भारतीय प्रबंध संस्थान बेंगलूर
INDIAN INSTITUTE OF MANAGEMENT BANGALORE

WORKING PAPER NO: 459

Revenue Sharing and Network Size

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Year of Publications – April 2014

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Abstract:

This paper investigates the relationship between network size and license fees, revenue sharing, uncertainty and the nature of competition in telecommunications. We also investigate whether profits are higher with revenue sharing under uncertain demand.

Keywords: License fees, revenue sharing, uncertainty, duopoly, monopoly

1. Introduction

This paper has as its motivation a rather puzzling feature in the history of telecommunications in India. At the time of introduction of competition in mobile telephony the firms that bid for the right to provide telephony in their respective circles, did so by bidding to pay license fees. Later they discovered that their assessment of the market had been too ambitious and they would have to default on their license fees. At that time, faced with the imminent demise of the mobile telephony market, the government amended the rules to allow the firms to pay for the license fees through revenue sharing. Later this model of revenue sharing was applied to a whole host of different areas. It now seems received wisdom that in introducing competition in a new market the government would do so through revenue sharing rather than through license fees. Standard microeconomic theory would suggest that revenue sharing would lead to a reduction in output, which would correspond to a smaller network size in the case of telecommunications, in comparison with license fees. License fees are non-distorting. However, the Indian experience seems to suggest that revenue sharing can be "good" for the growth of the market. This appears to be a contradiction.

I try to investigate this issue by contrasting a Cournot duopoly with and without revenue sharing and show that network size would be lower with revenue sharing, under certainty. Under uncertainty, the results are ambiguous; they depend on the nature of competition. The results depend on whether the strategic variable is a strategic substitute or complement as defined by Bulow, Genekeapolous and Klemperer. So the results should depend on the degree and type of price competition being present in the market.

2. Model

Modeling telecommunications demand is a somewhat tricky affair since the sector is likely to be characterized by network externalities. Network externalities exist when a consumer joining a particular network bestows a benefit to the other members of the network. The classic example is that of telephones. Owning a telephone would be useless if no one else owned one. As more people own telephones it is now possible to connect to more people. Further, all the individuals on the network can also now call the individual who has joined the network. So joining a network in beneficial not only to the individual who joins but to those already on the network. So there is an external benefit that the persons who joins gives to others. Also, the more people there are on the network the more valuable it is to join.

This creates a problem in terms of standard analysis of demand. The value of the product depends not only on my level of consumption but on that of others as well. Consequently, there may be multiple equilibriums. The earliest work on modeling telecommunications is due to Rolfs (1972); a more general version was developed later by Economides and Himmelberg (1995). We will present a simple version of the Rolfs' model, which we will use to derive some simple results.

Let us assume that there is a market of size η. Consumers are uniformly distributed on the interval [0,1] and indexed by x, with lower x implying higher willingness to pay. Let

$$U_x = \begin{cases} (1-x)q^e - p \text{ if he subscribes} \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

where $q^e$is the expected number of consumers. Let $\hat{x}$ be a consumer who is indifferent between subscribing and not subscribing.

Then, $0=(1-\hat{x})q^{e}-p,$ or $\hat{x}=\frac{q^{e}-p}{q^{e}}$ (2)

The number of consumers who subscribe is then $q=\eta\hat{x}$

Assume consumers have perfect foresight, so that $q^{e}=q=\eta\hat{x}$

Substitute in (2) to get $p=(1-\hat{x})\eta\hat{x}$, which is a quadratic in $\hat{x}$. We can now derive the demands at a price p as

$$\hat{x}_{L}=\frac{\eta-\sqrt{\eta(\eta-4p)}}{2\eta}\ and\ \hat{x}_{H}=\frac{\eta+\sqrt{\eta(\eta-4p)}}{2\eta} \qquad (3)$$

There are consequently three possible network sizes at a given price. First, there is the possibility of a network of size zero. If everyone expects that no one will join the network, then with fulfilled expectations no body actually joins the network. The other two possibilities are the ones given above in (3). There is a small size network and a large size network. We can show that the small size network is unstable and that the size of the network will tend to either the large size or zero.

We will illustrate the more general model due to Economides and Himmelberg (1995). The important difference is that the externality is modeled as $f(q^e)$, with $0<q^e<1$ and $f(0)=0, f'>0, f''<0.$ Consumers are as before indexed by $x\in[0,1]$ now with low x implying low willingness to pay, x distributed according to the cumulative distribution function $G(x)$

The utility function for the consumer is given by

$$U(x,q^{e})=xf(q^{e}) \qquad (4)$$

Consequently the indifferent consumer is such that $xf(q^{e})=p,$ or $x^{*}=\frac{p}{f(q^{e})}$

Then $q = 1 - G(x^*)$, or $G^{-1}(1-q) = x^* = \frac{p}{f(q^e)}$

Then $p(q, q^e) = G^{-1}(1-q) f(q^e)$ is the demand curve and (5)

$$p_q = \frac{-f(q^e)}{G'(1-q)} < 0, \; p_{q^e} = G^{-1}(1-q) f'(q^e) > 0$$

(6)

If expectations are fulfilled then $p(q) = G^{-1}(1-q)f(q)$.

3. License fees and revenue sharing under monopoly and duopoly

We will first look at the effect of revenue sharing under monopoly using the Rolfs' model.
For the purpose of comparison we will first look at a model without license fees or connection costs. The monopolist would maximize

$$\pi(x) = p\eta x = (1-x)(\eta x)^2$$

The first order conditions would be $\frac{d\pi}{dx} = (2x - 3x^2)\eta^2 = 0$, or $x(2-3x)\eta^2 = 0$

Then, $x = 0$ or $\frac{2}{3}$.

$$\frac{d^2\pi}{dx^2} = (2x - 6x)\eta^2 < 0, \text{ for } x > \frac{1}{3}, \text{ so } x = \frac{2}{3} \text{ is optimal}$$

Substitute $x = \frac{2}{3}$ into $p = (1-x)\eta x$ to get

$$p = \frac{2\eta}{9}, \; \pi = (1-x)(\eta x)^2 = \frac{4\eta^2}{27},$$

$$U_x = (1-x)\frac{2}{3}\eta - \frac{2\eta}{9} = \frac{2\eta(2-3x)}{9} \text{ for } x \in [0, \frac{2}{3}]$$

Include a connection cost c and a license fee F

$$\pi = \{(1-x)\eta x - c\}\eta x - F$$

The first order condition is $2\eta^2 x - 3\eta^2 x^2 - \eta c = 0$

Then $x_F^* = \frac{1}{3} + \frac{1}{3}\sqrt{(1 - \frac{3c}{\eta})}$. (5)

Note that x^* does not depend on F, license fees do not distort the size of the network.

Now consider a situation where the government takes a share $s \in [0,1]$ of the revenue. The corresponding profit is

$$\pi = \{(1-s)(1-x)\eta x - c\}\eta x$$

The first order condition is $(1-s)\{2\eta^2 x - 3\eta^2 x^2\} - \eta c = 0$

Then $x_S^* = \frac{1}{3} + \frac{1}{3}\sqrt{(1 - \frac{3c}{\eta(1-s)})}$ (6)

Proposition 1: The size of the network with revenue sharing is smaller than that with license fees under monopoly.

Let us now investigate duopoly in the Rolfs model. We will assume two identical firms with connection costs c and Cournot competition.

The demand curve we will consider is $p = (1-X)\eta X$, where $X = x_1 + x_2$. As before to ease comparison we will assume that there is no connection cost. An individual firm will maximize

$$\pi = (1 - x_1 - x_2)(x_1 + x_2)\eta^2 x_1$$

The first order condition will be $\eta^2\{-3x_1^2 + 2x_1 - 4x_1x_2 - x_2^2 + x_2\} = 0$

By symmetry, $x_1 = x_2$, $x_1(8x_1 - 3) = 0$, or $x_1^* = \frac{3}{8}$

Thus $X = \frac{3}{4}$, $P = \frac{3\eta}{16}$.

As expected output is higher and prices lower under duopoly.

With a connection cost and license fee $x_i^* = \frac{3 + \sqrt{9 - \frac{32c}{\eta}}}{16}$

With revenue sharing: $x_i^* = \dfrac{3+\sqrt{9-\dfrac{32c}{\eta(1-s)}}}{16}$

As before we note that revenue sharing leads to a lower network size.

We can shown the same results in the more general model.

$$p = p(q, q^e) = G^{-1}(1-q)f(q^e)$$

With fulfilled expectations, $q = q^e$, $p = p(q, q)$

The monopolist will maximize $\pi = qp(q, q) - cq - F$

The first order conditions are, $p(q, q) + q\dfrac{dp}{dq} - c = 0$ to get q_{mF}^*

With revenue sharing the firm will maximize $\pi = (1-s)qp(q, q) - cq$

$$(1-s)\{p(q, q) + q\frac{dp}{dq}\} - c = 0 \text{ to get } q_{ms}^*$$

Clearly, $q_{ms}^* < q_{mF}^*$, the size of the network with revenue sharing is smaller than that under license fees.

We can use the general model to show that this is true under duopoly as well. The demand curve will be

$$p(Q, Q^e) = G^{-1}(1-Q)f(Q^e)$$

Then firm 1 will maximize $\pi = q_1\{G^{-1}(1-q_1-q_2)f(q_1+q_2^e\} - cq_1 - F$

The first order condition will be $p(q_1+q_2, q_1+q_2^e) + q_1(p_1+p_2) - c = 0$

In equilibrium all expectations are fulfilled
$p(Q, Q) + q_1(p_1+p_2) - c = 0$

and by symmetry $q_1^* = q_2^* = \dfrac{Q^*}{2}$, then, $Q_{dF}^* > Q_{mF}^*$

With revenue sharing, the first order condition will be

$$(1-s)\{p(Q,Q)+q_1(p_1+p_2)\}-c=0,$$

which will imply that $Q^*_{dF} > Q^*_{ds}$

Proposition 2: The size of the network with Cornot duopoly with revenue sharing will be larger than with license fees.

4. The role of uncertainty

We will now look at the effect of uncertainty on the results that have been derived. The argument for considering uncertainty is that with variable demand firms will have a less variable income stream with revenue sharing than with license fees. This would suggest that in industries that are nascent and where the size of the market is unknown the government might be better off by adopting a regime of revenue sharing than relying on license fees.

Let us therefore assume that the utility function being faced by a consumer is

$$U(x,q^e)=\theta x f(q^e),$$

where θ is a random variable with mean μ and variance σ^2.

Then the demand curve is $p_u=\theta G^{-1}(1-q)f(q^e)$

Consider a risk neutral monopolist with license fee. He would maximize

$$E[q\theta G^{-1}(1-q)f(q)-cq-F] = \mu q p_u(q,q)-cq-F \quad \text{.........}(7)$$

Differentiate to get q^*_{mFu}

Under revenue sharing, the firm will maximize

$$E[(1-s)q\theta G^{-1}(1-q)f(q)-cq-F]$$

$$= (1-s)\mu q p_u(q,q)-cq$$

= $\mu q p_u(q,q) - cq - s\mu q p_u(q,q)$……….(8)

Solve to get q^*_{msu}, and as before $q^*_{msu} < q^*_{mFu}$

Proposition 3: The size of the network under uncertainty and with revenue sharing would be smaller than with license fees.

To find out whether profits are higher under revenue sharing we need to know the relationship between s and F. We can assume that the expected revenue under revenue sharing would be equal to the license fee.

Then $s\mu q_{msu} p_u(q_s, q_s) = F$, comparing (2) and (3) we get

$\pi_{su} < \pi_{Fu}$

Further, $\theta q p_u(q,q) - cq - s\theta q p_u(q,q)$

= $\theta q_{msu} p_u - c q_{msu} - \frac{F}{\mu q^*_{msu} p_u} \theta q_{msu} p_u$

$= \theta q_{msu} p_u - c q_{msu} - F\frac{\theta}{\mu}$

which gives us our next result.

Proposition 4: Profits under revenue sharing stochastically dominates profits under license fees

If however the monopolist is risk averse he would maximize

$EU[q\theta p(q,q) - cq - F]$, where $U$is a von Neumann – Morgenstern utility function with $U'(\pi) > 0$ and $U''(\pi) < 0$

The first and second order conditions are

$E[U'(\pi)(MR - c)] = 0$

$$E[U''(\pi)(MR-c)^2+U'(\pi)(\frac{\partial MR}{\partial q})]<0$$

Denote $q^*_{mFu}=q_\mu$

Let $\theta>\mu$, then $\pi_\theta>\pi_\mu$, so $U'(\pi_\theta)<U'(\pi_\mu)$…..(9)

By the PIU $MR(\theta,q_\mu)$ > $MR(q_\mu)=c$……..(10)

Combining (4) and (5),

$$[MR(\theta,q_\mu)-c]U'(\pi_\theta)<[MR(\theta,q_\mu)-c]U'(\pi_\mu)$$

$$E[MR(\theta,q_\mu)-c]U'(\pi_\theta)]<E[MR(\theta,q_\mu)-c]U'(\pi_\mu)$$

Now, $E[MR(\theta,q_\mu)=MR(q_\mu)$

Then $E[MR(\theta,q_\mu)-c]U'(\pi_\theta)]<0$, so $q<q_\mu$

Thus with risk aversion and license fees network size decreases. In fact one would suspect that with risk aversion the size of the network would be even smaller than under risk neutrality. This is similar to results obtained by Ormiston (1992)

Proposition 5: The result that the size of the network is smaller hold under risk aversion as well.

5. Future work

The aim of this paper is to understand the relationship between license fees, revenue sharing and uncertainty. The problem is that the government has a number of policy instruments to choose. It has to decide on the level of the license fee, the revenue share if it is adopted and the number of players that it will allow. The larger the number of firms and with players playing Cournot the larger is the size of the network. Revenue sharing has a detrimental effect on the network size while license fees are neutral. The government would typically not like the firms to go bankrupt but would like a large network with low prices. It is not clear which combination of policy choices would be appropriate.

The obvious direction in which to travel would be to consider a duopoly with risk aversion.

Gradstein, Nitzan and Slutsky (1992) show that for strategic substitutes output decreases with increase in uncertainty. With strategic complements as in Bertrand competition the result would be reversed.

6. Conclusion

This paper investigates the relationship between license fees, revenue sharing, the nature of competition and uncertainty and the impact on the network size and profits. This work is at a preliminary stage and will be expanded at a later date.

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