOPMD. Interestingly, there is no correlation between the random effects from the propensity to visit a hospital and the random effect from the conditional OOPMD model. This suggests that while the conditional count of hospital visits and the conditional OOPMD are closely related to each other, the propensity to visit a hospital at all is determined differently.

Finally, we look at the effect of age on health care demand and how it varies as people age and with gender. Figure 2 plots the effect of aging on each components of the 4 part

Table 7: Correlation between Random Effects Across Models

| | mean | 95% Credible Interval |
|---|---|---|
| corr between logit and log of ZIP | -0.20 | [-0.49,-0.07] |
| corr between logit of ZIP and logit of semi-continuous | 0.00 | [-1.89,1.78] |
| corr between log of ZIP and logit of semi-continuous | -0.28 | [-1.91,-0.09] |
| corr between logit of ZIP and log of semi-continuous | 0.02 | [-0.09,0.16] |
| corr between log of ZIP and log of semi-continuous | 0.66 | [0.11,1.25] |
| corr between logit and log of semi-continuous | 0.00 | [-0.13,0.16] |

Figure 2: Non-linear Effects of Aging for each part of the 4PM




*Note*: On each plot the x-axis measures age in years. The top two plots capture the gender effects of the Poisson Hurdle model. The top left captures the difference in the propensity for any hospital visit and no hospital visit for women (dotted line) and men (straight line). The top right captures the conditional count of hospital visits. The bottom two captures gender effects in semicontinuous model with the bottom left capturing gender differences in propensity for any OOPMD while the bottom right captures the gender difference in conditional OOPMD.

model. Each of these diagrams show that the demand for health care varies significantly across a person's life and across gender; note this is not apparent from the base effects that we would see in the regression tables. Section (a) shows that there is a large difference in the baseline levels of demand for health care with women having higher propensity for any hospital visits. For women the baseline demand for health care does not change until the age

of 40 after which it rises linearly till the age of 60. Post the age of 60, further aging appears to have almost no additional impact on the propensity to use hospital facilities. Men on the other hand, have no change in baseline propensity to visit a hospital until the age of almost 60. Thereafter, the propensity to visit a hospital at least once increases exponentially. The conditional demand for health care in terms of the count of visits behaves somewhat differentially - women visit more frequently over the entire life time, while men maintain their baseline rates of hospitalization almost until the age of 60. Thereafter men start visiting a hospital more frequently than they had in the past, however the increase is slower than the increase in conditional counts seen for women.

Similarly, with OOPMD we find that women are more likely to incur costs and they also tend to incur larger costs than men at each of their life cycle. From the age of 40 the propensity to incur costs rises rapidly till the age of 60 and thereafter it increases at a much more modest rate for women. For men there is no change in the baseline propensity of incurring OOPMD until the age of 60. Thereafter, there is a modest increase in the propensity for incurring any OOPMD. In terms of conditional OOPMD expenditure given that there has been some, it is clear that women incur substantially higher costs throughout there lifetime than men, with a modest increase after the age of 40. Consistent with the Poisson hurdle model, men have a much lower level of baseline conditional OOPMD expenditure till the age of 60. After the age of 60, conditional OOPMD expenditure increases very rapidly and the gap between men and women expenditure declines rapidly, but does not fully go away.

# 5 Conclusion

In this paper we estimate healthcare demand for an aging population using a Bayesian semi-parametric joint modeling framework. We bring a number of interesting adaptations to this joint model to ensure that our model is both robust and yet allows us to flexibly estimate a key covariate for an aging population - the effects of age itself. In a Bayesian framework

we allow for zero-inflation that is a key characteristic for each healthcare demand endpoint without which our estimates are problematic (Duan et al. 1982; Olsen and Schafer 2001; Liu et al. 2010). Thus, our model is a four-part model (4PM) that differentially captures the propensities for usage as well we levels of use differentially across the two end points. This enables us to uncover complex patterns of correlations across a range of covariates and at different portions of the distribution of each outcome. Using DP priors to specify random effects for each participant allows us to reliably estimate health care demand after accounting for unobserved heterogeneity. Finally the correlation across the components allows us to borrow information across endpoints to better understand the co-movement in our joint model in a way that has not been done for healthcare demand.

The 4PM model also provides us a way to capture a number of important aspects of how aging will influence healthcare demand. Age splines and it's interaction with gender allows us to show that at younger age healthcare demand is higher for women, however past 60 years of healthcare demand for men increases very rapidly. This affects not only the need for hospital visits but it also affects out-of-pocket medical expenses. These have different inferences - with increasing aging there is need for greater profiling of men as they near 60 and has implications for the hospital industry, while greater out-of-pocket medical expenses will have important implications for financial planning as well as insurance systems. The robustly specified non-parametric random effects enable us to control for sample wide heterogeneity providing greater confidence in these estimates. While modeling healthcare demand for this population presents several challenges, this type of semicontinuous data is not unique to this sample. Zero inflated and semi-continuous data are common in the insurance sector, in modeling loan default and in many other situations. In many of these scenarios, multiple outcomes jointly allow for interesting and deeper understanding of the data. For example, micro-finance firms observe borrower's repayment profiles as a marker for potential default, however much more could be learnt by modeling both repayment profiles and the likelihood

of default jointly as the unobserved effects that determine either outcome are surely likely to be highly correlated and informative.

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