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A Generic Model for Public Provision of Insurance In the Presence of Externalities

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Abstract

Emerging markets are subject to exogenous shocks that are more frequent and bigger in size compared to the developed countries. Structural weaknesses such as currency mismatches in their balance sheets make these shocks even costlier. Yet, often times these countries are found having insufficient coverage against such shocks. Using a general equilibrium framework this paper looks at alternative government policies to encourage adequate provision of insurance when private decisions are not optimum socially. It also studies the impact of such policies on growth performance of the economy. A tax cum transfer scheme is found to be more effective in encouraging private provision of insurance compared to direct supply of hedging against shocks by the government. The optimal tax rate in our model depends on the extent to which decentralized level of insurance is socially sub-optimal.

Keywords: Public Insurance, Externality, Constrained Inefficiency, Growth

[1] Its Initial preliminary draft of the paper comments are welcome

1. Introduction

Emerging economies are subject to shocks that are both larger in size and more frequent compared to the developed countries (Pallage and Robe, 2003). Structural weaknesses in these economies increase the cost of these shocks even further thus making the role of government stabilization policies even more important. One example of such rigidities is the relative dominance of `hard currency` debt in the balance sheets of these economies even though their income streams are largely in domestic currency. This results in a currency mismatch between the liabilities and assets in their balance sheet. Balance sheet mismatches in subsectors of the economy, such as the banking system, the corporate sector, or the public sector have been at the heart of several emerging market crisis including the recent ones (e.g., Mexico 1994, East Asia 1997–98, Russia 1998, Argentina 2000-01, Brazil 2001). At the same time these currency mismatches can provide much needed capital to the financially constrained firms in the emerging markets and hence promote growth (see Ranciere, Tornell, and Vamvakidis, 2010). Emerging markets, therefore, face a trade-off between higher risks associated with greater currency mismatches and higher growth.

In case of India, the recent episode of depreciation in Rupee starting around the middle of 2011 brought the danger posed by such balance sheet mismatches in to sharp focus. Between July 2011 and August 2012, Rupee depreciated by around 20 percent against the US Dollar (see fig 1). For a country like India, such a sharp decline in rupee had an impact on the overall economy through higher costs of imports (particularly for items such as crude oil) and an increase in the external debt burden. The corporates in India had been increasingly tapping overseas loans, mostly in the US dollar, to save costs arising out of higher interest rates and liquidity constraints within the country. However, a significant proportion of these loans were un-hedged causing repayment problems for many of the corporate borrowers due to exchange rate depreciation and indirectly exposing domestic banks too.

Ranciere et. al. (2011) provides a new measure of currency mismatch in a recent paper that adjusts for un-hedged borrowing by household and firms. They provide this measure for a group of 10 Emerging European Economies and find that their de-facto measure of currency mismatch is much larger than other measures of currency mismatch that do not control for indirect mismatches in the banks' balance sheets. They conclude by saying that, in assessing systemic risk, policymakers should monitor not only mismatches in banks' balance sheets, but also indirect imbalances via the ability of banks' borrowers to repay foreign currency debts.

Evidence from India and emerging markets in general therefore point towards the presence of externalities in the individual firms` decision to insure themselves against exogenous shocks to their balance sheets. Further, indirect exposure of the financial system to un-hedged currency risks of their borrowers can be a significant source of increased financial vulnerability.

The purpose of our paper is to provide a generalized framework for studying the role of alternative government stabilization policies in the presence of such externalities and its impact on the long-run growth. We begin our analysis with a stochastic endowment economy having access to state contingent securities. Individual agents maximize private utility while deciding upon the level of insurance coverage but the probability of an `adverse` endowment shock or a `crisis` depends upon the overall level of insurance coverage in the economy. We compare the decentralized and social planner's solution and show that the decentralized solution can be constrained `inefficient` because decentralized agents do not take in to account the impact of their decisions on the vulnerability of the economy to `crisis`. We show that public provision of insurance to compensate for insufficient insurance in the market economy ends up `crowding out` private insurance while leaving the overall level of insurance in the economy unchanged. A tax –cum-transfer scheme on the other hand manages to bring the decentralized insurance level close to the socially optimum. We also calculate the level of tax that brings decentralized solution close to the social planner's outcome. The optimal tax rate is a function of externality parameter and the output cost of bad states.

We then extend the model to study the impact of individual's insurance decisions on economic growth using the linear production technology given by Rebelo (1991). The idea is that the economy combines the stochastic endowment of an intermediate good with capital to produce a `final good`. Once again, decentralized solution is `constrained inefficient`. In situations where the social planner's solution implies higher insurance coverage compared to the decentralized one, a tax-cum-transfer scheme can be used to bring centralized and decentralized outcomes together. A tax –cum – transfer scheme increases consumption growth during `bad` states at the expense of lower growth during `good` states. For a risk-averse individual this leads to higher welfare. Finally, the tax rate that equates decentralized and social planner's solution varies between 3 to 15 percent based on the values of the externality parameters.

Rest of the paper is organized a follows – Section II describes the model and compares the decentralized and centralized solutions. Section III discusses alternative stabilization policies. Section IV extends the endowment economy model to include growth. Section V concludes.

2. Model

Consider a small endowment economy inhabited by a large number of infinitely lived identical individuals distributed uniformly over the unit interval [0 1]. Each individual gets a stochastic endowment of a single good every period. These endowments are only subject to systemic risk (any idiosyncratic risk across individuals is diversified away). Endowment can take two values – e_N and e_C where N denotes 'normal' times while C denotes `crisis` or `bad` period. Endowment process can thus be described as follows:

$$\begin{aligned} e_N &= 1/n \\ e_C &= \phi/n, \ 0 < \phi < 1 \end{aligned} \quad (1)$$

The above specification is a simplified way of describing an economy subject to periodic crisis that reduces its per capita income significantly. Aggregate income of the economy during normal times is 1 while the aggregate income during the crisis period is ϕ. All idiosyncratic risks to income are diversified away and hence only the aggregate or systemic risks remain.

An important feature of the model is the ability of individual agents to buy insurance against the crisis situation. Agents can buy insurance from the rest of the world at the cost c per unit. Insurance is supplied in-elastically to all the agents in the economy at cost c.

The insurance contract can be described as follows: at the beginning of the period individual promises to pay c units of consumption good during `normal` times in return for one unit of consumption good if the crisis hits the economy. Of course, we assume that the contract can be enforced fully.

If i is the amount of insurance bought by an individual agent then his/her consumption stream can be described as:

$$C_N = 1/n - c*i \\ C_C = \phi/n - c*i + i \qquad (2)$$

Probability of a crisis hitting the economy is a function of the overall insurance level in the economy, i.e.

$$\pi(I_n) = f\left(\int_{n=0}^{1} i_n d_n\right), \ \pi^{'} < 0, \pi^{''} < 0 \qquad (3)$$

Higher the overall insurance level in the economy, lower the probability of crisis hitting the economy. Probability of a crisis is therefore endogenously determined in the model. This also implies that individual agent's decision regarding how much insurance to buy imposes an `externality` on the rest of the economy in the form of a lower or a higher probability of crisis.

One example where such an externality might be in play is the case of un-hedged foreign currency exposure of the commercial sector. Fluctuations in exchange rate can put the net revenues and balance sheets of the commercial sector under severe stress. This would be especially true if the liabilities and the assets of the firms are in different currency. Presence of such currency mismatches was one of the key reasons behind the Mexican crisis in 1994. A large part of the problem was bank's exposure to un-hedged borrowers who earned income denominated in local currency but had liabilities denominated in dollars. Devaluation of Peso made it difficult for the un-hedged borrower firms to pay back their loans thereby burdening the bank's balance sheets with non-performing assets and increasing the financial sector fragility.

Under such circumstances, un-hedged foreign currency liabilities of individual firms can cause systemic distress in the banking sector and the economy as a whole. Fire sales of un-hedged firms' assets and decline in aggregate demand are the other channels by which crisis can spread throughout the banking sector and rest of the economy. Hedging decisions by individual firms can therefore increase or reduce the vulnerability of the economy to systemic crisis.

Next section describes the optimization problem in a decentralized economy.

Optimization Problem in the Decentralized Economy

The representative individual wants to maximize his/her lifetime utility given the stochastic endowment process and the cost of insurance. Individual has CRRA preferences given by:

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma} \text{ if } \gamma \neq 1, \quad (4)$$
$$= \log(x) \text{ if } \gamma = 1$$

At the beginning of every period individual makes certain conjecture regarding the aggregate level of insurance in the economy I_{DC} and hence the probability of a crisis. He/she chooses the optimal level of insurance to buy so as to maximize the expected level of utility over the two states of the economy. Individual's maximization problem can be described as follows:

$$\max_{\{i\}} E[U] = (1-\pi(I_{DC})) \times \frac{(1/n - c \times i_{DC})^{1-\gamma}}{1-\gamma} + \pi(I_{DC}) \times \frac{(\phi/n - c \times i_{DC} + i_{DC})^{1-\gamma}}{1-\gamma} \quad (5)$$

Subject to: $1/n - c \times i_{DC} > 0$ and $\phi/n - c \times i + i > 0$

It is important to note that in the decentralized setting, individual agents do not take in to account the impact of their decisions on the aggregate level of insurance and hence the probability of a crisis. As we will see later on, this can result in the decentralized equilibrium being potentially inefficient. First order necessary conditions for optimization of this problem are:

$$\frac{\partial E[U]}{\partial i} = (1-\pi(I_{DC})) \times (1/n - c \times i_{DC})^{-\gamma}(-c) + \pi(I_{DC}) \times (\phi/n - c \times i_{DC} + i_{DC})^{-\gamma}(1-c) = 0 \quad (6)$$
$$\Rightarrow (1-\pi(I_{DC})) \times (1/n - c \times i_{DC})^{-\gamma}(c) = \pi(I_{DC}) \times (\phi/n - c \times i_{DC} + i_{DC})^{-\gamma}(1-c)$$

Equation (6) says that the welfare maximizing individual will buy insurance so that the expected marginal cost of insurance in terms of utility during `normal` states is equal to the expected marginal increase in utility due to insurance during `bad` states.

A higher level of insurance reduces the loss in consumption during the bad states at the cost of lower consumption during normal periods. At the same time, a higher level of insurance in the economy reduces the probability of a `bad` state.

CRRA preferences ensure that the welfare maximizing consumption is positive in both the states. This implies:

$$i_{DC} < 1/(n \times c) \text{ (i)}$$

Next, we define the decentralized equilibrium as follows:

Competitive equilibrium for the economy is a decision rule $i(I_{DC})$ such that the household maximization problem given by (5) is solved and the actual level of insurance in the economy is equal to the conjectured level ($\int_0^1 i_{DC} \times dn = I_{DC}$).

As discussed in the beginning, decentralized agents do not internalize the impact of their decisions on the overall probability of a crisis occurring in the economy. It is therefore important to ask whether a social planner's solution to this problem would be different. This is what we try to do in the next section.

Social Planner's Solution

Suppose that instead of individual agents, a benevolent social planner decides upon the level of insurance to be bought by individual agents in the economy in a way that maximizes the aggregate welfare in the economy. Social planner faces the same cost of insurance and the same endowment distribution as the private sector. The social planner's maximization problem can be expressed as follows:

$$\max_{\{i\}} \left(1-\pi\left(\int_0^1 i \times dn\right)\right) \times \frac{(1/n - c \times i)^{1-\gamma}}{1-\gamma} + \pi\left(\int_0^1 i \times dn\right) \times \frac{(\phi/n - c \times i + i)^{1-\gamma}}{1-\gamma} \qquad (7)$$
$$s.t.\ 1/n - c \times i > 0;\ \text{and}\ \phi/n - c \times i + i > 0$$

Key difference between equations 5 and 7 is the fact that the social planner internalizes the impact of his/her decision on the probability of a crisis hitting the economy. First order necessary conditions for the social planner are as follows:

$$\begin{aligned} &-\pi'\left(\int_0^1 i \times dn\right) \times \frac{(1/n - c \times i)^{1-\gamma}}{1-\gamma} + \left(1-\pi\left(\int_0^1 i \times dn\right)\right) \times (1/n - c \times i)^{-\gamma} \times (-c) \\ &+\pi'\left(\int_0^1 i \times dn\right) \times \frac{(\phi/n - c \times i + i)^{1-\gamma}}{1-\gamma} + \pi\left(\int_0^1 i \times dn\right) \times (\phi/n - c \times i + i)^{-\gamma} \times (1-c) = 0 \end{aligned} \qquad (8)$$

Rearranging the terms in (8) we can write:

$$\begin{aligned} &\left(1-\pi\left(\int_0^1 i \times dn\right)\right) \times (1/n - c \times i)^{-\gamma} \times (c) = \pi\left(\int_0^1 i \times dn\right) \times (\phi/n - c \times i + i)^{-\gamma} \times (1-c) \\ &+\pi'\left(\int_0^1 i \times dn\right) \times \left(\frac{(\phi/n - c \times i + i)^{1-\gamma}}{1-\gamma} - \frac{(1/n - c \times i)^{1-\gamma}}{1-\gamma}\right) \end{aligned} \qquad (9)$$

Equation (9) is similar to equation (6) except for the last term. Last term in equation (9) captures the fact that a higher level of insurance reduces the probability of economy hitting a crisis state which in turn affects the expected utility of the individual. Given that $\pi^{'}\left(\int_0^1 i \times dn\right)$ is negative by assumption, second term in (9) is positive (negative) if the term inside the brackets is negative (positive).

Is the decentralized equilibrium '*Constrained Efficient*'

We next ask the question whether the decentralized solution presented in (6) is 'constrained efficient'.

'Constrained Efficiency' is defined as follows: The decentralized equilibrium is constrained efficient if a social planner, who chooses the actual level of insurance subject to the overall resource constraint of the economy in the two states cannot improve the expected welfare of the individual agents in the economy.

In other words we want to ask whether the social planner can change the decentralized equilibrium at the margin and obtain an improvement in the welfare. If yes, then the decentralized equilibrium is not constrained efficient otherwise it is.

In order to answer the above question we start by comparing the first order conditions of decentralized agent and the social planner.

$$(1-\pi(I_{DC}))\times(1/n-c\times i_{DC})^{-\gamma}(c)=\pi(I_{DC})\times(\phi/n-c\times i_{DC}+i_{DC})^{-\gamma}(1-c) \qquad \text{(\_DC)}$$

$$\begin{aligned}&\left(1-\pi\left(\int_0^1 i_{DC}\times dn\right)\right)\times(1/n-c\times i_{DC})^{-\gamma}\times(c)=\pi\left(\int_0^1 i_{DC}\times dn\right)\times(\phi/n-c\times i_{DC}+i_{DC})^{-\gamma}\times(1-c)\\&+\pi^{'}\left(\int_0^1 i_{DC}\times dn\right)\times\left(\frac{(\phi/n-c\times i_{DC}+i_{DC})^{1-\gamma}}{1-\gamma}-\frac{(1/n-c\times i_{DC})^{1-\gamma}}{1-\gamma}\right)\end{aligned} \qquad \text{(\_SP)}$$

Consider a reallocation of consumption across the two states by the social planner, starting from the privately optimal allocation in the decentralized equilibrium. In particular, consider the impact of an increase in i by a very small amount.

The term on the left hand side of the decentralized Euler equation gives the loss in expected utility during good states resulting from such a decision while the term on the right hand side gives the resultant gain in bad states. The two are equal along the decentralized optimum. Looking at the social planner's Euler equation we can see that, even though the marginal cost of increasing the level of insurance by one unit is the same for the social planner, the marginal benefit given by the right hand side expression is different because of the additional term $\pi'\left(\int_0^1 i_{DC}\times dn\right)\times\left(\frac{(\phi/n-c\times i_{DC}+i_{DC})^{1-\gamma}}{1-\gamma}-\frac{(1/n-c\times i_{DC})^{1-\gamma}}{1-\gamma}\right)$ representing the impact of higher insurance on the probability of a crisis state.

To continue our analysis further, notice that the second term on the right hand side of (_SP) is positive if the term inside the bracket is negative i.e. if $\frac{(1/n-c\times i_{DC})^{1-\gamma}}{1-\gamma}>\frac{(\phi/n-c\times i_{DC}+i_{DC})^{1-\gamma}}{1-\gamma}$ or, equivalently, if $1/n-c\times i_{DC}>\phi/n-c\times i_{DC}+i_{DC}$ (This is because $\pi'\left(\int_0^1 i_{DC}\times dn\right)$ is negative by assumption) . In other words, social planner's marginal benefit from higher insurance is greater than the marginal benefit of the private agent in the decentralized set up as long as the level of consumption chosen by the decentralized economy in the good state is higher than the level of consumption during the crisis.

Proposition 1: Assume that the consumption in the good state is higher than the consumption in the bad state along the decentralized equilibrium. In this case the decentralized equilibrium is 'constrained inefficient'. Central planner can improve upon the decentralized equilibrium by choosing a higher level of insurance compared to the decentralized solution.

Proof:

Step 1. Marginal cost of an extra unit of insurance is the same for the social planner and the decentralized economy to begin with.

Step 2. Marginal benefit of an extra unit of insurance is higher for the social planner when consumption in the good state is higher than the consumption in the bad state along the decentralized equilibrium.

Step 3. Since the marginal benefit and marginal cost of an additional unit of insurance are equal along the decentralized equilibrium, 1 and 2 imply that the marginal cost of an extra unit of insurance is less than the marginal benefit of insurance for the social planner's problem.

Thus, social planner can improve upon the decentralized solution by increasing the level of insurance. This implies that the decentralized equilibrium is constrained inefficient.

Q.E.D.

It is easy to see that decentralized equilibrium will be constrained inefficient even when $1/n-c\times i_{DC}>\phi/n-c\times i_{DC}+i_{DC}$. In this case social planner can improve the decentralized equilibrium by reducing the level of insurance and raising the level of consumption in `normal` states. The only situation when the decentralized equilibrium is constrained efficient is when $1/n - c \times i_{DC} = \phi / n - c \times i_{DC} + i_{DC}$, i.e. when decentralized consumption in normal and crisis states is equal.

To understand the intuition behind this result, consider the example of insurance against exchange rate risk on foreign currency transactions. Our analysis implies that if the decentralized market economy results in less than complete hedging of exchange rate risk as measured by the relative levels of consumption in `normal` and `bad` states then the social planner can improve the overall welfare of the economy by choosing a higher level of insurance as long as level of insurance reduces the probability of economy hitting a crisis state. On the other hand, if the decentralized economy ‘over-insures’, say by buying currency hedging instruments for speculation, thereby making the level of consumption during crisis state greater than that during normal times, then the social planner can improve the welfare of the economy by choosing a smaller level of insurance.

The only case when the decentralized solution is constrained efficient is when the amount of insurance completely and exactly covers the risk from the crisis state. In that case the decentralized solution will be the same as the social planner’s solution. In other words, social planner cannot improve upon the decentralized outcome by changing the level of insurance in the decentralized economy marginally.

Inefficiency in the de-centralized set up in our model results from the fact that individual agents do not take in to account the impact of their decisions on the probability of the economy hitting the crisis state. Consequently, whenever there is a divergence between the consumption levels in `normal` and `crisis` states; social planner’s marginal benefit from additional insurance differs from that of individual agents making the decentralized solution constrained inefficient.

3. Impact of Government Stabilization Schemes

Public Provision of Insurance

A logical question that follows from the above analysis is the following. Can the social planner provide for the difference between market based and socially optimum level of insurance whenever former is less than the latter? Here we are assuming that in cases where the opposite is true, social planner can reduce the level of insurance in the economy to the socially desirable level by imposing a tax on insurance that increases the cost of purchasing insurance for the individual agents.

In other words we want to find out the impact on decentralized equilibrium of a benevolent social planner buying insurance on behalf of private individuals whenever the private level of insurance is less than the social optimum. Social planner finances his purchase of additional insurance is through state contingent taxes. Social planner imposes a tax on income in good states at the rate τ and uses the proceeds to buy insurance against bad states at cost c. Government’s contract is the same as that of the private individual except that the government promises to pay the insurer in good states on the basis of its ability to raise taxes from private individuals in the economy during good states.

Total tax collection by the government per person is equal to the cost of insurance per person. Let τ be the tax rate of the government and i_g be the amount of insurance bought by the government on behalf of private individual. Budget constraint of the government is given by:

$$\frac{\tau}{n} = c \times i_g \qquad (10)$$

For private individual consumption in the modified set up is given by:

$$\begin{aligned} C_N &= \frac{(1-\tau)}{n} - c \times i_{pvt} \\ C_C &= \frac{\phi}{n} - c \times i_{pvt} + i_{pvt} + i_g \end{aligned} \qquad (11)$$

Subscripts N and C denote 'normal' and 'crisis' states. Ignoring the question of optimal level of taxation for now we express the modified individual maximization problem as follows:

$$\max_{\{i_{pvt}\}} E[u] =$$

$$\left(1-\pi\left(\int_0^1 (i_{pvt}+i_g)\times dn\right)\right)\times\left(\frac{\left(\frac{(1-\tau)}{n}-c\times i_{pvt}\right)^{(1-\gamma)}}{(1-\gamma)}\right)+\pi\left(\int_0^1 (i_{pvt}+i_g)\times dn\right)\times\left(\frac{\left(\frac{\phi}{n}-c\times i_{pvt}+i_{pvt}+i_g\right)^{(1-\gamma)}}{(1-\gamma)}\right) \qquad (12)$$

Subject to (10) and (11).

Once again, private agents take π and i_g as given. First order necessary condition for optimization is given by:

$$\frac{\partial E[u]}{\partial i_{pvt}} = (1-\pi)\times\left(\frac{(1-\tau)}{n} - c\times i_{pvt}\right)^{(-\gamma)} \times(-c) + \pi\times\left(\frac{\phi}{n} - c\times i_{pvt} + i_{pvt} + i_g\right)^{(-\gamma)}\times(1-c) = 0 \qquad (13)$$

Or

$$(1-\pi)\times\left(\frac{(1-\tau)}{n} - c\times i_{pvt}\right)^{(-\gamma)} \times(c) = \pi\times\left(\frac{\phi}{n} - c\times i_{pvt} + i_{pvt} + i_g\right)^{(-\gamma)}\times(1-c) \qquad (13')$$

Once again, optimization requires that the marginal cost of insurance in 'Normal' states is equal to its marginal benefit in good states.

We would now like to ask the question whether public provision of insurance can 'crowd out' private insurance and if yes then under what conditions? It is worth noting once again that we are assuming a situation where the decentralized economy by itself provides for less than socially 'efficient' insurance level.

Proposition 2:

Introduction of public insurance reduces the private level of insurance in the economy by more than the provision for public insurance thereby reducing the overall level of insurance in the economy.

Proof:

We prove proposition 2 in three steps. First we show that the overall level of insurance in the economy with the provision of public insurance cannot be higher than the decentralized equilibrium. Next we show that the overall level of insurance with public provisioning cannot be equal to the decentralized equilibrium. Step 3 then combines the results from the first two steps to show that the overall level of insurance in the economy declines with public provision for insurance.

Step 1: *Overall insurance level in the presence of public provisioning cannot be greater than that without it.* Suppose that the overall level of insurance in the economy with public provision of insurance is greater than the decentralized equilibrium, i.e., $i_{pvt} + i_g > i_{DC}^{OLD}$ where i_{DC}^{OLD} is the solution to (6) and i_{pvt} is the solution to (13). Then using (11) and (13') we can write:

$$\begin{aligned} E[MCI_{PI}] &= (1-\pi(I_{PI}))\times\left(\frac{(1-\tau)}{n} - c\times i_{pvt}\right)^{(-\gamma)} \\ &= (1-\pi(I_{PI}))\times\left(\frac{1}{n} - \frac{\tau}{n} - c\times i_{pvt}\right)^{(-\gamma)} \\ &= (1-\pi(I_{PI}))\times\left(\frac{1}{n} - c\times(i_{pvt} + i_g)\right)^{(-\gamma)} \end{aligned} \tag{14}$$

Here $E[MCI_{PI}]$ is the expected marginal cost of insurance and I_{PI} is the aggregate level of insurance in the economy with public provision of insurance. Since, by assumption $i_{pvt} + i_g > i_{DC}^{OLD}$ (14) implies

$$E[MCI_{PI}] = (1-\pi(I_{PI}))\times\left(\frac{1}{n} - c\times(i_{pvt}+i_g)\right)^{(-\gamma)}$$
$$> (1-\pi(I_{DC}))\times\left(\frac{1}{n} - c\times(i_{DC}^{old})\right)^{(-\gamma)} = E[MCI_{DC}] \qquad (15)[2]$$

Equation (15) implies that the marginal cost of insurance with public provision of insurance is much greater than the marginal cost of insurance without it. Similarly, looking at the marginal benefit of insurance equations (6) and (13`) imply that

$$E[MBI_{PI}] = \pi(I_{PI})\times\left(\frac{\phi}{n} - c\times(i_{pvt}) + i_{pvt} + i_g\right)^{(-\gamma)}$$
$$< \pi(I_{DC})\times\left(\frac{\phi}{n} - c\times(i_{DC}^{old}) + i_{DC}^{old}\right)^{(-\gamma)} = E[MBI_{DC}] \qquad (16)[3]$$

$E[MBI_{PI}]$ is the expected marginal benefit of insurance with public provision and $E[MBI_{DC}]$is the expected marginal benefit of insurance in the decentralized set up without public provisioning. Equation (16) implies that the expected marginal benefit of insurance with public provisioning is higher than that without it. From (15) and (16) we can write:

$$E[MCI_{PI}] > E[MCI_{DC}] = E[MBI_{DC}] > E[MBI_{PI}] \qquad (17)$$

The equality in (17) follows from the fact that along the equilibrium path marginal benefits and marginal cost of insurance are the same. However, equation (17) contradicts the first order optimality condition expressed in (13`) according to which expected marginal benefit and expected marginal cost of private insurance have to be equal at the optimum with public provisioning of insurance.

Hence our assumption - $i_{pvt} + i_g > i_{DC}^{OLD}$ cannot be true, or, in other words the overall insurance level in the presence of public provisioning cannot be greater than that without it.

Step 2: _Overall level of insurance with public provisioning of insurance cannot be equal to that without it_. Suppose that $i_{pvt} + i_g = i_{DC}^{OLD}$, i.e. there is no change in the level of insurance due to public provisioning. This implies:

$$C_N^{PI} = \frac{(1-\tau)}{n} - c\times i_{pvt} = \frac{1}{n} - \frac{\tau}{n} - c\times i_{pvt} = \frac{1}{n} - c\times i_g - c\times i_{pvt}$$
$$= \frac{1}{n} - c\times(i_g + i_{pvt}) = \frac{1}{n} - c\times i_{DC}^{OLD} = C_N^{DC} \qquad (18)$$

[2] Both the terms of the right hand side of the inequality in (15) are smaller than the corresponding terms on the left hand side
[3] Again, both the terms of the right hand side of the inequality in (16) are smaller than the corresponding terms on the left hand side

C_N^{PI} is the consumption in 'normal' state with public insurance while C_N^{DC} is the consumption in 'normal' state without public insurance. Consumption level in good states is same with and without public provisioning. Since the overall level of insurance in the economy and therefore the probability of a crisis state is the same, (18) implies that

$$E[MCI_{PI}] = E[MCI_{DC}] \qquad (19)$$

Further:

$$C_C^{PI} = \frac{\phi}{n} - c \times i_{pvt} + i_{pvt} + i_g = \frac{\phi}{n} - c \times i_{pvt} + i_{DC}^{OLD} > \frac{\phi}{n} - c \times i_{DC}^{OLD} + i_{DC}^{OLD} = C_C^{DC} \quad (20)$$

C_C^{PI} is the consumption in 'crisis' state with public insurance while C_C^{DC} is the consumption in 'crisis' state without public insurance. The last inequality follows from the fact that $i_{DC}^{OLD} = i_{pvt} + i_g > i_{pvt}$ by assumption. Once again, (20) implies that

$$E[MBI_{PI}] < E[MBI_{DC}] \qquad (21)$$

Combining (19) and (21)

$$E[MCI_{PI}] = E[MCI_{DC}] = E[MBI_{DC}] > E[MBI_{PI}] \qquad (22)$$

Equation (22) contradicts (13`) according to which $E[MCI_{PI}] = E[MBI_{PI}]$.

Once again, this implies that our assumption - $i_{pvt} + i_g = i_{DC}^{OLD}$ cannot be true or, the overall level of insurance with public provisioning of insurance cannot be equal to that without it.

Step 3: *<u>Public provision of insurance induces private agents to reduce their own purchase of insurance by more than the amount of insurance provided by the social planner.</u>* Together the first two steps imply that the overall level of insurance in an economy with public provision of insurance cannot be greater than or equal to the level of insurance in an economy without such public insurance. This leaves us with only one possibility - overall level of insurance in an economy with public provision of insurance is actually lower than the level of insurance in an economy without such a provision or $i_{pvt} + i_g < i_{DC}^{OLD}$.

This implies that public provision of insurance induces private agents to reduce their own purchase of insurance by more than the amount of insurance provided by the social planner.

The above result is easy to understand. When the social planner tries to increase the level of insurance in the economy by taxing the endowment in good states and purchasing insurance from the proceeds, it reduces the expected after tax consumption of the individual in `good` states while increasing it `bad` states *ceteris paribus*. This in turn implies a higher marginal utility of consumption in `good` states and lower marginal utility of consumption in `bad` states. Clearly, under these circumstances the individual agent can benefit from shifting consumption from good states to bad states by reducing his own purchase of insurance.

A Tax cum Transfer Scheme

As shown above, government effort to try and increase the overall level of insurance in the economy by taxing the income in good states and purchasing insurance from the proceeds will result in a 'crowding out' of private insurance. An alternative to the above scheme is what we refer to as a `tax-cum-transfer` scheme. Social Planner imposes a proportional consumption tax in 'Normal' state which it returns in the form of lump-sum transfers. To be more specific:

$$C_{TT}^{N} = \left(1/n - c \times i_{TT}\right) \times \left(1 - \tau\right) + Tr \qquad (23)$$

Where ` $\tau$ ` is the proportional tax rate on consumption and Tr is the lump sum transfer. Social planner's budget constraint requires:

$$\left(1/n - c \times i_{TT}\right) \times \left(1 - \tau\right) = Tr \qquad (24)$$

We show that the tax-cum-transfer scheme described above will necessarily increase the overall level of insurance in the economy and can be used in situations where the decentralized economy chooses less than socially optimum level of insurance.

<u>**Proposition 3:**</u> A proportional consumption tax combined with lump-sum transfer in `normal` times will necessarily increase the aggregate level of insurance in the decentralized economy.

Proof:

Once again, we prove proposition 3 by contradiction in three steps. First note that the first order necessary condition for individual's welfare maximization problem is:

$$\begin{aligned} &\left(1 - \pi\left(I_{TT}\right)\right) \times \left(\left(1/n - c \times i_{TT}\right) \times \left(1 - \tau\right) + T\right)^{-\gamma} \times \left(1 - \tau\right) \times \left(c\right) = \\ &\left(\pi\left(I_{TT}\right)\right) \times \left(\phi / n - c \times i_{TT} + i_{TT}\right)^{-\gamma} \times \left(1 - c\right) \end{aligned} \qquad (25)$$

We want to prove that $I_{TT} > I_{DC}$.

Step 1: <u>*Overall level of insurance in an economy with the tax-cum-transfer scheme described above cannot be less than the level of insurance in an economy without such a scheme.*</u> Suppose that the converse is true, i.e. $I_{TT} < I_{DC}$. In this case:

$$1 - \pi\left(I_{TT}\right) < 1 - \pi\left(I_{DC}\right) \text{ by assumption.}$$

$$\text{and } \left(1/n - c \times i_{TT}\right) \times \left(1 - \tau\right) + Tr = 1/n - c \times i_{TT} > 1/n - c \times i_{DC}$$

Here we have used the fact that the tax cum transfer scheme leaves the disposable income in `Normal` states unchanged. This implies:

$$MCI_{TT} = (1-\pi(I_{TT}))\times((1/n-c\times i_{TT})\times(1-\tau)+T)^{-\gamma}\times(1-\tau)\times(c) < (1-\pi(I_{DC}))\times(1/n-c\times i_{TT})^{-\gamma}\times(c) = MCI_{DC} \quad \text{(a)}$$

Similarly,

$$MBI_{TT} = (\pi(I_{TT}))\times(\phi/n-c\times i_{TT}+i_{TT})^{-\gamma}\times(1-c) > (\pi(I_{DC}))\times(\phi/n-c\times i_{DC}+i_{DC})^{-\gamma}\times(1-c) = MBI_{DC} \quad \text{(b)}$$

Given that $MCI_{DC} = MBI_{DC}$, (a) and (b) imply:

$$MCI_{TT} < MCI_{DC} = MBI_{DC} < MBI_{TT} \quad \text{(c)}$$

This, however, contradicts the first order optimality condition given in (25). Thus, $I_{TT} < I_{DC}$ cannot be true. Overall level of insurance in an economy with the tax-cum-transfer scheme cannot be less than the level of insurance in an economy without such a scheme.

Step 2: _Overall level of insurance in an economy with the tax-cum-transfer scheme described above cannot be equal to the level of insurance in an economy without such a scheme._ Suppose that the converse is true, i.e. $I_{TT} = I_{DC}$. In this case:

$$MCI_{TT} = (1-\pi(I_{TT}))\times((1/n-c\times i_{TT})\times(1-\tau)+T)^{-\gamma}\times(1-\tau)\times(c) < (1-\pi(I_{DC}))\times(1/n-c\times i_{TT})^{-\gamma}\times(c) = MCI_{DC} \quad \text{(a)}$$

Here, once again, we use the fact that the disposable income in `Normal` states remains unchanged with the tax and transfer scheme.

$$MBI_{TT} = (\pi(I_{TT}))\times(\phi/n-c\times i_{TT}+i_{TT})^{-\gamma}\times(1-c) = (\pi(I_{DC}))\times(\phi/n-c\times i_{DC}+i_{DC})^{-\gamma}\times(1-c) = MBI_{DC} \quad \text{(b)}$$

Again, (a) and (b) imply: $MCI_{TT} < MCI_{DC} = MBI_{DC} = MBI_{TT}$ (c)

This contradicts the optimality condition laid out in (25). Thus, $I_{TT} = I_{DC}$ cannot be true. Overall level of insurance in an economy with the tax-cum-transfer scheme cannot be equal to the level of insurance in an economy without such a scheme.

Step 3 _Overall level of insurance in a decentralized economy with the tax-cum-transfer scheme described above must be greater than the level of insurance in an economy without such a scheme._

Together the first two steps imply that the overall level of insurance in an economy with tax-cum-transfer scheme cannot be less than or equal to the level of insurance in an economy without such a scheme. This leaves us with only one possibility - overall level of insurance with tax-cum-transfer scheme is higher than the level of insurance in an economy without such a scheme or $I_{TT} > I_{DC}$.

Once again, intuition behind the results is simple. A consumption tax-cum-transfer scheme reduces the marginal cost of insurance in `Normal` states while keeping the disposable income in 'Normal` states and the marginal benefit of insurance during `bad` states (at the same level of insurance) unchanged. Individual agents can therefore benefit from buying more insurance; thereby shifting consumption towards the `crisis` states.

In the context of un-hedged foreign currency exposure this suggests that the government could try to limit the risk taking tendency of firms by imposing a state contingent tax coupled with a lump-sum transfer of equivalent amount.

Model Calibration and Simulation

We calibrate the above model to study the welfare impact of sharp currency devaluations in emerging market countries where firms tend to borrow in foreign currency due to various reasons and are therefore subject to potential exchange rate risks. Cost of hedging implies that often these firms do not hedge their currency exposures adequately especially when there is an implicit guarantee by the government in the form of `pegged` exchange rate[4]. Under such circumstances, unexpected sharp devaluations in exchange rates can lead to systemic risks.

Cost of hedging - c is calibrated using the Mumbai inter-bank forward offer rate of 6.5%. Coefficient of risk aversion γ is set at 2. Cost of depreciation- ϕ is calibrated using estimates of exposure of Indian firms to exchange rate risks by Patnaik and Shah (2010) (0.063%). We normalize n to 1 without loss of generalization.

It is necessary to pay some attention to the form of probability function. We choose the following functional form for the probability of bad state: $\pi(I) = 1 - \chi I^{\eta}$. Parameter χ is calibrated to match the average probability of crisis in emerging countries. η is the elasticity of $1-\pi(.)$, or the probability of a good state with respect to the level of insurance. If η is equal to 1 then it implies that a one percent increase in the level of insurance raises the probability of `good` state by 1 percent. Coefficient η is calibrated to match the equilibrium level of insurance to the level of hedging observed in the Indian data (60 percent). While $\pi(I)$ falls as I increases, it falls at a declining rate as I increases. Table A in the appendix gives the benchmark parameter values.

[4] Pegged is being loosely used to denote any regime where central bank tries to control `excessive` exchange rate volatility or change. In case of Brazil the Central Bank directly provided foreign exchange hedge through net placements of USD linked securities and foreign exchange derivatives in order to help the economy face large exchange rate fluctuations.

Simulation Results

We first solve the model for the decentralized economy and get the equilibrium value of insurance and the probability of shock. Then we estimate the solution for the social planner and compare it with the decentralized solution. Next, we try to find the optimal tax-cum-transfers scheme that will bring the decentralized solution close to the social planner's optimum. Tables 1 and 2 give the results from this exercise.

Text Table [1]

η	Prob. Bad (Decentralized)	Prob. Bad (Social Planner)	Coverage (Decentralized)	Coverage (Social Planner)
1.54	0.063	0.044	73 %	83%
1.55	0.0634	0.0455	74%	84%
1.56	0.0635	0.047	75.5%	84.4%
1.57	0.0636	0.049	76.8%	85.2%
1.58	0.0638	0.050	78.4%	85.8%
1.59	0.0639	0.0519	79.7%	86.5 %
1.6	0.064	0.0534	80.7%	87.3%

The first column of Table 1 gives the equilibrium probability of `crisis` state under the decentralized set up for a given value of η while the second column gives the equilibrium probability of `crisis` state under the social planner. As we can see, social planner's solution implies a lower probability of `bad` state for relevant values of η. Columns 3 and 4 give the insurance coverage defined as the percentage of output loss during `crisis` states covered by insurance. Once again, for the relevant values of η, social planner chooses a higher level of insurance compared to the decentralized economy. These results reflect the presence of externality in the economy that is not taken in to account by the private agent.

Table 2 shows the level of consumption tax that will bring decentralized solution close to the social planner's solution for different values of η. As we can see; the `optimal` consumption tax varies between 18 and 33 percent for our choice of externality parameter η

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Text Table [2]

η	τ (0.063)
1.54	0.33 (73%)
1.55	0.30 (74%)
1.56	0.28 (75%)
1.57	0.25 (77%)
1.58	0.20 (78%)
1.59	0.20 (79%)
1.6	0.18 (81%)

Simulations

This section presents the welfare implications of `insurance externality` by comparing the results from simulating the social planner's solution and the decentralized equilibrium. We are particularly interested in how the externality affects the volatility of consumption/returns and the welfare of an economy. We simulate the economy 5000 times to get the distribution of consumption and welfare in the decentralized economy and compare it with social planner's economy. For benchmark parameters, volatility of consumption is reduced by more than one-fourth and welfare cost due to consumption volatility is halved under social planner's solution. One caveat in this result is that financial crisis often lead to a reduction in the trend output itself (Barlevy (2004), Epaulard and Pommeret (2003)) in which case the loss in welfare would be much greater and hence the reduction in welfare loss under social planner much higher compared to those reported in this paper. In the next section we extend the above framework to incorporate growth.

4. Insurance and Growth

In this section we extend the endowment economy model presented above to incorporate long term growth. We do it using the endogenous growth model of Rebelo (1991). The basic framework draws on the approach of Mendoza (1997). Details of our approach are presented below.

Economy is inhabited by a large number of identical household consumers consuming a single good. Households have access to a single asset or a linear production technology that gives stochastic returns. This is similar to Mendoza (1997). However, the returns in the model are a function of the stochastic endowment process described in the previous section. Once again, household can insure against the 'bad' states in endowment at a cost. Household preferences are given by the standard CRRA utility function. They chose consumption and savings so as to maximize their life time utility. We can summarize these assumptions as follows:

Household's problem:

$$\max \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$$

$$\text{Subject to: } A_{t+1} = R_t \times (A_t - C_t)$$

$$R_t = m_t + 1$$

$$m_t = \frac{1}{n} - c \times i_t \text{ with probability } p = 1 - \pi(I)$$

$$= \frac{\phi}{n} - c \times i_t + i_t \text{ with probability } 1\text{-}p = \pi(I)$$

Time line of the events in the economy is as follows:

1. Households choose insurance based on their expectations of I, the level of asset A_t and knowledge of ϕ.
2. They then choose the level of consumption and savings.
3. Finally, the `state` of the economy is realized and household knows A_{t+1}.

We solve the above problem in two steps. First we solve for the choice of consumption and saving for a given level of insurance i_t. We then use the value function obtained in the first step to obtain the optimal level of insurance. Bellman equation for the optimal consumption choice can be written as:

$$V(A_t, s_t) = \max_{\{i_t\}} u(C_t) + \beta \times E_t[V(A_{t+1}, s_{t+1})]$$

$$\text{Subject to: } A_{t+1} = R_t(A_t - C_t)$$

The Euler equation for this problem is given by:

$$u`(C_t) = \beta \times E[R_t \times u`(c_{t+1})] \quad (1)$$

As shown by Mendoza (1997), the consumption and value functions of this problem are given by: $C_t = \lambda \times A_t$ and $V(A_t, s_t) = \dfrac{\lambda^{-\gamma} A_t^{1-\gamma}}{1-\gamma}$, where $\lambda = 1 - \beta^{1/\gamma} \times \left[E\left[R_t^{1-\gamma}\right]\right]^{1/\gamma}$.

For the solution to be meaningful we must have $\beta^{1/\gamma} \times \left[E\left[R_t^{1-\gamma}\right]\right]^{1/\gamma} < 1$ or $\left[E\left[R_t^{1-\gamma}\right]\right] < \beta^{-1}$. This condition ensures that the level of consumption is a positive fraction of the household's asset holdings. Growth rate of consumption in this model is given by: $\dfrac{C_{t+1}}{C_t} = (1-\lambda)R_t \quad (2)$

Optimal level of insurance

Households choose the optimal level of insurance so as to maximize the value function $V(A_t, s_t) = \dfrac{\lambda^{-\gamma} A_t^{1-\gamma}}{1-\gamma}$ for a given level of A_t. It is clear from the expression that maximizing $V(A_t, s_t)$ is the same as minimizing λ which in turn is the same as maximizing $E\left[R_t^{1-\gamma}\right]$. The problem of choosing the optimal level of insurance can thus be written as:

$$\max_i E\left[R_t^{1-\gamma}\right]$$

$$\Rightarrow \max_i p \times \left(R_t^{Good}\right)^{1-\gamma} + (1-p) \times \left(R_t^{Bad}\right)^{1-\gamma} = \Phi(i)$$

First order necessary conditions for the optimization are given by:

$$\frac{\partial \Phi(i)}{\partial i} = p \times \left(R_t^{Good}\right)^{-\gamma} \times \frac{\partial R_t^{Good}}{\partial i} + (1-p) \times \left(R_t^{Bad}\right)^{-\gamma} \times \frac{\partial R_t^{Good}}{\partial i} = 0$$

$$\Rightarrow p \times \left(R_t^{Good}\right)^{-\gamma} \times c = (1-p) \times \left(R_t^{Bad}\right)^{-\gamma} \times (1-c) \quad \_DC$$

Left hand side of the above equation gives the expected loss in $\Phi(i)$ due to an extra unit of insurance in good states while the right hand side is the gain in $\Phi(i)$ due to an extra unit of insurance in `bad` states. The above equation assumes that the individuals do not take in to account the impact of their purchase of insurance on the output cost in `bad` states.

We next try to see whether the social planner will choose a different solution to the problem described above.

Social Planner's Solution

Social planner's problem can be described as follows:

$$\max \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$$

$$\text{Subject to: } A_{t+1} = R_t \times (A_t - C_t)$$

$$R_t = m_t + 1$$

$$m_t = \frac{1}{n} - c \times i_t \text{ with probability } p$$

$$= \frac{\phi}{n} - c \times i_t + i_t \text{ with probability } 1\text{-}p$$

This is the same as the decentralized setting except that the social planner takes in to account the impact of the aggregate level of insurance on the output cost during bad states. The consumption function and value function for the social planner are also the same as those under the decentralized set up. The social planner's problem is therefore once again:

$$\max_i E\left[R_t^{1-\gamma}\right] \quad (5)$$

$$\Rightarrow \max_i p \times \left(R_t^{Good}\right)^{1-\gamma} + (1-p) \times \left(R_t^{Bad}\right)^{1-\gamma} = \Phi(i)$$

First order necessary condition for the social planner is:

$$\frac{\partial\Phi(i)}{\partial i}=0\Rightarrow\left(1-\pi\left(\int_0^1 i\times dn\right)\right)\times\left(R_t^{Good}\right)^{-\gamma}\times c=$$

$$\left(\pi\left(\int_0^1 i\times dn\right)\right)\times\left(R_t^{Bad}\right)^{-\gamma}\times(1-c)+\pi^{'}\left(\int_0^1 i\times dn\right)\times\left(\left(R_t^{Bad}\right)^{1-\gamma}-\left(R_t^{Good}\right)^{1-\gamma}\right) \qquad \text{\_SP}$$

Left hand side of the above equation is the same as the left hand side of decentralized optimization condition (_DC). It represents the loss in $\Phi(i)$ due to an extra unit of insurance in good states. However, the right hand side of the above equation, which gives the benefit of an extra unit of insurance during bad times, is different from the right hand side of (_SP). Given that $\pi^{'}(I)$is negative, the right hand side of (_SP) is in fact greater than (_DC) for any given level of I such that $\left(\left(R_t^{Bad}\right)^{1-\gamma}-\left(R_t^{Good}\right)^{1-\gamma}\right)<0$ or $\left(R_t^{Bad}<R_t^{Good}\right)$.

Is the decentralized equilibrium '*Constrained Efficient*'

Once again we ask the question whether the decentralized solution presented in (_DC) is 'constrained efficient', i.e., can the social planner; starting from the decentralized equilibrium, change the level of insurance marginally so as to improve the welfare of the economy.

Comparing the decentralized and social planner's solution we can see that as long as $R_t^{Bad}\neq R_t^{Good}$ the decentralized solution will be different from the Social Planner`s solution.

As we saw in the case of an endowment economy, it is easy to show that in this case too, the social planner can change the value of insurance in the decentralized economy marginally so as to achieve a higher level of overall welfare whenever $R_t^{Bad}\neq R_t^{Good}$. Once again, a tax-cum-transfer scheme can be used to raise the overall level of insurance in a decentralized economy when the decentralized insurance level is below the socially optimum. Such a scheme will raise the decentralized insurance level closer to the socially desirable level. (See Appendix).

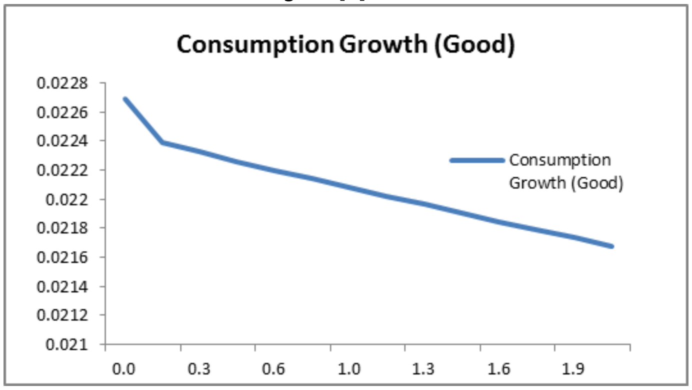
Behavior of Growth and Marginal Propensity to Consume

As shown above, the presence of externality implies that the decentralized solution is not `Constrained Efficient`. However, is it possible for us to draw any conclusions regarding the behavior of consumption growth in this setting? Figures 1 and 2 show the behavior of growth rate in `good` and `bad` states for different levels of insurance coverage.

As we can see, a higher level of insurance coverage increases the consumption growth rate during bad states while reducing it in the good states. This makes intuitive sense because a higher level of insurance coverage implies higher returns to savings in `bad` states and lower returns to savings in `good` states.

Text Figure [1]

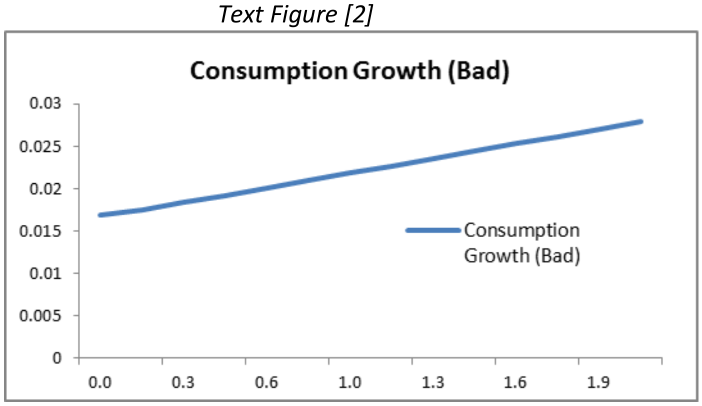
Consumption Growth



Insurance Coverage

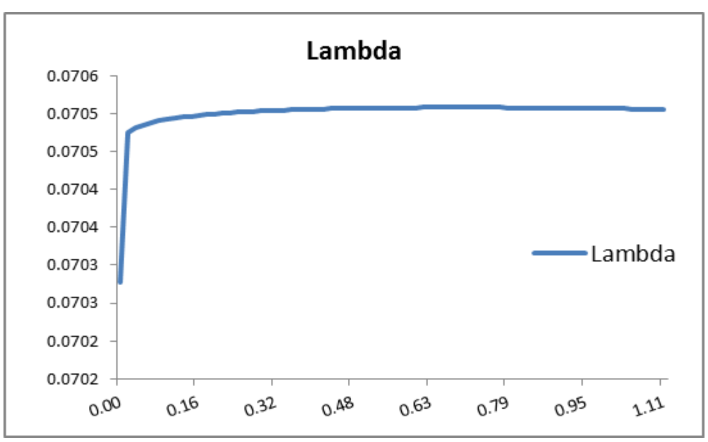
Text Figure [2]

Consumption Growth



Insurance Coverage

Text Figure [3]

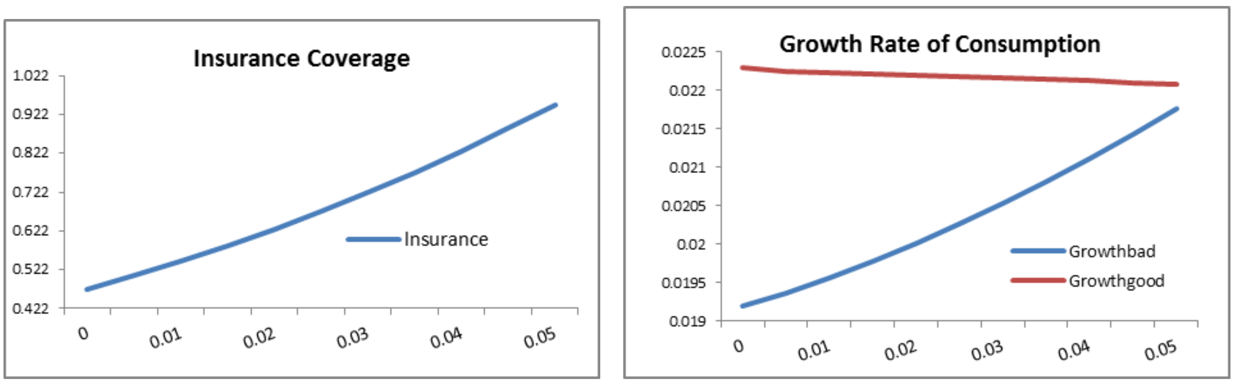


Insurance Coverage

Figure 3 plots the value of lambda, the marginal propensity to consume out of wealth for different levels of insurance coverage. It indicates that a higher level of insurance increases the marginal propensity to consume out of wealth. A higher level of insurance reduces the uncertainty in income and therefore encourages consumption out of wealth. The increase in marginal propensity to consume due to higher insurance is much larger at smaller levels of insurance coverage than at higher levels of insurance coverage.

How does tax-cum-transfer scheme affect growth

Text Figure [4]

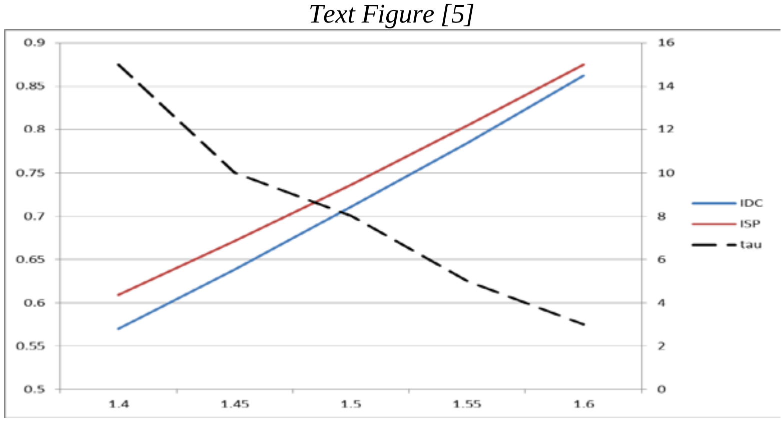


Tax Rate *Tax Rate*

Figure 4 plots the behavior of insurance coverage and growth rates for alternative levels of tax rates under the tax-cum-transfer scheme. As we can see, a tax-cum-transfer scheme increases the growth rate of consumption during bad states while lowering it slightly during good states. The level of insurance coverage increases monotonically with a higher tax rate on consumption.

Constrained `Inefficiency` and Optimal Tax Rate

Text Figure [5]



η

As discussed above, the presence of externality implies that the decentralized solution is constrained `inefficient` and a tax-cum-transfer scheme can be used to bring the decentralized solution closer to the social planner's solution. Figure 5 gives the level of insurance coverage under decentralized and centralized set up for different values of η.

Greater the elasticity of π with respect to *i* the higher is the optimal level of insurance coverage. For the relevant values of η, social planner always chooses a higher level of insurance coverage as it takes in to account the impact of higher insurance on the probability of a `bad` state. The broken line in Figure 5 gives the tax rate that brings decentralized solution close to the central planner's solution (Scale on the right hand side). As we can see, the optimal level of consumption tax falls as η increases since the difference between the decentralized and centralized solution is smaller for higher values of η. For the values of parameter η under consideration, the optimal tax rate varies between 3 and 15 percent. Compared to other studies looking at the question of `Tobin` tax, our estimates are much higher.

5. Conclusions

This paper explores the question of optimal level of insurance against economy wide shocks in the presence of externalities. In particular our focus was on the possible impact of public sector trying to provide insurance in situations where the decentralized economy under-provides for the bad outcomes due to the presence of externality. We find that direct public provision of insurance might not be the best solution in situations like these. Instead a tax-cum-transfer scheme is more likely to bring the decentralized solution closer to the social planner's preferred outcome. This has implications for a number of situations where futures' contract exists and has an impact on the spot values of the asset or the commodity in question.

There are several directions in which the above analysis can be extended. In particular presence of heterogeneous agents instead of a single representative agent can make the model richer. Similarly, explicit treatment of `moral hazard` and `agency` problems associated with such insurance contracts can give further interesting insights.

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Appendix I

Tax Cum Transfer Scheme

This section provides a formal proof of how a tax-cum-transfer scheme can encourage the demand for insurance in a decentralized economy. Social Planner imposes a proportional tax on m_t in `Normal` state which it returns in the form of lump-sum transfers. To be more specific:

$$m_{TT}^{N} = (1/n - c \times i) \times (1-\tau) + Tr \qquad (8)$$

Social planner's budget constraint requires: $(1/n - c \times i) \times (\tau) = Tr$ (9)
The first order condition for the decentralized economy becomes:

$$\Rightarrow p \times \left(R_{TT}^{Good}\right)^{-\gamma} \times c \times (1-\tau) = (1-p) \times \left(R_{TT}^{Bad}\right)^{-\gamma} \times (1-c) \qquad (10)$$

Or

$$\begin{aligned} &\Rightarrow p \times (1 + (1/n - c \times i_{TT}) \times (1-\tau) + Tr)^{-\gamma} \times c \times (1-\tau) \\ &= (1-p) \times (1 + \phi/n - c \times i_{TT} + i_{TT})^{-\gamma} \times (1-c) \end{aligned} \qquad (11)$$

From the government's budget constraint this implies:

$$p \times (1 + (1/n - c \times i_{TT}))^{-\gamma} \times c \times (1-\tau) = (1-p) \times (1-c) \times (1 + \phi/n - c \times i_{TT} + i_{TT})^{-\gamma} \qquad (12)$$

Proposition: $i_{TT} > i_{DC}$.

We will rewrite (_DC) as:

$$(1 + (1/n - c \times i_{DC}))^{-\gamma} = (1 + \phi/n - c \times i_{DC} + i_{DC})^{-\gamma}$$

We prove the above proposition in three steps.

Step 1: *Overall level of insurance in an economy with the tax-cum-transfer scheme described above cannot be less than the level of insurance in an economy without such a scheme.* Suppose that the converse is true, i.e. $I_{TT} < I_{DC}$. In this case:

$$\pi(I_{TT}) < \pi(I_{DC}) \text{ by assumption.}$$

$$\text{and } (1/n - c \times i_{TT}) \times (1-\tau) + Tr = 1/n - c \times i_{TT} > 1/n - c \times i_{DC}$$

Here we have used the fact that the tax cum transfer scheme leaves the disposable income in `Normal` states unchanged. This implies:

$$MCI_{TT} = (1-\pi(I_{TT}))\times(1+(1/n - c\times i_{TT})\times(1-\tau)+Tr)^{-\gamma}\times(1-\tau)\times(c) <$$
$$(1-\pi(I_{DC}))\times(1+1/n - c\times i_{DC})^{-\gamma}\times(c) = MCI_{DC} \qquad \text{(a)}$$

Similarly,

$$MBI_{TT} = (\pi(I_{TT}))\times(1+\phi/n - c\times i_{TT} + i_{TT})^{-\gamma}\times(1-c) >$$
$$(\pi(I_{DC}))\times(1+\phi/n - c\times i_{DC} + i_{DC})^{-\gamma}\times(1-c) \qquad \text{(b)}$$
$$= MBI_{DC}$$

Given that $MCI_{DC} = MBI_{DC}$, (a) and (b) imply:

$$MCI_{TT} < MCI_{DC} = MBI_{DC} < MBI_{TT} \quad \text{(c)}$$

This, however, contradicts the first order optimality condition given in (25). Thus, $I_{TT} < I_{DC}$ cannot be true. Overall level of insurance in an economy with the tax-cum-transfer scheme cannot be less than the level of insurance in an economy without such a scheme.

Step 2: *Overall level of insurance in an economy with the tax-cum-transfer scheme described above cannot be equal to the level of insurance in an economy without such a scheme.* Suppose that the converse is true, i.e. $I_{TT} = I_{DC}$. In this case:

$$MCI_{TT} = (1-\pi(I_{TT}))\times(1+(1/n - c\times i_{TT})\times(1-\tau)+Tr)^{-\gamma}\times(1-\tau)\times(c) <$$
$$(1-\pi(I_{DC}))\times(1+1/n - c\times i_{DC})^{-\gamma}\times(c) = MCI_{DC} \qquad \text{(a)}$$

Here, once again, we use the fact that the disposable income in `Normal` states remains unchanged with the tax and transfer scheme.

$$MBI_{TT} = (\pi(I_{TT}))\times(1+\phi/n - c\times i_{TT} + i_{TT})^{-\gamma}\times(1-c)$$
$$= (\pi(I_{DC}))\times(1+\phi/n - c\times i_{DC} + i_{DC})^{-\gamma}\times(1-c) \qquad \text{(b)}$$
$$= MBI_{DC}$$

Again, (a) and (b) imply: $MCI_{TT} < MCI_{DC} = MBI_{DC} = MBI_{TT}$ (c)

This contradicts the optimality condition laid out in (25). Thus, $I_{TT} = I_{DC}$ cannot be true. Overall level of insurance in an economy with the tax-cum-transfer scheme cannot be equal to the level of insurance in an economy without such a scheme.

Step 3 *Overall level of insurance in a decentralized economy with the tax-cum-transfer scheme described above must be greater than the level of insurance in an economy without such a scheme.*

Together the first two steps imply that the overall level of insurance in an economy with tax-cum-transfer scheme cannot be less than or equal to the level of insurance in an economy without such a scheme. This leaves us with only one possibility - overall level of insurance with tax-cum-transfer scheme is higher than the level of insurance in an economy without such a scheme or $I_{TT} > i_{DC}$.

Q.E.D.

Once again, intuition behind the results is simple. A tax-cum-transfer scheme reduces the `marginal cost` of insurance in `Normal` states while keeping the disposable income in ‘Normal` states and the marginal benefit of insurance at the same level of insurance unchanged. Individual agents can therefore benefit from buying more insurance; thereby shifting consumption towards `crisis` states.

Figure 1

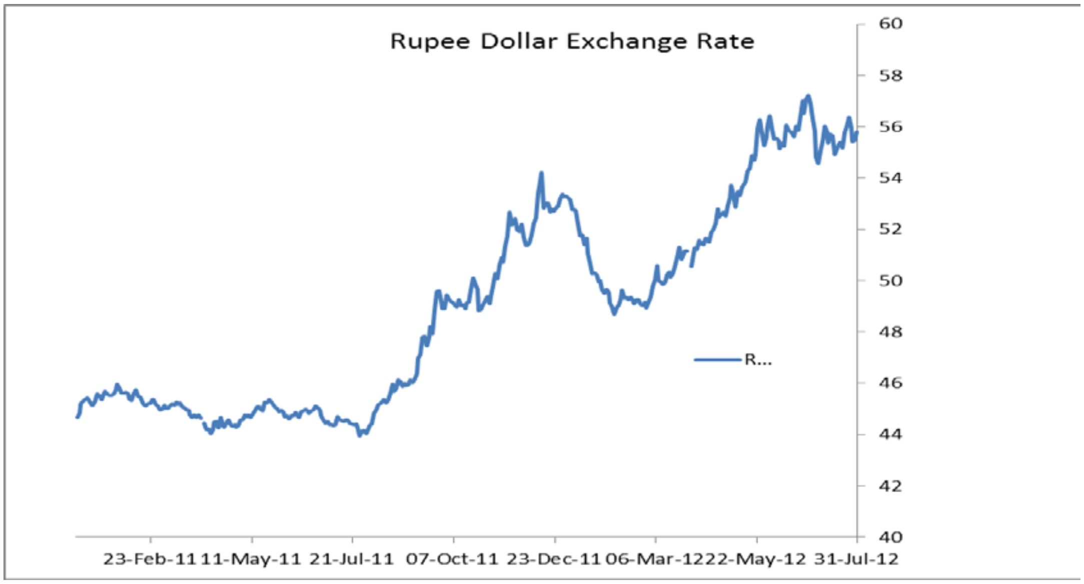

Table A

Benchmark Parameters

Parameter	Benchmark Value
Coefficient of Risk Aversion, γ	2
Cost of Insurance, c	6.5%
Output loss, ϕ	6.3%
Externality Parameter, η	1.5