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# ON AN EVALUATION AND OPTIMIZATION MODEL FOR MEDIA PLANNING

by

A.K. RAO & M.R. RAO*

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INDIAN INSTITUTE OF MANAGEMENT
BANGALORE

**Professors, Indian Institute of Management, Bangalore*

# On An Evaluation and Optimization Model for Media Planning

**A.K. Rao**

**M.R. Rao***

Indian Institute of Management, Bangalore.

**Abstract**

In media planning, the media planner has to decide on the optimal number of insertions to be made in various media vehicles so as to maximize some measure of effectiveness. The parameters such as Reach, Gross Opportunities-To-See, Average Opportunities-To-See, Cost Per Person Reached etc. have to be evaluated for arriving at the optimal decision. In the past, researchers have developed an optimization model for determining the ideal combination of advertisement insertions that maximizes Reach. The constraints considered consist of upper bounds on the number of insertions in a vehicle and an upper bound on the total number of insertions in all vehicles. The optimal solution was obtained using dynamic programming. In this paper it is shown that the optimal solution can be obtained easily without using dynamic programming. A solution method is developed for the general case when there are lower and upper bounds on the number of insertions in each vehicle and also an upper bound on the total number of insertions. The problem of maximizing Reach when there is a budget constraint is also considered. A simple solution method is developed for some specified values of the budget. For other values of the budget, dynamic programming can be used to obtain an optimal solution.

## 1. INTRODUCTION

In a recent paper, Raghavendra [1] developed an optimization model to determine the number of insertions in various media vehicles to maximize Reach. The optimal solution is derived using dynamic programming. He also described a method of evaluation of various measures of advertising effectiveness like Reach, Gross Opportunities- To- See (GOTS), Average Opportunities-To-See (AOTS),

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* On leave from New York University, New York.

and Cost Per Person Reached. An evaluation model based on binomial distribution for computing these basic measures is also given. These models for evaluation and optimization were illustrated with the National Readership Survey (NRS) data base of India. In the Evaluation Model for Media Planning in India, Raghavendra estimated the unduplicated readership of a vehicle sequentially, after ranking and arranging the vehicles in descending order of the target group readership. It was also mentioned that any procedure of ordering of vehicles can be followed. In this note, it is shown that the ordering of vehicles will have a bearing on the evaluations and the optimal solution. For solving the optimization problem of maximizing Reach, dynamic programming method was used. Here it is shown that Reach is a concave function and consequently the optimal value can be obtained more easily by calculating incremental Reach. It is also shown that the table given in [1] for maximum Reach for a specified total number of insertions in all vehicles and the corresponding cost cannot be used to find the optimal number of insertions in the vehicles which maximizes Reach for a given budget.

## 2. NOTATION

The same notation as used in [1] is followed with slight modification.

M = Number of media vehicles considered in the advertisement campaign.

$i,j$ = 1,2,...,M; Media vehicles ,ranked in the order of target group readership.

$N_i$ = Target group readership of vehicle i.

$M_i$ = Unduplicated target readership of vehicle i.

$d_{ij}$ = Duplication percentage, expressed as a fraction of readers of vehicle i who also read vehicle j.

$f_{ik}$ = Number of sampled readers (from NRS) who read on an average, k issues of the total possible $k_i$ issues of vehicle i in a prespecified period for $k = 0,1,2,\ldots,k_i$ ( $k_i = 7$ for dailies, 4 for weekly, 6 for fortnightlies and monthlies).

$p_i$ = probability of readership of vehicle i. We assume, without loss of generality that $0 < p_i < 1$.

$X_i$ = Number of insertions in vehicle i.

$C_i$ = Cost per insertion in vehicle i.

$Z_i(r,X_i)$ = Probability of r OTS if $X_i$ insertions are made in vehicle i.

$D_i(r,X_i)$ = Number of expected readers seeing r insertions if $X_i$ insertions are made in vehicle i. $D_i'(r,X_i)$ is the expected number of unduplicated readership of vehicle i after the first (i-1) vehicles are considered.

$R_i(X_i)$ = Reach from vehicle i if $X_i$ insertions are made in it.

## 3. EFFECT OF RANKING VEHICLES ON OPTIMAL SOLUTION

A method for estimating unduplicated reach is suggested in [1]. After the media vehicles are ranked and arranged in descending order of target group readership, the unduplicated readership (Reach) of a vehicle is estimated sequentially as follows, with $d_{ij}$ expressed as a fraction. It may be noted that this is an approximation to calculate Reach.

$$M_1 = N_1$$

$$M_i = N_i \prod_{j=1}^{i-1} (1-d_{ij}) \ ; \ i = 2,3,\ldots,M.$$

The optimal solution for the problem of maximizing Reach depends on the ranking of the vehicles.

This is illustrated with the same example given in [1] for two specific values of s = 5 and 10.

For s = 5, the optimal solution is $X_1 = 2$, $X_2 = 1$, $X_3 = 1$, $X_4 = 1$ and the other $X_i$ 's are zero i.e. 2 insertions in ILL Wkly, one insertion each in Indian Express, Screen and Sunday Mid-Day.

For s = 10, the optimal solution is $X_1 = 3$, $X_2 = 2$, $X_3 = 2$, $X_4 = 1$, $X_5 = 1$, $X_7 = 1$ and the other $X_i$'s are zero i.e. 3 insertions in ILL Wkly, 2 insertions each in Indian Express and Screen, and one insertion each in Sunday Mid-Day, Daily and Hitvada.

Suppose these 8 media vehicles are arranged in some other order, say, Daily, Economic Times, Hitavada, Indian Express, ILL Wkly India, Screen, Sunday Mid-Day and Bombay. This ordering is the same as in Table 3 of [1]. The duplication of readership, $d_{ij}$ expressed as a fraction is taken from the lower diagonal of

Table 3 in [1]. This is reproduced below in Table 1.

## Table 1

**Duplication of Readership among the Chosen Publications**

| Publication | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1. Daily | 0.00 | | | | | | | |
| 2. Economic Times | 0.26 | 0.00 | | | | | | |
| 3. Hitavada | 0.00 | 0.00 | 0.00 | | | | | |
| 4. Indian Express | 0.15 | 0.07 | 0.01 | 0.00 | | | | |
| 5. Ill Wkly India | 0.17 | 0.07 | 0.03 | 0.32 | 0.00 | | | |
| 6. Screen | 0.08 | 0.04 | 0.03 | 0.22 | 0.20 | 0.00 | | |
| 7. Sunday Mid-Day | 0.29 | 0.09 | 0.00 | 0.33 | 0.27 | 0.07 | 0.00 | |
| 8. Bombay | 0.22 | 0.20 | 0.00 | 0.47 | 0.38 | 0.07 | 0.33 | 0.00 |

The unduplicated Reach values for this ordering of vehicles as calculated by the method given in [1] is given in Table 2.

**Table 2: Unduplicated Readership for Media Vehicles After Reordering**

| S.No. | Media Vehicle | Prob.of Exposure | Target Group Readership (in '000) | Unduplicated Reach (in '000) |
|---|---|---|---|---|
| 1 | Daily | 0.70 | 247 | 247 |
| 2 | Economic Times | 0.72 | 78 | 58 |
| 3 | Hitavada | 0.74 | 134 | 134 |
| 4 | Indian Express | 0.71 | 492 | 385 |
| 5 | ILL Wkly India | 0.69 | 513 | 261 |
| 6 | Screen | 0.60 | 283 | 151 |
| 7 | Sunday Mid-Day | 0.77 | 253 | 74 |
| 8 | Bombay | 0.59 | 144 | 18 |

In the above table, the unduplicated Reach values are rounded to the nearest thousand.

For this ordering of vehicles, the optimal solutions are :

For s = 5,

$X_1 = 1$, $X_2 = 0$, $X_3 = 1$, $X_4 = 1$, $X_5 = 1$, $X_6 = 1$, $X_7 = 0$, $X_8 = 0$ i.e. 1 insertion each in Daily, Hitvada, Indian Express, ILL Wkly and Screen.

For s = 10,

$X_1 = 2$, $X_2 = 1$, $X_3 = 1$, $X_4 = 2$, $X_5 = 2$, $X_6 = 1$, $X_7 = 1$, $X_8 = 0$ i.e. 2 insertions each in Daily, Indian Express and ILL Wkly and one insertion each in Economic Times, Hitvada, Screen and Sunday Mid-Day.

This solution is different from the one obtained earlier.

This shows that the ordering of the vehicles is important and that the optimal solution depends on the ordering of publications.

## 4. MAXIMIZING REACH

The Reach from vehicle i if $X_i$ insertions are made in it is given by:

$$R_i(X_i) = \sum_{r=1}^{X_i} M_i \; Z_i(r, X_i) = M_i \; [1 - Z_i(0, X_i)]$$

As $Z_i(0, X_i) = (1-p_i)^{X_i}$, we have

$$R_i(X_i) = M_i \; (1 - q_i^{X_i}) \quad \text{where } q_i = 1-p_i.$$

Taking derivatives of $R_i(X_i)$, the second derivative is

$$- M_i \; (\ln q_i)^2 \; q_i^{X_i}$$

which is negative since $0 < q_i < 1$ and $M_i$ is positive.

Hence $R_i(X_i)$ is a concave function and

$$R(X) = \sum_{i=1}^{M} R_i(X_i)$$

is a concave function of X.

Raghavendra [1], suggested a solution procedure for maximizing Reach using dynamic programming. He considered constraints consisting of upper limits on the number of insertions in each vehicle and also an upper limit on the total number of insertions in all vehicles.

A direct method of arriving at the optimal solution without using dynamic programming is given below for a more general situation when there are lower and upper bounds on the number of insertions in each vehicle and an upper bound on the total number of insertions. We also give a solution method when there is a constraint on the budget.

## 4.1 SOLUTION PROCEDURE

Let $L_i$ and $U_i$ be the lower and upper bounds on the number of insertions in vehicle i, i = 1,2,...,M and U the upper bound on the number of insertions on the total number of insertions in all vehicles. Let C be the total budget available.

We consider the following two problems.

Problem 1:

$$\text{Maximize } R(X) = \sum_{i=1}^{M} M_i \ (1 - q_i^{x_i})$$

subject to:

$$\sum_{i=1}^{M} X_i \le U$$

$$L_i \le X_i \le U_i \text{ and integer } ; \; i = 1,2,\ldots,M.$$

We assume, without loss of generality, that

$$U < \sum_{i=1}^{M} U_i$$

for otherwise the optimal solution is $X_i = U_i$ ; $i = 1,2,\ldots,M$.

Problem 2:

$$\text{Maximize } R(X) = \sum_{i=1}^{M} M_i \; (1 - q_i^{x_i})$$

subject to:

$$\sum_{i=1}^{M} C_i \; X_i \le C$$

$$L_i \le X_i \le U_i \text{ and integer } , \; i = 1,2,\ldots,M.$$

We assume , without loss of generality, that

$$C < \sum_{i=1}^{M} C_i \; U_i$$

for otherwise the optimal solution is $X_i = U_i$ ; $i = 1,2,\ldots,M$.

**4.1 OPTIMAL SOLUTION TO PROBLEM 1:**

We first transform the problem by introducing integer variables $Y_i$ such that

$$Y_i = X_i - L_i \quad \text{for } i = 1,2,\ldots,M.$$

Let $\quad s_i = U_i - L_i \quad$ for $i = 1,2,\ldots,M$ and

$$s = U - \sum_{i=1}^{M} L_i$$

Note that $s < \sum_{i=1}^{M} s_i$.

Problem 1 now becomes:

$$\text{(P1)}: \quad \text{Maximize } \overline{R}(Y_i) = \sum_{i=1}^{M} \overline{R}_i(Y_i) = \sum_{i=1}^{M} M_i\,(1-q_i^{L_i+Y_i})$$

subject to :

$$\sum_{i=1}^{M} Y_i \le s$$

$$0 \le Y_i \le s_i \quad \text{and integer} \; ; \; i = 1,2,\ldots,M.$$

Taking derivatives, it follows that $\overline{R}_i(Y_i)$ is a concave function and hence $\overline{R}(Y)$ is a concave function.

In order to solve (P1), we start with all $Y_i = 0$. We successively increase the value of some $Y_j$ by 1 from its current value. This process is repeated until $\sum_{i=1}^{M} Y_i$ is equal to s.

The steps involved are given below.

Step 0: Let $Y_i = 0$ , $i = 1,2,\ldots,M$.

Step 1: If $\sum_{i=1}^{M} Y_i = s$ then stop.

Otherwise, go to step 2.

Step 2: Let j be such that

$$M_j\, q_j^{L_j+Y_j}(1-q_j) = \max_{\substack{i\in(1,2,\ldots,M)\\ Y_i < s_i}} \{ M_i\, q_i^{L_i+Y_i}(1-q_i) \}$$

Set $Y_j = Y_j + 1$ and return to step 1.

<u>Lemma 1 :</u> The values of $Y_i$ arrived at by the above procedure are optimal for (P1).

Proof:

Replace $Y_i$ by $s_i$ zero-one variables, $Z_{ik}$ ; $k = 1,2,\ldots,s_i$ such that $Z_{ik} \geq Z_{i(k+1)}$ ; $k = 1,2,\ldots,(s_i - 1)$.

Now

$$\bar{R}_i(Y_i) = \sum_{i=1}^{s_i} \bar{R}_{ik} Z_{ik} \quad \text{where} \quad \bar{R}_{ik} = M_i q_i^{L_i+k-1}(1-q_i)$$

Here, $\bar{R}_{ik}$ is the incremental Reach obtained by setting $Y_i = k$ instead of $Y_i = k-1$.

Then (P1) is equivalent to

$$\text{Maximize} \sum_{i=1}^{M} \sum_{k=1}^{s_i} \bar{R}_{ik} Z_{ik}$$

subject to :

$$Z_{ik} \geq Z_{i(k+1)}, \quad k = 1,2,\ldots,s_i-1, \; i = 1,2,\ldots,M$$

$$\sum_{i=1}^{M} \sum_{k=1}^{s_i} Z_{ik} \leq s$$

$$Z_{ik} = 0 \text{ or } 1, \quad i = 1,2,\ldots,M, \; k = 1,2,\ldots,s_i.$$

But since $\bar{R}_i(Y_i)$ is a concave function, it follows that $\bar{R}_{ik} \geq \bar{R}_{ik+1}$, $k = 1,2,\ldots s_i-1$. The constraints, $Z_{ik} \geq Z_{i(k+1)}$ are not necessary and the optimal solution is given by the above procedure.

The above procedure is illustrated for the problem which was solved using dynamic programming in [1]. Here $L_i = 0$ for all i.

The rank order of 8 media vehicles, probability of exposure $p_i$, target group readership, unduplicated target group readership and the maximum number of insertions $s_i$ in each vehicle are given in Table 3 below.

**Table 3:Probability of Exposure and Unduplicated Readership of Publications**

| Rank order No. | Publication | Probability of exposure | Target Group Reader ship('000) | Unduplicated Target Reader ship('000) | Max. No.of insertions |
|---|---|---|---|---|---|
| 1 | Ill.Wkly | 0.69 | 513 | 513 | 6 |
| 2 | Indian Express | 0.71 | 492 | 354 | 5 |
| 3 | Screen | 0.60 | 283 | 176 | 4 |
| 4 | Sunday Mid-Day | 0.77 | 253 | 115 | 3 |
| 5 | Daily | 0.70 | 247 | 66 | 4 |
| 6 | Bombay | 0.59 | 144 | 22 | 2 |
| 7 | Hitavada | 0.74 | 134 | 83 | 2 |
| 8 | Economic Times | 0.72 | 78 | 7 | 2 |

From Table 3, the incremental reach, $\overline{R}_{ik}$, for each vehicle is calculated by

$$\overline{R}_{ik} = M_i q_i^{k-1} (1-q_i) \quad , \quad \begin{array}{l} k = 1,2,\ldots,s_i-1 \\ i = 1,2,\ldots,M. \end{array}$$

Table 4 gives the values of reach for each vehicle for different number of insertions and Table 5 gives the values of $\overline{R}_{ik}$ .

**Table 4: Values of Reach**

| Rank order | Publication | Number of Insertions | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | Ill Wkly | 354 | 464 | 498 | 508 | 512 | 513 |
| 2 | Indian Express | 251 | 324 | 345 | 351 | 353 | --- |
| 3 | Screen | 106 | 148 | 165 | 171 | --- | --- |
| 4 | Sunday Mid-day | 89 | 109 | 114 | --- | --- | --- |
| 5 | Daily | 46 | 60 | 64 | 65 | --- | --- |
| 6 | Bombay | 13 | 18 | --- | --- | --- | --- |
| 7 | Hitvada | 61 | 77 | --- | --- | --- | --- |
| 8 | Economic Times | 5 | 6 | --- | --- | --- | --- |

**Table 5: Incremental Reach**

| Rank order | Publication | Number of Insertions | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | Ill Wkly | 354 | 110 | 34 | 10 | 4 | 1 |
| 2 | Indian Express | 251 | 73 | 21 | 6 | 2 | --- |
| 3 | Screen | 106 | 42 | 17 | 6 | --- | --- |
| 4 | Sunday Mid-day | 89 | 20 | 5 | --- | --- | --- |
| 5 | Daily | 46 | 14 | 4 | 1 | --- | --- |
| 6 | Bombay | 13 | 5 | --- | --- | --- | --- |
| 7 | Hitvada | 61 | 16 | --- | --- | --- | --- |
| 8 | Economic Times | 5 | 1 | --- | --- | --- | --- |

The procedure given above to maximize Reach is equivalent to arranging the values of incremental Reach in descending order of magnitude and the maximum Reach for any specified number of insertions s is given by the sum of the first s values of incremental Reach. The values of the incremental reach arranged in descending order are : 354, 251, 110, 106, 89, 73, 61, 46, 42, 34, 21, 20, 17, 16, 14, 13, 10, 6, 6,, 5, 5, 5, 4, 4, 2, 1, 1, 1. For instance, for s = 5, the maximum Reach is equal to 910 which is the sum of the first five values. The first value of incremental Reach corresponds to the first insertion in vehicle 1, the second value corresponds to the first insertion in vehicle 2, the third value corresponds to the second insertion in vehicle 1 and so on. Thus the optimal solution is given by $X_1 = 2$, $X_2 = 1$, $X_3 = 1$ and $X_4 = 1$. By following the sequence of steps detailed above, the optimal Reach for a given number of cumulative insertions is calculated and given in Table 6.

**Table 6: Optimal Reach**

| Cumulative No. of insertions | Optimal Reach | No. of insertions in Vehicles | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ |
| 1 | 354 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 354+251=605 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 605+110=715 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 715+106=821 | 2 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 5 | 821+89=910 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 6 | 910+73=983 | 2 | 2 | 1 | 1 | 0 | 0 | 0 | 0 |
| 7 | 983+61=1044 | 2 | 2 | 1 | 1 | 0 | 0 | 1 | 0 |
| 8 | 1044+46=1090 | 2 | 2 | 1 | 1 | 1 | 0 | 1 | 0 |
| 9 | 1090+42=1132 | 2 | 2 | 2 | 1 | 1 | 0 | 1 | 0 |

| Cumulative No. of insertions | Optimal Reach | No. of insertions in Vehicles | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ |
| 10 | 1132+34=1166 | 3 | 2 | 2 | 1 | 1 | 0 | 1 | 0 |
| 11 | 1166+21=1187 | 3 | 3 | 2 | 1 | 1 | 0 | 1 | 0 |
| 12 | 1187+20=1207 | 3 | 3 | 2 | 2 | 1 | 0 | 1 | 0 |
| 13 | 1207+17=1224 | 3 | 3 | 3 | 2 | 1 | 0 | 1 | 0 |
| 14 | 1224+16=1240 | 3 | 3 | 3 | 2 | 1 | 0 | 2 | 0 |
| 15 | 1240+14=1254 | 3 | 3 | 3 | 2 | 2 | 0 | 2 | 0 |
| 16 | 1254+13=1267 | 3 | 3 | 3 | 2 | 2 | 1 | 2 | 0 |
| 17 | 1267+10=1277 | 4 | 3 | 3 | 2 | 2 | 1 | 2 | 0 |
| 18 | 1277+6=1283 | 4 | 4 | 3 | 2 | 2 | 1 | 2 | 0 |
| 19 | 1283+6=1289 | 4 | 4 | 4 | 2 | 2 | 1 | 2 | 0 |
| 20 | 1289+5=1294 | 4 | 4 | 4 | 3 | 2 | 1 | 2 | 0 |
| 21 | 1294+5=1299 | 4 | 4 | 4 | 3 | 2 | 2 | 2 | 0 |
| 22 | 1299+5=1304 | 4 | 4 | 4 | 3 | 2 | 2 | 2 | 1 |
| 23 | 1304+4=1308 | 5 | 4 | 4 | 3 | 2 | 2 | 2 | 1 |
| 24 | 1308+4=1312 | 5 | 4 | 4 | 3 | 3 | 2 | 2 | 1 |
| 25 | 1312+2=1314 | 5 | 5 | 4 | 3 | 3 | 2 | 2 | 1 |
| 26 | 1314+1=1315 | 6 | 5 | 4 | 3 | 3 | 2 | 2 | 1 |
| 27 | 1315+1=1316 | 6 | 5 | 4 | 3 | 4 | 2 | 2 | 1 |
| 28 | 1316+1=1317 | 6 | 5 | 4 | 3 | 4 | 2 | 2 | 2 |

**4.2 OPTIMAL SOLUTION TO PROBLEM 2:**

Introducing variables $Y_i = X_i - L_i$ ; $i = 1,2,\ldots,M$ as in section 4.1, Problem 2 now becomes

$$(P2):\quad \text{Maximize } \overline{R}(Y) = \sum_{i=1}^{M} M_i\,(1-q_i^{L_i+Y_i})$$

magnitude and the maximum Reach for any specified number of insertions s is given by the sum of the first s values of incremental Reach. The values of the incremental reach arranged in descending order are : 354, 251, 110, 106, 89, 73, 61, 46, 42, 34, 21, 20, 17, 16, 14, 13, 10, 6, 6,, 5, 5, 5, 4, 4, 2, 1, 1, 1. For instance, for s = 5, the maximum Reach is equal to 910 which is the sum of the first five values. The first value of incremental Reach corresponds to the first insertion in vehicle 1, the second value corresponds to the first insertion in vehicle 2, the third value corresponds to the second insertion in vehicle 1 and so on. Thus the optimal solution is given by $X_1 = 2$, $X_2 = 1$, $X_3 = 1$ and $X_4 = 1$. By following the sequence of steps detailed above, the optimal Reach for a given number of cumulative insertions is calculated and given in Table 6.

**Table 6: Optimal Reach**

| Cumulative No. of insertions | Optimal Reach | No. of insertions in Vehicles | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ |
| 1 | 354 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 354+251=605 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 605+110=715 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 715+106=821 | 2 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 5 | 821+89=910 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 6 | 910+73=983 | 2 | 2 | 1 | 1 | 0 | 0 | 0 | 0 |
| 7 | 983+61=1044 | 2 | 2 | 1 | 1 | 0 | 0 | 1 | 0 |
| 8 | 1044+46=1090 | 2 | 2 | 1 | 1 | 1 | 0 | 1 | 0 |
| 9 | 1090+42=1132 | 2 | 2 | 2 | 1 | 1 | 0 | 1 | 0 |

| Cumulative No. of insertions | Optimal Reach | No. of insertions in Vehicles | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ |
| 10 | 1132+34=1166 | 3 | 2 | 2 | 1 | 1 | 0 | 1 | 0 |
| 11 | 1166+21=1187 | 3 | 3 | 2 | 1 | 1 | 0 | 1 | 0 |
| 12 | 1187+20=1207 | 3 | 3 | 2 | 2 | 1 | 0 | 1 | 0 |
| 13 | 1207+17=1224 | 3 | 3 | 3 | 2 | 1 | 0 | 1 | 0 |
| 14 | 1224+16=1240 | 3 | 3 | 3 | 2 | 1 | 0 | 2 | 0 |
| 15 | 1240+14=1254 | 3 | 3 | 3 | 2 | 2 | 0 | 2 | 0 |
| 16 | 1254+13=1267 | 3 | 3 | 3 | 2 | 2 | 1 | 2 | 0 |
| 17 | 1267+10=1277 | 4 | 3 | 3 | 2 | 2 | 1 | 2 | 0 |
| 18 | 1277+6=1283 | 4 | 4 | 3 | 2 | 2 | 1 | 2 | 0 |
| 19 | 1283+6=1289 | 4 | 4 | 4 | 2 | 2 | 1 | 2 | 0 |
| 20 | 1289+5=1294 | 4 | 4 | 4 | 3 | 2 | 1 | 2 | 0 |
| 21 | 1294+5=1299 | 4 | 4 | 4 | 3 | 2 | 2 | 2 | 0 |
| 22 | 1299+5=1304 | 4 | 4 | 4 | 3 | 2 | 2 | 2 | 1 |
| 23 | 1304+4=1308 | 5 | 4 | 4 | 3 | 2 | 2 | 2 | 1 |
| 24 | 1308+4=1312 | 5 | 4 | 4 | 3 | 3 | 2 | 2 | 1 |
| 25 | 1312+2=1314 | 5 | 5 | 4 | 3 | 3 | 2 | 2 | 1 |
| 26 | 1314+1=1315 | 6 | 5 | 4 | 3 | 3 | 2 | 2 | 1 |
| 27 | 1315+1=1316 | 6 | 5 | 4 | 3 | 4 | 2 | 2 | 1 |
| 28 | 1316+1=1317 | 6 | 5 | 4 | 3 | 4 | 2 | 2 | 2 |

## 4.2 OPTIMAL SOLUTION TO PROBLEM 2:

Introducing variables $Y_i = X_i - L_i$ ; $i = 1,2,...,M$ as in section 4.1, Problem 2 now becomes

$$(P2): \text{ Maximize } \overline{R}(Y) = \sum_{i=1}^{M} M_i \left(1-q_i^{L_i+Y_i}\right)$$

subject to :

$$\sum_{i=1}^{M} C_i Y_i \le B$$

$$0 \le Y_i \le s_i \quad \text{and integer}, \; i = 1,2,\ldots,M$$

where $B = C - \sum_{i=1}^{M} C_i L_i$ .

It has been shown above that $\overline{R}(Y)$ is a concave function of Y. The optimal solution for the above problem can be obtained easily for some specified values of the budget B.

Define $\overline{R}_{ik} = M_i q_i^{L_i+k-1} (1-q_i)$ ; $k = 1,2,\ldots,s_i-1$, $i = 1,2,\ldots M$.

Note that $\overline{R}_{ik}$ is the incremental Reach obtained by setting $Y_i = k$ instead of k-1.

Define $\hat{R}_{ik} = \overline{R}_{ik}/C_i$ ; $i = 1,2,\ldots,M$, $k = 1,2,\ldots,s_i$.

Arrange $\hat{R}_{ik}$ in descending order, breaking ties arbitrarily. Let the nth value be denoted by $\hat{R}_{i(n),k(n)}$, $n = 1,2,\ldots$.. Suppose the budget value B is such that

$B = C_{i(1)} + C_{i(2)} + C_{i(3)} + \ldots + C_{i(n)}$ for some n. Then the optimal solution to (P2) can be easily obtained by a process similar to the one given in section 4.1. The solution procedure is given below.

Step 0: Let $Y_i = 0$, $i = 1,2,\ldots,M$.

Step 1: If $\sum_{i=1}^{M} C_i Y_i = B$ , then stop. We have the optimal solution.

Otherwise go to step 2.

Step 2: Let j be such that

$$M_j q_j^{L_j+Y_j}(1-q_j)/C_j = \max_{\substack{i\in(1,2,\ldots,M)\\ Y_i < s_i}} \{ M_i q_i^{L_i+Y_i}(1-q_i)/C_i \}$$

In case there is a tie, select any one j.

Set $Y_j = Y_j + 1$ and return to step 1.

Claim 2 : The solution arrived at by the above procedure is optimal to problem (P2).

The proof is similar to the one given for Claim 1.

By following the sequence of steps detailed above, the optimal Reach for specified budget values is calculated and given in Table 9. Table 7 gives the assumed cost per insertion for the various publications. Note that these values are the same as given in Table 1 of [1].

**Table 7: Assumed Cost Per Insertion**

| Sl.No | Publication | Cost per Insertion in Rs. (in '000) |
|---|---|---|
| 1 | Ill Wkly India | 16 |
| 2 | Indian Express | 14 |
| 3 | Screen | 21 |
| 4 | Sunday Mid-Day | 25 |
| 5 | Daily | 10 |
| 6 | Bombay | 28 |
| 7 | Hitavada | 20 |
| 8 | Economic Times | 12 |

Table 8 gives the incremental Reach values $\hat{R}_{ik}$.

**Table 8 : Incremental Reach ($R_{ik}$) per Rs. thousand**

| Rank Order | Publication | Number of Insertions | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | ILL Wkly India | 22.12 | 6.88 | 2.12 | 0.62 | 0.25 | 0.06 |
| 2 | Indian Express | 17.93 | 5.21 | 1.50 | 0.43 | 0.14 | --- |
| 3 | Screen | 5.05 | 2.00 | 0.81 | 0.29 | --- | --- |
| 4 | Sunday Mid-Day | 3.56 | 0.80 | 0.20 | --- | --- | --- |
| 5 | Daily | 4.60 | 1.40 | 0.40 | 0.10 | --- | --- |
| 6 | Bombay | 0.46 | 0.18 | --- | --- | --- | --- |
| 7 | Hitavada | 3.05 | 0.80 | --- | --- | --- | --- |
| 8 | Economic Times | 0.42 | 0.08 | --- | --- | --- | --- |

Table 9 gives the optimal number of insertions for specified budgets.

**Table 9 : Optimal Number of Insertions for Specified Budgets**

| Specified Budget in Rs. (in '000) | Optimal Reach | Number of Insertions | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ |
| 16 | 354 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 30 | 354+251=605 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 46 | 605+110=715 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 60 | 715+73=788 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 81 | 788+106=894 | 2 | 2 | 1 | 0 | 0 | 0 | 0 | 0 |
| 91 | 894+46=940 | 2 | 2 | 1 | 0 | 1 | 0 | 0 | 0 |
| 115 | 940+89=1029 | 2 | 2 | 1 | 1 | 1 | 0 | 0 | 0 |
| 135 | 1029+61=1090 | 2 | 2 | 1 | 1 | 1 | 0 | 1 | 0 |
| 151 | 1090+34=1124 | 3 | 2 | 1 | 1 | 1 | 0 | 1 | 0 |
| 172 | 1124+42=1166 | 3 | 2 | 2 | 1 | 1 | 0 | 1 | 0 |
| 186 | 1166+21=1187 | 3 | 3 | 2 | 1 | 1 | 0 | 1 | 0 |

| Specified Budget in Rs. (in '000) | Optimal Reach | Number of Insertions | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ |
| 196 | 1187+14=1201 | 3 | 3 | 2 | 1 | 2 | 0 | 1 | 0 |
| 217 | 1201+17=1218 | 3 | 3 | 3 | 1 | 2 | 0 | 1 | 0 |
| 237 | 1218+16=1234 | 3 | 3 | 3 | 1 | 2 | 0 | 2 | 0 |
| 262 | 1234+20=1254 | 3 | 3 | 3 | 2 | 2 | 0 | 2 | 0 |
| 278 | 1254+10=1264 | 4 | 3 | 3 | 2 | 2 | 0 | 2 | 0 |
| 306 | 1264+13=1277 | 4 | 3 | 3 | 2 | 2 | 1 | 2 | 0 |
| 320 | 1277+6=1283 | 4 | 4 | 3 | 2 | 2 | 1 | 2 | 0 |
| 332 | 1283+5=1288 | 4 | 4 | 3 | 2 | 2 | 1 | 2 | 1 |
| 342 | 1288+4=1292 | 4 | 4 | 3 | 2 | 3 | 1 | 2 | 1 |
| 363 | 1292+6=1298 | 4 | 4 | 4 | 2 | 3 | 1 | 2 | 1 |
| 379 | 1298+4=1302 | 5 | 4 | 4 | 2 | 3 | 1 | 2 | 1 |
| 404 | 1302+5=1307 | 5 | 4 | 4 | 3 | 3 | 1 | 2 | 1 |
| 432 | 1307+5=1312 | 5 | 4 | 4 | 3 | 3 | 2 | 2 | 1 |
| 446 | 1312+2=1314 | 5 | 5 | 4 | 3 | 3 | 2 | 2 | 1 |
| 456 | 1314+1=1315 | 5 | 5 | 4 | 3 | 4 | 2 | 2 | 1 |
| 468 | 1315+1=1316 | 5 | 5 | 4 | 3 | 4 | 2 | 2 | 2 |
| 484 | 1316+1=1317 | 6 | 5 | 4 | 3 | 4 | 2 | 2 | 2 |

For other values of the budget B, we can solve (P2) by dynamic programming where the stages correspond to media vehicles 1,2,...M and the states correspond to the budget available.

In [1], a table [Table 6] of results of Optimization for maximizing Reach is given. The total costs corresponding to specified total number of insertions were also calculated. It was stated that if the limits on budget are known then the table could be used to arrive at a media plan which gives maximum Reach for a given budget. But, this is not correct. For example, suppose we have a budget of Rs. 92,000. As per the Table 6 in [1], the maximum Reach corresponding to this budget is 908,000 which is attained by having a total of 5 insertions, i.e $X_1 = 2$, $X_2 = 1$, $X_3 = 1$, $X_4 = 1$, $X_5 = X_7 = X_7 = X_8 = 0$. It can be seen from Table 9 above that if we have 6 insertions so that $X_1 = 2$, $X_2 = 2$, $X_3 = 1$, $X_5 = 1$, $X_4 = X_6 = X_7 = X_8 = 0$, then the total cost will be Rs. 91,000 and the corresponding Reach will be 940,000. Hence, a more systematic procedure as given in section 4.2 is required for arriving at a media plan which maximizes Reach for a given budget.

## 5. SUMMARY

In this paper it is shown that the ordering of vehicles has a bearing on the optimal solution which maximizes Reach. A simple and direct method of obtaining a media plan which maximizes unduplicated Reach is given when the constraint set consists of lower and upper bounds on the total number of insertions. The method is extended to the case when we have a budget constraint.

## 6. REFERENCES

[1] Raghavendra B.G.(1989), 'An Evaluation and Optimization Model for Media Planning', International Journal of Management and Systems, Vol. 5, No. 1.