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Consumer Uncertainty and Pack Size

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Abstract

Packaging is among the most visible features of modern day marketing. Goods which once used to be sold without any packaging such as fruits, vegetables, cereals and milk are now very likely to be sold in packs of fixed sizes. This is true for developed and emerging economies. In this paper we investigate the effect of uncertainty in consumption on pricing of pack sizes. We emphasize the role of uncertainty in consumption as the main issue driving the choice of pack sizes and their pricing. A typical consumer facing a purchase decision does not usually know how much she will later consume. The purchase decision precedes consumption. Buying too much could lead to a large stock of unconsumed goods. This would either have to be consumed later with possibly reduced levels of satisfaction. It would certainly occupy scarce shelf space. We will include both these effects under the common term of storage costs. Buying too little could lead to facing the prospect of having to do without or another trip to the store. Consumers, also differ in the type of uncertainty they face. The producer can then design pack sizes and their corresponding prices to discriminate across consumers and maximize profits.

Keywords: Consumer Uncertainty, Package Size, Pricing

1. Introduction

Packaging is among the most visible features of modern day marketing. Goods which once used to be sold without any packaging such as fruits, vegetables, cereals and milk are now very likely to be sold in packs of fixed sizes. This is true for developed and emerging economies. In this paper we investigate the effect of uncertainty in consumption on pricing of pack sizes. When we use the term pack size or packaging we will concentrate only on the size of the package. A larger pack will have more, either in terms of numbers, volume or weight, of the product in question. The attractiveness of the package or any other feature such as durability will not feature in our analysis.

There has been some research into different aspects of pack sizes. The research can be roughly categorized into theoretical or applied; or also as to whether it emphasizes demand side issues versus supply side issues. Some researchers combine some or all of these aspects. One of the earliest papers that discusses packaging is by Adams and Yellen (1976), who emphasized the bundling aspect. Restricting customers to the purchase of packs of sizes fixed by the producer (or the retailer) rather than allowing her to buy as much as she desires is seen as an attempt to extract consumer surplus through bundling. Consumers typically differ in terms of their reservation prices for different products which presents opportunities for bundling. Adams and Yellen argue that even though bundling is usually thought of in relation to different products, such as razors and razor blades, the same analysis can be extended to packs of the same product. The analysis in this paper is theoretical and since the successful application of bundling would require knowledge of reservation prices it emphasises the demand side. We will provide a synopsis of the kind of research that exists in the next section.

There has been little theoretical work on pack sizes. Besides Adams and Yellen mentioned above the other noteable paper has been by Gerstner and Hess (1987). They build a theoretical model of consumers who are forced to choose from two different pack sizes. Further, consumers face a transaction cost when purchasing the product and also face storage costs for storing the product. Different consumers face different storage costs. The source of heterogeneity here is different storage and transaction costs rather than reservation prices. Conseqently, consumers choose the package appropriate for them given their individual transaction and storage costs. The producer can then design the package size and price appropriately to maximize his profits. One of the interesting features of their research is that there may be quantity discounts as well as quantity surcharges. Larger pack sizes may be sold at lower, higher or the same unit prices. The possibility of quantity surcharges is intriguing since we are more familiar with quantity discounts. However, there is strong empirical evidence for the presence of these in different markets in the USA and Europe (Agarwal, Grimm and Narasimhan (1993), Abdulai, Kuhlgatz and Schmitz (2009)).

We also present a theoretical paper and use the concept of transaction costs and storage costs. We emphasize the role of uncertainty in consumption as the main issue driving the choice of pack sizes and their pricing. A typical consumer facing a purchase decision does not usually know how much she will later consume. The purchase decision precedes consumption. Buying too much could lead to a large stock of unconsumed goods. This would either have to be consumed later with possibly reduced levels of satisfaction. It would certainly occupy scarce shelf space. We will include both these effects under the common term of storage costs. Buying too little could lead to facing the prospect of having to do without or another trip to the store. Consumers, also differ in the type of uncertainty they face. The producer can then design pack sizes and their corresponding prices to discriminate across consumers and maximize profits. The next section provides a literature review. After that we present our model in Section 3. We go on to discuss the results in Section 4. Section 5 provides directions for further research and the conclusion.

2. Literature Review

The litearure on package sizes is an ecelectic mix of applied and theoretical work. One particular issue that has intriguied researchers is the existence of quantity sucharge ((Widrick (1979a, 1979b), Nason and Della Bitta (1983), Cude and Walker (1984), Agarwal, Grimm and Narasimhan (1993), Sprott, Manning and Miyazaki (2003), Abdulai, Kuhlgatz and Schmitz (2009)). There have been a number of studies that show that while discounts for larger package sizes is the norm, for a substantially large number of product categories the opposite holds true. Larger package sizes are more expensive in terms of the price per unit. The average percentage of brands exhibiting quantity surcharges varied between 12% to 34% (Gupta and Rominger (1996)). One natural response to this phenomenon is to suggest that this happens because consumers cannot or do not (Salop (1977)) calculate unit prices. A natural corrolary to this idea is that providing information on unit prices would lead to consumers altering their purchasing behaviour and incresing their welfare through substantial savings. Indeed, mandatory labelling of unit prices is the law in the U.S. and Europe. Empirical investigation into this hypothesis, either through surveys and experiments or through econometric methods, have produced mixed results.

A number of studies have used econometric methods to try to isolate the factors which are conducive to quantity surcharges. Agarwal, Grimm and Narasimhan (1993) divide these into demand side factors such as consumer ignorance, search costs, storage costs and transaction costs. These factors affect the cunsumers choice of pack sizes. On the supply side producers may find it costlly to provide packs of larger sizes. Though, the reverse should be the case, it is possible that for items such as frozen foods there could be diseconomies of size. They find that quantity surcharges are more likely for products with a higher level of usage of the product and a propensity to buy the larger size. In store storage costs also affect quantity surcharges positively. Sprott, Manning and Miyazaki (2003) also study quantity surcharges and arrive at somewhat different conclusions. They contend that grocery stores compete in prices of high volume units. For low volume units store managers see little benefit in lowering prices. If the low volume units turn out to be packages of larger size then this behaviour leads to quantity surcharges. Abdulai, Kuhlgatz and Smitz (2009) use a probit model to investigate the likelihood of quantity surcharges in Germany. Their findings show that the larger the number of pack sizes the higher is the likelihood of quantity surcharges. The same is true of large differences in pack sizes and in pack sizes of noninteger values. They also show that frozen or refrigerated products show a greater probability of quantity surcharges.

Theoritical work on pack sizes has been rather limited. We have earlier refered to Adams and Yellen(1976) and Gerstner and Hess (1987). Salop (1977) contended that consumers with high search costs would not spend time and effort searching for the lowest prices. Consequently, retrailers could charge them a higher price. Thus quantity surcharges arise because of wilful ignorance about unit prices. This notion gains some empirical support from Mitchell, Lennard and McGoldrick (2003) who found in a sample of 1000 people that 31% did not understand the concept of unit pricing, 35% did not use it and 28% found it too difficult to use. Gerstner and Hess, on the other hand, assume that consumers are fully informed, but some are still willing to pay a premium for a larger size package. The reason behind this supposedly perverse behaviour is heterogeneity among consumers. They differ in terms of transaction costs and storage costs. The busy executive with little time on her hands and a giant fridge would willingly buy a large size pack even if it is sold at a higher unit price compared to smaller packs, ceteris paribus. The point is that different pack sizes are not perfect substitutes and are in fact different goods. This is a notion supported by Granger and Billson (1972) and more recently Chouinard (2004) has argued along similar lines. Koenigsberg, Kohli and Montoya (2010) concentrate on packaging of perishable goods such as food and drugs. They posit that pack size depends on costs of packaging, consumer usage rates and the utility from consumption. There is no quantity discount or surcharge in their model since there is only one package size. Since the consumer would like to match her purchases as closely as possible to her consumption, so that there is no wastage or unfulfilled demand, she would find small packs best suited to her and the market would provide that size.

There are a number of papers which comment on the issue of pack size tangentially. For instance, Guadagni and Little (1983) present a logit model of consumer choice using scanner data and in passing remark on the importance of package size as well as branding. Similarly, Allenby, Shivley, Yang and Garrett (2004) accept packaging as a fact and discuss the challenges of modelling consumer choice when consumers buy discrete quantities and first order conditions do not hold. The situation is further complicated by the presence of quantity discounts, they note. Besanko, Dube and Gupta (2003) investigate the possibility of price discrimination using retail data. They perform their estimation using ketchup data which exhibits quantity surcharges. Again, in a different vein, Wansink(1996) discusses the issue of whether larger package sizes lead to more usage; an issue of tangential interest to us. Similarly, Gupta and Rominger (1996) dwell on the ethics of quantity surcharges.

The theory of mechanism design has been of much interest in the last few decades and has been instrumental in developing a good understanding of price discrimination. Early work focused mostly on information asymmetry between the seller and buyers and implications for non linear pricing. More recently, pricing strategies adopted in the airlines industry have interested reserachers (Courty & Li, 2001) where contracts are first selected and consumption decisions are made later. This aspect allows for sequential price discrimination. The telecom industry too is charcterized by seperation of choice and consumption which makes sequential screening possible (Miravete, 2002). Buyer uncertainty about future consumption and learning over time are the primary drivers of pricing strtategies in these cases. However, as Shugan and Xie (2000) point out buyer uncertainty about future valuations is not restricted to products such as air travel but likely for most services. We argue that such uncertainty exists for all products where the purchase decision preceeds consumption and the consumer has to plan ahead as to how much she should buy. could exist with respect to likely quantity of consumption in a planning period for any category. Heterogeneity in quantity uncertainty creates price discrimination possibilities where a price premium for larger package is possible even if the consumer pays attention to unit prices.

3. Model

In our model consumers face uncertainty over the amount they may consume over a certain period. A consumer, when she embarks on her weekly shopping trip does not typically know in advance how much she is going to consume during the coming week. So when making the decision as to how much to buy she has to bear in mind the possibility that she might run out of the good in question and then have to do without. Alternatively she might have to bear the transaction cost of going down to the grocery store again or visit convenience stores where goods are typically more expensive. So there are conseqences of misestimating consumption at the time of purchase. We presume that all consumers face this problem though some face less variation than others. A lonely widow with few friends would typically face less uncertainty in consumption than a bunch of college kids.

Consumers can also differ in other ways. They can differ in terms of storage costs and also in terms of transaction costs. A well paid but busy executive could have ample storage space but face a very high transaction cost of shopping. Finally their problem is compounded by the firms offerring only a limited number of package sizes to choose from. So consumers who differ in terms uncertainty, storage costs and transaction costs have to choose the package size that is best for them. We assume that consumers face a monopoly seller who can then design package sizes and their corresponding prices to discriminate among the consumers to maximize profits. To keep our model simple we will assume that there are no transaction costs and that the consumer can only purchase once in the current period.

Specifically, we assume that consumers can be of two types, indexed by $\{1,2\}$ who receive an utility $U(x_i)$ from consuming an amount x_i. Consumers are constrained to choose from two pack sizes, large and small denoted Q_l and Q_s, respectively. The utility function is written as

$$u(x_i) = \begin{cases} \theta_i x_i \ if \ x_i \leq Q_k \\ \theta_i Q_k \ if \ x_i > Q_k \\ 0 \ if \ x_i = 0 \end{cases} \tag{1}$$

If a consumer buys a pack of size Q_k and consequently finds that she wishes to consume an amount x_i then her utility is the quantity x_i multiplied by her taste parameter θ_i as long as her desired consumption is less than the pack she bought. If she wishes to consume more than she is constrained to consume the amount that is available in the pack. We do not allow reselling. We will, further assume that x_i is a random variable that is uniformly distributed on $(0, \overline{x_i})$ and that $\theta_1 < \theta_2$, so that type 2 has a higher marginal utility of consumption. The expected utility from consumption is then

$$U = \int_0^{Q_k} \frac{\theta_i x_i}{\overline{x_i}} dx_i + \int_{Q_k}^{\overline{x}_i} \frac{\theta_i Q_k}{\overline{x_i}} dx_i = \theta_i Q_k - \frac{\theta_i Q_k^2}{2\overline{x_i}} \tag{2}$$

Note that this allows us to represent the consumer's utility in terms of pack size and that the consumer's utility increases with an increase in pack size, $U' > 0$ for $Q_k < \overline{x_i}, U' = 0 \; if \; Q_k = \overline{x_i}$. The expected utility function is concave since $U'' < 0$. Also, the second term reflects the disutility of not being able to consume the desired amount. We further assume that consumers face a holding or storage cost $h_i(Q_k - x_i)$ for the amount left over. So the expected holding cost is H and is convex

$$H = \int_0^{Q_k} \frac{h_i(Q_k - x_i)}{\overline{x_i}} dx_i = \frac{h_i Q_k^2}{2\overline{x_i}} \tag{3}$$

It is useful to derive the ideal pack size for each type since it will facilitate our discussions later. Each individual, if they could choose their preferred pack size, would choose a size that would maximize

$$NEU_i \equiv v_i = \theta_i Q_k - \frac{\theta_i Q_k^2}{2\overline{x_i}} - \frac{h_i Q_k^2}{2\overline{x_i}} \tag{4}$$

The result of this maximization problem would be

$$Q_k^p = \left(\frac{\theta_i}{\theta_i + h_i}\right)\overline{x}_i \tag{5}$$

Note that in the absence of holding costs the consumer would buy the largest amount that she is likely to consume. The presence of holding costs induces the consumer to buy less than the maximum. It is also useful to note that the second and third expression represent the cost of buying a larger size package and the consequent cost of wastage and storage. If we combine the two we can write the *marginal disutility* of a larger package as $\left(\frac{\theta_i+h_i}{\bar{x}_i}\right)Q_k$. To ensure that the indifference curves cross once we will assume that the marginal utility of a larger pack is higher for type 2 for all pack sizes. The condition for this is

$$\left\{\theta_2 - \left(\frac{\theta_2 + h_2}{\bar{x}_2}\right)Q_k\right\} > \left\{\theta_1 - \left(\frac{\theta_1 + h_1}{\bar{x}_1}\right)Q_k\right\} \tag{6}$$

A sufficient condition for this ineqality to be satisfied is

$$\left(\frac{\theta_1 + h_1}{\bar{x}_1}\right) > \left(\frac{\theta_2 + h_2}{\bar{x}_2}\right) \tag{7}$$

This condition implies that the marginal expected disutility from buying a larger package is greater for type 1

The producer demands a transfer $T_k$for each package. Since the large package is meant for the consumer with the higher marginal utility of consumption, the following two conditions must hold.

$$v_2(Q_l) - T_l \geq 0..(IR2) \tag{4}$$

$$v_2(Q_l) - T_l \geq v_2(Q_s) - T_s..(IC2) \tag{5}$$

These are called the individual rationality (IR) and the incentive compatibility constraints (IC) respectively. Similarly type 1 will also face two constraints.

$$v_1(Q_s) - T_s \geq 0..(IR1) \tag{6}$$

$$v_1(Q_s) - T_s \geq v_1(Q_l) - T_l..(IC1) \tag{7}$$

The producer chooses to maximize profits.

$$\pi = \alpha T_s + (1-\alpha)T_l - \alpha c_s Q_s - (1-\alpha)c_l Q_l \tag{8}$$

The number of customers are normalized to one. A proportion α is of type 1 and the cost of producing packs of different sizes is c_k. Efficient levels of Q_s and Q_l are found by maximising welfare $(v_1 + v_2 - \pi)$ and the efficient quantities are

$$Q_s^e = \frac{(\theta_1 - c_s)}{\frac{(\theta_1+h_1)}{\overline{x_1}}} \tag{9}$$

and

$$Q_l^e = \frac{(\theta_2 - c_l)}{\frac{(\theta_2+h_2)}{\overline{x_2}}} \tag{10}$$

Note that efficient pack sizes are smaller than the preferred pack size of the consumers. Also, the efficient pack size for type 2 is larger than that for type 1because of (7).

3. Some results

The profit maxmizing quantities and prices are given in Proposition 1. They follow the standard pattern , where the high type gets the efficient amount and the low type is has to make do with less than its efficient amount. Transfers are set in such a way so that the low type is left without a surplus while the high type is better off.

Proposition 1: The producer sets quantity and prices as given below.

$$Q_s^* = \begin{cases} \dfrac{(\theta_1 - \theta_2) + \alpha(\theta_2 - \ c_s)}{\left(\frac{(\theta_1+h_1)}{\overline{x}_1} - (1-\alpha)\frac{(\theta_2+h_2)}{\overline{x_2}}\right)} \text{ for } \alpha > \dfrac{\theta_2 - \theta_1}{\theta_2 - c_s} \\ 0 \ otherwise \end{cases}$$

$$Q_l^* = \frac{(\theta_2 - c_l)}{\frac{(\theta_2+h_2)}{\overline{x_2}}} \text{for } \alpha \neq 1$$

$$T_s^* = \theta_1 Q_s^* - \frac{(\theta_1 + h_1) Q_s^{*^2}}{2\overline{x_1}} \text{ and } T_l^* = T_s^* + \theta_2 (Q_l^* - Q_s^*) - \left(\frac{\theta_2 + h_2}{2\overline{x}_2}\right)(Q_l^{*^2} - Q_s^{*^2})$$

We can also see that the market is not always covered. The producer would produce a smaller size only if there is a significant proportion of the low type in the market. Otherwise, profits are higher if the producer caters only to the higher type and extracts the full surplus from this type. In a sense introducing a small size is troublesome, not only does it have to be manufactured but the size and price has to be configured so that the high type is not attracted. The condition for selling to the lower type turns out to be quite simple and easy to calculate in our model. It is also interesting to note that the high type's consumption depends on parameters specific to that type whereas the low types quantity depend on both its and the high types's parameters. One can think of an algorithm where the producer first sets the quantity for the high type, since that is straight forward and then the quantity for the low type. Setting the transfer for the low type is again straightforward but that for the high type is a bit more complicated. The effect on the quantities and transfers from changes in the different parameters are given below.

Proposition 2: The impact of changes of various parameters on Q_s^* and Q_l^* are given below.

$$(i) \frac{\partial Q_l^*}{\partial c_l} < 0, (ii) \frac{\partial Q_l^*}{\partial h_2} < 0, (iii) \frac{\partial Q_l^*}{\partial \overline{x_2}} > 0, (iv) \frac{\partial Q_l^*}{\partial \theta_2} = \frac{(h_2 + c_l)}{(\theta_2 + h_2)^2} \overline{x}_2 > 0$$

$$(v) \frac{\partial Q_s^*}{\partial c_s} < 0, (vi) \frac{\partial Q_s^*}{\partial h_1} < 0, (vii) \frac{\partial Q_s^*}{\partial \overline{x}_1} > 0, (viii) \frac{\partial Q_s^*}{\partial h_2} > 0, (ix) \frac{\partial Q_s^*}{\partial \overline{x_2}} < 0,$$

$$(x) \frac{\partial Q_s^*}{\partial \alpha} = \frac{\theta_2 \left\{\left(\frac{\theta_1 + h_1}{\overline{x}_1}\right) - \left(\frac{\theta_2 + h_2}{\overline{x}_2}\right)\right\} + \alpha c_s \left\{\left(\frac{\theta_2 + h_2}{\overline{x_2}}\right)\right\}}{\left\{\left(\frac{\theta_1 + h_1}{\overline{x}_1}\right) - (1 - \alpha)\left(\frac{\theta_2 + h_2}{\overline{x}_2}\right)\right\}^2} > 0$$

$$(xi) \frac{\partial Q_s^*}{\partial \theta_1} = \frac{\left\{\frac{(\theta_1 + h_1)}{\overline{x}_1} - (1 - \alpha)\frac{(\theta_2 + h_2)}{\overline{x_2}}\right\} - \frac{(\theta_1 - \theta_2) + \alpha(\theta_2 - c_s)}{\overline{x}_1}}{\left\{\frac{(\theta_1 + h_1)}{\overline{x}_1} - (1 - \alpha)\frac{(\theta_2 + h_2)}{\overline{x_2}}\right\}^2}$$

$$(xii)\,\frac{\partial Q_s^*}{\partial \theta_2}=\frac{\left[\left\{\frac{(\theta_1+h_1)}{\overline{x}_1}-(1-\alpha)\frac{(\theta_2+h_2)}{\overline{x_2}}\right\}-\frac{(\theta_1-\theta_2)+\alpha(\theta_2-c_s)}{\overline{x_2}}\right](\alpha-1)}{\left\{\frac{(\theta_1+h_1)}{\overline{x}_1}-(1-\alpha)\frac{(\theta_2+h_2)}{\overline{x_2}}\right\}^2}$$

As expected, an increase in the cost of producing the large size reduces the pack size for the high type. Similarly, an increase in holding costs reduces the large pack size. An increase in $\overline{x_2}$ will have the opposite effect, since it will increase the range of uncertainty that the high type faces. Since the high type is now more uncertain about its actual consumption it would prefer to buy a larger amount. In the same way an increase in θ_2, the high type's taste parameter, will increase the large size. The effect on the small size package for the corresponding parameters is the same.

As mentioned earlier, the samll size package is also affected by other parameters. We find that an increase in the holding cost of the high type will increase the small size. In fact, since an increase in h_2 also decreases the large pack size, the two sizes will become closer. The effect of an increase in $\overline{x_2}$ has the opposite effect: the difference in terms of size increases. The large size becomes larger while the small size becomes smaller. This phenomenon can be called the incentive effect. Since the large size has increased the high type has to be disuadded from buying the small size by making it even smaller.

The proportion of the low type, α, only affects the size of the small pack, and the effect of an increase is positive. The effect of $\theta_1$and $\theta_2$on the small pack size is ambiguous. An increase in $\theta_1$should increase the small pack size. However, that might make it more attractive to the high type. In fact, to keep incentives properly aligned the small pack size might be reduced. Similarly, an increase in θ_2 could lead an increase or decrease in the small pack size.

Proposition 3: The effect on the transfers can be deduced from the following three relations and

Proposition 2.

$$(i)\frac{\partial T_s^*}{\partial Q_s^*} > 0, (ii)\frac{\partial T_l^*}{\partial Q_l^*} > 0 \; and \; \frac{\partial T_l^*}{\partial Q_s^*} < 0$$

Any parametric change can be considered to first manifest itself in pack sizes and then on transfers. For example an increse in the cost of producing the large size package will lead to a smaller sized package from proposition 2. This in turn will mean a lower transfer from (ii) in Proposition 3. Note that the transfer payment for the small pack increases with the size of the pack. The transfer for the large size though depends both the size of the large pack as well as that of the small pack. In fact as the size of the small pack increases it leads to a lower transfer for the large pack.

It will be useful to consider a parametric change that affects both pack sizes and therefore transfers. Take h_2, the holding cost for type 2. When it increases the large pack becomes smaller and the small pack larger. Consequently, the small pack becomes more expensive. Since the large pack has become smaller the transfer for the large pack should reduce by (ii). Also since the small pack has increased in size the transfer for the large pack should also reduce by (iii).

Proposition 4: The effect on profits of any parameter β can be derived from the expression

$$\frac{\partial \pi}{\partial \beta} = \frac{\partial Q_s^*}{\partial \beta}\left\{\frac{\partial v_1({Q_s}^*)}{\partial Q_s^*} - \frac{\partial v_2({Q_s}^*)}{\partial Q_s^*} + \alpha\left(\frac{\partial v_2({Q_s}^*)}{\partial Q_s^*} - c_s\right)\right\}$$

The effect on profits acts entirely through the small size. Any parametric change that only affects the large pack size will not change profit levels. So if the cost of producing a larger pack increases then the large pack will become smaller and so will the transfer and there will be no change in profits. The sign of this expression is ambiguous. If we restrict ourselves to the terms within brackets then the first two terms are negative since the first term is smaller than the second term. An increase in the small size provides more benefit to the type 2 customer if she is consricted to consume the small size pack. The next term in brackets is positive since in equilibrium the type 1 consumer gets less than the efficient amount and the type 2 consumer gets the efficient amount (see appendix). If the cost of producing the small pack size increases the small pack will become smaller and become cheaper. However, the large pack can become more expensive even though its size does not change. Profits can then go up or down depending on the values of the parameters.

Proposition 5: The effect on welfare can be derived from the following expression

$$\frac{\partial W}{\partial \beta} = \frac{\partial Q_s^*}{\partial \beta}\left(\frac{\partial v_1({Q_s}^*)}{\partial Q_s^*} - c_s\right)$$

Again, the effect is only through the small quantity. The term within brackets is positive so the effect can be gauged by proposition 2. This makes sense, since the high value type gets the efficient amount. Any change that increase the size of the small pack, an increase in uncertainty for the lower type or a decrease in the cost of producing the smaller size, increases welfare.

4. Size ratios, transfer ratios and unit prices

One intrigueing issue is the difference in pack sizes and transfers across different products and brands. For some products the large size is often close to twice the size of the small pack. In other cases the ratio can be as large as four times. Its is also noticed that different brands provide packages of different sizes and the the size ratio also varies, probably, because of competitive pressures. The same is true of transfers and unit prices. Further, there is the issue of premiums and discounts as already discussed earlier. We begin our discussion on these ratios with Proposition 6 which deals with size ratios.

Proposition 6: The effect of any parameter β on ratio of the two pack sizes $\frac{Q_l^*}{Q_s^*}$ will depend on the sign of

$$Q_s^* \frac{\partial Q_s^*}{\partial \beta} - Q_l^* \frac{\partial Q_l^*}{\partial \beta}$$

Obviously, the effect of any parameter on the size ratio depends on the effect of the change in the parameter on the small and large size, which is given in proposition 2. An increase in c_l will reduce the large size and have no effect on the small size. The two sizes will therefore become closer in size and the size ratio will decrease. Conversely, an increase in type 2's uncertainty, $\overline{x_2}$, will increase the large size while decreasing the small size, leading to an increase in the size ratio. So for products which are valued by the high value type, mainly luxaries, there will be a significant difference in the pack sizes.

Proposition 7: The effect of any parameter β on ratio of the two pack transfers $\frac{T_l^*}{T_s^*}$ will depend on the sign of

$$T_s^* \frac{\partial T_l^*}{\partial Q_l^*} \frac{\partial Q_l^*}{\partial \beta} + \frac{\partial Q_s^*}{\partial \beta} \left(T_s^* \frac{\partial T_l^*}{\partial Q_s^*} - T_l^* \frac{\partial T_s^*}{\partial Q_s^*} \right)$$

The term within brackets is negative and $\frac{\partial T_l^*}{\partial Q_l^*}$ positive from Proposition 3. If the parametric change contemplated has a negative effect on the small size then the whole term will be unambiguously positive. For instance an increase in c_s, will lead to a larger small size and a higher transfer.

Proposition 8: The effect of an increase in β on the unit price of a small size pack, $\frac{T_s^*}{Q_s^*} = p_s^*$, is given by

$$\frac{1}{Q_s^*} \frac{\partial Q_s^*}{\partial \beta} \left\{ \frac{\partial v_1(Q_s^*)}{\partial Q_s^*} - \frac{v_1(Q_s^*)}{Q_s^*} \right\}$$

The second term can be seen as the difference between the marginal valuation and the average valuation. Since the value of the small pack to the low value consumer would be increasing in pack size, this difference should always be positive. Then the sign of this term would depend on the first term which can be found from proposition 2.

Proposition 9: (i) The effect of an increase in β on the unit price of a large size pack, $\frac{T_l^*}{Q_l^*} = p_l^*$, is given by

$$\frac{1}{Q_l^*}\frac{\partial Q_l^*}{\partial \beta}\left[\left\{\frac{\partial v_2(Q_l^*)}{\partial Q_l^*} - \frac{v_2(Q_l^*)}{Q_l^*}\right\} + \left\{\frac{v_2(Q_s^*) - v_1(Q_s^*)}{Q_l^*}\right\}\right] + \frac{1}{Q_l^*}\frac{\partial Q_s^*}{\partial \beta}\left\{\frac{\partial v_1(Q_s^*)}{\partial Q_s^*} - \frac{\partial v_2(Q_s^*)}{\partial Q_s^*}\right\}$$

The effect of a change in any of the parameters on the price of the large pack is not that straightforward. It depends on the change of the price of the small pack, the second term, as well as the change in the price of the large pack.

Proposition 10: The large pack will be sold at a premium to the small pack size if

$$\frac{v_2(Q_l^*) - v_2(Q_s^*)}{Q_l^* - Q_s^*} > \frac{v_1(Q_s^*)}{Q_s^*}$$

Proposition 11: Define p = $p_l^* - p_s^*$. Then

$$\begin{aligned}\frac{dp}{d\beta} = \frac{1}{Q_l^*}\frac{\partial Q_l^*}{\partial \beta}&\left[\left\{\frac{\partial v_2(Q_l^*)}{\partial Q_l^*} - \frac{v_2(Q_l^*)}{Q_l^*}\right\} + \left\{\frac{v_2(Q_s^*) - v_1(Q_s^*)}{Q_l^*}\right\}\right] \\ &+ \frac{\partial Q_s^*}{\partial \beta}\left[\frac{1}{Q_l^*}\left\{\frac{\partial v_1(Q_s^*)}{\partial Q_s^*} - \frac{\partial v_2(Q_l^*)}{\partial Q_l^*}\right\} - \frac{1}{Q_s^*}\left\{\frac{\partial v_1(Q_s^*)}{\partial Q_s^*} - \frac{v_1(Q_s^*)}{Q_s^*}\right\}\right]\end{aligned}$$

Note that the first term in square brackets is always positive and the second is always negative. So if $\frac{\partial Q_s^*}{\partial \beta}$ is negative and $\frac{\partial Q_l^*}{\partial \beta}$ is positive the whole term will become positive and then the premium will increase. The only possibility for this seems increased high type uncertainty.

5. Directions for further research

As can be seen from the preceding sections, pack sizes and their unit prices have a complex relationship. As an addition to this paper we will try and tease out this relationship through a series of examples. We will also show how pack sizes and prices vary for different categories of goods. We hope that this would provide a sort of a ready reckoner for pricing.

There are two other directions that we could travel in future work. In our model we deal with a single period. In a way this is akin to assuming that the good is perishable. Durability raises the possibility of multiple periods and can change the nature of uncertainty faced by the consumer. In a similar vein we can consider the situation where consumers buy different goods at the same time with different storage and depletion rates. Substitutability and complementarity in storage and transaction costs may alter pricing and size decisions.

The other direction that we can travel is to look at competitive issues. Different producers can choose packs of different sizes and their corresponding prices. Clearly there are problems similar to location issues a la Hotelling. If both producers choose similar pack sizes then it intensifies competition at that size. Choosing a different size might also be detrimental in that it might face lower competition but could be unpopular. Thus, there are interesting strategic issues to consider.

6. Conclusion

This paper investigates the optimal choice of pack sizes and transfers that a monopolist would choose to maximise profits. There are two types of consumers, high value and low value and consequently two pack sizes, large and small. We assume that the difference between consumers is due to their different storage and transactions costs and that their consumption rates are uncertain. So, faced with the choice of buying a particular pack size the consumer has to consider prices as well as other costs. We derive the effects of changes in various parameters on optimal sizes and unit prices. We show that under some circumstances unit prices for the large pack may exceed that of the small pack. Our work can be of practical value to retailers.

Appendix

Proposition 1. The two binding constraints are (IR1) and (IC2)
$v_1(Q_s) - T_s = 0$
and
$v_2(Q_l) - T_l = v_2(Q_s) - T_s$

We substitute these two constraints into (8) to get

$$\pi = \alpha\left\{\theta_1 Q_s - \left(\frac{\theta_1 + h_1}{2\overline{x_1}}\right)Q_s^2\right\}$$

$$+(1-\alpha)\left\{\theta_2 Q_l - \left(\frac{\theta_2 + h_2}{2\overline{x_2}}\right)Q_l^2 - \theta_2 Q_s + \left(\frac{\theta_2 + h_2}{2\overline{x_2}}\right)Q_s^2 + \theta_1 Q_s - \left(\frac{\theta_1 + h_1)}{2\overline{x_1}}\right)Q_s^2\right\}$$

$$-\alpha c_s Q_s - (1-\alpha)c_l Q_l \tag{11}$$

Then the first order conditions are

$$\pi_{Q_s} = \alpha\left\{\theta_1 - \left(\frac{\theta_1 + h_1}{\overline{x_1}}\right)Q_s\right\} + (1-\alpha)\left\{-\theta_2 + \left(\frac{\theta_2 + h_2}{\overline{x_2}}\right)Q_s + \theta_1 - \left(\frac{\theta_1 + h_1}{\overline{x_1}}\right)Q_s\right\} - \alpha c_s = 0 \tag{12}$$

and

$$\pi_{Q_l} = (1-\alpha)\left\{\theta_2 - \left(\frac{\theta_2 + h_2}{\overline{x_2}}\right)Q_l\right\} - (1-\alpha)c_l = 0 \tag{13}$$

Then

$$Q_s^* = \frac{(\theta_1 - \theta_2) + \alpha(\theta_2 - c_s)}{\left(\frac{(\theta_1 + h_1)}{\overline{x}_1} - (1-\alpha)\frac{(\theta_2 + h_2)}{\overline{x_2}}\right)} \tag{14}$$

and

$$Q_l^* = \frac{(\theta_2 - c_l)}{\frac{(\theta_2 + h_2)}{\overline{x_2}}} \tag{15}$$

Substitution in the binding constraints gives us

$$T_s^* = \theta_1 Q_s^* - \frac{(\theta_1 + h_1){Q_s^*}^2}{2\overline{x_1}} \tag{16}$$

and

$$T_l^* = T_s^* + \theta_2(Q_l^* - Q_s^*) - \left(\frac{\theta_2 + h_2}{2\overline{x}_2}\right)({Q_l^*}^2 - {Q_s^*}^2) \tag{17}$$

Note that the the parameters need to fulfill certain conditions for the values of Q_k to represent an optimum. Specifically we require that

$$\pi_{Q_sQ_s} < 0 \; or - \alpha\left\{\left(\frac{\theta_1 + h_1}{\overline{x_1}}\right)\right\} + (1-\alpha)\left\{\left(\frac{\theta_2 + h_2}{\overline{x_2}}\right) - \left(\frac{\theta_1 + h_1}{\overline{x_1}}\right)\right\} < 0$$

$$or, -\alpha\left(\frac{\theta_1 + h_1}{\overline{x_1}}\right) + (1-\alpha)\left(\frac{\theta_2 + h_2}{\overline{x_2}}\right) - (1-\alpha)\left(\frac{\theta_1 + h_1}{\overline{x_1}}\right) < 0$$

$$or, (1-\alpha)\frac{(\theta_2 + h_2)}{\overline{x_2}} - \frac{(\theta_1 + h_1)}{\bar{x}_1} < 0 \qquad (18)$$

This condition is automatically satisfied by (7). We also require that $\pi_{Q_sQ_s}\pi_{Q_lQ_l} - 2\pi_{Q_sQ_l} > 0$, which is fulfilled.

We may also expect that $Q_s^* < Q_l^*$. This can be proved as follows.

Write π_{Q_s} as follows

$$\alpha\left\{\theta_1 - \left(\frac{\theta_1 + h_1}{\overline{x_1}}\right)Q_s - \alpha c_s\right\} + \left\{(1-\alpha) - \theta_2 + \left(\frac{\theta_2 + h_2}{\overline{x_2}}\right)Q_s + \theta_1 - \left(\frac{\theta_1 + h_1}{\overline{x_1}}\right)Q_s\right\}$$

Let $Q_s = Q_s^e$. Then the first term in the expression is zero and the second term is negative by (7). Then it follows that $Q_s^* < Q_s^e < Q_l^e = Q_l^*$. Quite naturally it follows that $T_l^* > T_s^*$, by incentive compatibility.

Proposition 2: The results can be derived from the expressions in proposition 1.

Proposition 3: $T_s = v_1(Q_s)$, so $\frac{\partial T_s^*}{\partial Q_s^*} > 0$.
Similarly, $v_2(Q_l) - T_l = v_2(Q_s) - T_s \Longrightarrow T_l = v_1(Q_s) + v_2(Q_l) - v_2(Q_s)$,
then $\frac{\partial T_l^*}{\partial Q_s^*} = v_1'(Q_s^*) - v_2'(Q_s^*)$. Now $v_2'(Q_s^*) > v_1'(Q_s^*)$ by the single crossing property (??).
So $\frac{\partial T_l^*}{\partial Q_s^*} < 0$. Again, from above $\frac{\partial T_l^*}{\partial Q_s^*} < 0$.

Proposition 4: Profits $\pi = \alpha T_s + (1-\alpha)T_l - \alpha c_s Q_s - (1-\alpha)c_l Q_l$, which can be written as
$\alpha\{v_1(Q_s^*) - c_s Q_s^*\} + (1-\alpha)\{v_2(Q_l^*) - v_2(Q_s^*) + v_1(Q_s^*) - c_l Q_l^*\}$
Then, $\frac{\partial \pi}{\partial \beta} = \frac{\partial \pi}{\partial Q_s^*}\frac{\partial Q_s^*}{\partial \beta} + \frac{\partial \pi}{\partial Q_l^*}\frac{\partial Q_l^*}{\partial \beta}$

$$= \alpha\{v_1'(Q_s^*) - c_s\}\frac{\partial Q_s^*}{\partial \beta} + (1-\alpha)\{v_1'(Q_s^*) - v_2'(Q_s^*)\}\frac{\partial Q_s^*}{\partial \beta} + (1-\alpha)\{v_2'(Q_l^*) - c_l\}$$

$= \alpha\{v_1'(Q_s^*) - c_s\}\frac{\partial Q_s^*}{\partial \beta} + (1-\alpha)\{v_1'(Q_s^*) - v_2'(Q_s^*)\}\frac{\partial Q_s^*}{\partial \beta}$, since $v_2'(Q_l^*) = c_l$
The sign is ambiguous, since $v_1'(Q_s^*) - c_s > 0$ ($Q_s^* < Q_s^e$) and $v_1'(Q_s^*) - v_2'(Q_s^*) < 0$ ($v_1'(Q_s^*) < v_2'(Q_s^*)$)

Proposition 5: Welfare $W = v_1(Q_s^*) + v_2(Q_l^*) - c_s Q_s^* - c_l Q_l^*$. Differentiate to get

$$v_1'(Q_s^*)\frac{\partial Q_s^*}{\partial \beta} - c_s\frac{\partial Q_s^*}{\partial \beta} + v_2'(Q_s^*)\frac{\partial Q_l^*}{\partial \beta} - c_l\frac{\partial Q_l^*}{\partial \beta}$$

$= \frac{\partial Q_s^*}{\partial \beta}\{v_1'(Q_s^*) - c_s\} + \frac{\partial Q_l^*}{\partial \beta}\{v_2'(Q_s^*) - c_l\} = \frac{\partial Q_s^*}{\partial \beta}\{v_1'(Q_s^*) - c_s\}$ since $v_2'(Q_l^*) = c_l$

Proposition 6 and 7: Differentiate $\frac{Q_l^*}{Q_s^*}$ and $\frac{T_l^*}{T_s^*}$

$$\frac{\partial\left(\frac{Q_l^*}{Q_s^*}\right)}{\partial \beta} = \frac{Q_s^*\frac{\partial Q_l^*}{\partial \beta} - Q_l^*\frac{\partial Q_s^*}{\partial \beta}}{(Q_s^*)^2}$$

Note that $T_s^* = \theta_1 Q_s^* - \frac{(\theta_1 + h_1){Q_s^*}^2}{2\overline{x_1}}$ and $T_l^* = T_s^* + \theta_2(Q_l^* - Q_s^*) - \left(\frac{\theta_2 + h_2}{2\overline{x}_2}\right)({Q_l^*}^2 - {Q_s^*}^2)$

$$\frac{\partial\left(\frac{T_l^*}{T_s^*}\right)}{\partial \beta} = \frac{T_s^*\frac{\partial T_l^*}{\partial \beta} - T_l^*\frac{\partial T_s^*}{\partial \beta}}{(T_s^*)^2} = \frac{T_s^*\left\{\frac{\partial T_l^*}{\partial Q_l^*}\frac{\partial Q_l^*}{\partial \beta} + \frac{\partial T_l^*}{\partial Q_s^*}\frac{\partial Q_s^*}{\partial \beta}\right\} - T_l^*\frac{\partial T_s^*}{\partial Q_s^*}\frac{\partial Q_s^*}{\partial \beta}}{(T_s^*)^2}$$

$$= \frac{T_s^*\frac{\partial T_l^*}{\partial Q_l^*}\frac{\partial Q_l^*}{\partial \beta} + \frac{\partial Q_s^*}{\partial \beta}\left(T_s^*\frac{\partial T_l^*}{\partial Q_s^*} - T_l^*\frac{\partial T_s^*}{\partial Q_s^*}\right)}{(T_s^*)^2}$$

Proposition 8: Write $\frac{T_s^*}{Q_s^*}$ as $\frac{v_1(Q_s^*)}{Q_s^*}$ and differentiate.

Proposition 9: Write $\frac{T_l^*}{Q_l^*}$ as $\frac{v_2(Q_l^*) - v_2(Q_s^*) + v_1(Q_s^*)}{Q_l^*}$ and differentiate.

Proposition 10: There will be a premium if $\frac{T_l^*}{Q_l^*} > \frac{T_s^*}{Q_s^*}$ or, $T_l^* Q_s^* > T_s^* Q_l^*$, which we can write as

$$T_l^* Q_s^* - T_s^* Q_s^* + T_s^* Q_s^* > T_s^* Q_l^* \Longrightarrow (T_l^* - T_s^*)Q_s^* > (Q_l^* - Q_s^*)T_s^* \Rightarrow \frac{(T_l^* - T_s^*)}{(Q_l^* - Q_s^*)} > \frac{T_s^*}{Q_s^*}$$

Write T_s^* as $v_1(Q_s^*)$ and T_l^* as $v_2(Q_l^*) - v_2(Q_s^*) + v_1(Q_s^*)$.

Proposition 11: Use propositions 8 and 9.

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