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Image Measurement Errors and Missile Performance

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Image Measurement Errors and Missile Performance

Summary

The main objective of this study is to estimate the errors in the measurement of lengths of missiles or satellite launch vehicles from digital images available in the public domain when the scale or the pixel sizes of the images are not known.

Under the International Strategic and Security Studies Programme (ISSP) of the National Institute of Advanced Studies (NIAS), assessments of China's Ballistic and Cruise missiles as well as Pakistan's missiles have been made earlier. In those studies the dimensions of missiles were measured using ENVI 3.5 image processing software. Using these measured dimensions the propellant mass and the lift off mass were estimated using available engineering data and knowledge. This derived data set from the measurements was used to run typical trajectory profiles to estimate the ranges of the missiles.

The present report is an attempt to validate the methodology used in these studies by estimating the measurement error. We estimate the error by making measurements on images of missiles whose dimensions are known a priori with confidence. Images are selected carefully so that geometric and scale effects are minimal. In each of the images, the diameter of the missile or launch vehicle is measured in pixels and since the actual diameter is known, the pixel size is estimated. Pixel size is the actual diameter in meters divided by the diameter in pixels. Once the pixel size is known the relevant length is measured in terms of number of pixels and then converted to meters using the estimated pixel size. Since the actual length is known the measurement error is obtained as the difference between the actual length and the measured length.

An estimate of measurement error is useful to understand the changes observed in missile dimensions during the early stages of its development. Since the actual lengths of the missiles being flown during its development phase may not be known precisely, we would like to know whether the changes we are seeing in the lengths during different launchings of the same missile are real or whether they are the result of measurement errors. If we are able to estimate the likely measurement error for the missile then we could confidently group the various launchings into different classes based on measurements made from images of the missile.

The measurement error depends on the inherent characteristics of the imaging device (camera or digitizer) as well as the angle at which the picture is taken. Since these are not known a priori, we studied the dependence of the measurement error on various parameters such as the pixel size, the calibration diameter which is the diameter of the missile in meters and the actual length that was measured.

For this study 22 sample images were analysed. In this sample the diameter used as the basis for calibration varied from 1 m to 2.8 m with the lengths being measured varying from 10 m to 20 m. The sample size though not very large was adequate to place bounds on the measurement error.

We observed that the measurement error generally increases with increasing pixel size. The measurement error also decreases as the diameter of the missile used as the basis for calibration increases. For the same calibration diameter the measurement error decreases with decreasing pixel size. As the length of the missile increases, the measurement error also increases. However, this is compensated to some extent if the image has very small pixel sizes.

After trying various combinations of the relevant parameters affecting the measurement error, a best fit equation was determined for estimating the error in terms of the measured length of the missile, the pixel size in centimeters and the reciprocal of the calibration diameter in meters. This equation was used to estimate the measurement error in images of missiles of unknown dimensions.

The average error and the standard deviation of the error in measuring lengths up to 13 m from digital images were estimated to be 0.08 m and 0.36 m respectively. The analysis showed that the measurement error was between 3% and 6% when the calibration diameter was 1.0 m while it was between 0.2% and 3.2% when the calibration diameter was 2.8 m.

The empirical equation obtained was used to determine whether there are variants in the Shaheen 2 missile developments of Pakistan. The images of Shaheen 2 launched in 2004, 2005, 2006, and 2008 were analysed. The launches of 2004 and 2005 had similar lengths. The missile launched in 2006 was longer than those launched in 2004, 2005 and 2008. The pair of missiles launched in 2008 was the shortest; they were at least 30 cm shorter than the missiles of 2004 and 2005.

Applying the empirical equation to these measurements revealed the following:

- ❏ The difference in the lengths between the first two launches (2004 and 2005) when compared to the launch of 2006 is significantly more than the estimated measurement error, suggesting that the 2006 launch is a different variant of the first two launches.
- ❏ Similar conclusions could be drawn when the first two launches are compared with the pair of launches carried out in 2008.

Although no specific reasons can be given for such variations in lengths of these missiles, we can infer that there are at least three variants of missiles of the same class suggesting that some experimental development process is going on in Pakistan.

Introduction

Digital Image processing techniques have been used extensively to analyze remote sensing images as well as other photographic images. In many situations enhancement of the images to identify features and objects of interest is followed by making measurements on these objects. In the case of remote sensing images, estimates of area are often of interest. A variety of techniques are available to enhance and classify an image prior to making such estimations. In all these situations the inherent resolution or the pixel size of the image (which depends on the imaging or the digitizing instrument used) plays a significant role. The error associated with area estimates due to what are called boundary pixels have also been studied extensively.[1] In these studies the resolution of the image is known and the errors that occur are attributed to misclassification and inadequate spectral and spatial resolution. Measurements made on digital images of animal foot prints, eggs, animal sizes etc. have also been used to model other not easily measurable characteristics such as weight, health of an animal etc..[2,3,4]

Changes in missile dimensions are of interest to the researchers in the strategic community since they enable tracking and monitoring developments in the missile programmes of countries of interest to them.

In earlier studies we have used image analysis software (ENVI 3.5) to make measurements on images of various missiles.[5,6]. In those studies we had used a limited set of images to calibrate and validate this methodology. In some of these images features such as vehicles with known dimensions, humans whose heights are known within some range or other objects of known dimensions were used as calibration sources for making measurements on the missile. If such possibilities exist in an image we could get an independent assessment of the critical dimensions of a missile such as its diameter or length. This could help us to check whether these dimensions are in agreement with other sources of information available in the public domain. More often than not such calibration benchmarks may not be available in the image. In such cases the only way we can make inferences about the missile is to know a priori the name of the missile. If we do know the name of the missile we can use its most characteristic feature as a basis for estimating other dimensions of the missile. For a missile the most characteristic feature is its diameter. We can use the publicly known value of the diameter of the missile as a calibration benchmark to estimate

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[1] Crapper P.F., “An estimate of the number of boundary cells in a mapped landscape coded to grid cells”, *Photogrammetric Engineering and Remote Sensing* 50, no.10 (1984): 1497

[2] Bridge E.S, Boughton R.K Aldredge R.A,. Harrison, T.J.E, Bowman R and Schoech S.J, “Measuring egg size using digital photography: testing Hoyt’s method using Florida Scrub-Jay eggs”, *J. Field Ornithol.* 78, no. 1 (2007): 109

[3] Tanaka S, Yamauchi A and Kono Y, “Easily accessible method for root length measurement using an image analysis system”, *Japanese Journal of crop science* 64, no. 1 (1995): 144

[4] Tasdemir S, Yakar M, Urkmez A and Inal S, “Determination of Body Measurements of a Cow by Image Analysis in International Conference on Computer Systems and Technologies” *CompSysTech* (2008) V.8.1

[5] Chandrashekar S, Arvind Kumar and Nagappa R, “An Assessment of Pakistan’s Ballistic Missile Programme,- Technical and Strategic Capability”, NIAS Study R5-06 (2006)

[6] Chandrashekar S, Gupta S, Nagappa R and Arvind Kumar, 2007, “An Assessment of China’s Ballistic and Cruise Missiles”, NIAS Study, R4-07, (2007)

various other dimensions of the missile. Changes in missile dimensions are of interest to researchers in the strategic community since they enable tracking and monitoring developments in the missile programmes of countries of interest to them.

In order to validate this approach for looking at missiles it is necessary to get an estimate of the measurement error. Knowing the measurement error is crucial not only for separating one type of missile from another but is also required to look at different variants within the same missile family.

In this study we try to estimate the errors in measurement by carrying out measurements on images of missiles whose dimensions are known a priori with confidence. An estimate of the measurement error is obtained from sample set of images of missiles of known dimensions. These estimates are then used as a basis for grouping the changes that we observe in other missile families of interest. Such groupings of changes may aid us in making better inferences about the development status of the missile and its potential role in any conflict situation.

Motivation for this Study

Studies have been carried out to independently assess the capabilities of Chinese and Pakistani missiles.[7,8] These studies have used the dimensions of the missiles[9] to make estimates of the propellant carried by them. From the amount of propellant we can estimate the total weight of the rocket or stage using procedures derived from published data on a number of missiles. We can use these derived values of stage and propellant mass and run typical trajectory profiles to estimate the range of the missile.

An evaluation of the measurement error is important in using such a procedure. Changes in a particular missile especially during its early stages of development may take place because of several reasons. These could come about because different kinds of warheads may be carried by them or because of some development problems. Once a missile is operational other changes may also take place to improve performance. Since the actual length of the missile being flown during its development phase may not be known precisely, we would like to know whether the changes we are seeing in the images of different launchings of the same missile are real or whether they are the result of measurement errors. If we are able to estimate the likely measurement error for the missile then we could confidently group the various launchings into different classes based on measurements made from images of the missile. Measurement error estimates may also help us to assess the likely errors in estimating the range of the missile.

In the studies that we have carried out earlier, the length of the missile is inferred from measurements made on the known diameter of

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[7] n.5

[8] n.6

[9] The diameter and the length are important to determine the cylindrical volume that can be filled by the propellant. The densities of the propellants are known and there is enough information available from public domain data on volumetric efficiencies. This enables the estimation of propellant weights. The weight of the structures and other ancillary components for converting the propellant mass into a stage can also be estimated using engineering data available for a large number of rockets and missiles available in the public domain or through interaction with missile experts.

the missile. Wherever possible, measurements on other components of the missile such as the warhead, the stage lengths, the interface elements and the nozzle were also carried out. These inputs were then used for estimating the propellant and lift off masses of the missile. From these derived values we estimate the performance of a missile by running simple trajectory and range models.[10,11]

The present study estimates the measurement error using images of satellite launch vehicles, rockets, and missiles whose dimensions are known and well documented. Though the number of samples is not very large, placing bounds on the measurement errors are possible. An empirical equation is obtained for estimating the error in terms of the measured length of the missile in meters, the pixel size in centimeters and the reciprocal of the calibration unit in meters. Some specific problems involved in selecting and using images of missiles available in the public domain for assessing the performance are also discussed.

The quality of a digital image depends on the inherent characteristics of the imaging device such as lens quality, the sensitivity of the detectors, data format and the processing method for the conversion of raw data into digital values.

Theoretical Framework

Error in measuring the length (linear dimension) of an object from its digital image depends on the precision of the system forming the digital image.[12] We describe briefly how an image is created and how these errors occur. This also helps us to choose the appropriate images for our study.

The images that we have used as already mentioned are from the public domain. These images are of two kinds.

- The photographs of a missile taken by a conventional camera is scanned, digitized and converted to a digital image.
- The image of a missile is taken by a digital camera.

The instrument errors associated with each of the above cases may be slightly different. The differences arise due to the differences in the scanning or digitizing devices used to convert a photograph to a digital image. This feature is inbuilt in a digital camera. In a digital camera, the sensors read the intensity of light that enters through different colour filters and this is then converted into digital values called grey values. A detailed description of this imaging process is available in Gonzalez and Woods.[13]

A digital image for the purpose of our study could be viewed as a raster[14] image consisting of a matrix of grey values. The quality of a digital image depends on the inherent characteristics

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[10] n.5

[11] n.6

[12] Nikitaev V.G. and Pronichev A.N, “Analysis of measurement errors in metric and orientational object parameters in computerized image processing systems”, *Measurement Techniques* 33, no. 12 (1990): 1177

[13] Gonzalez R.C and R.E. woods, *Digital image processing*, 2nd Edition, India: Pearson Education Asia, 2002.

[14] A raster image is a collection of dots called pixels. Each pixel is a tiny colored square. When an image is scanned, the image is converted to a collection of pixels called a raster image. Scanned graphics and web graphics (JPEG and GIF files) are the most common forms of raster images.

of the imaging device such as lens quality, the sensitivity of the detectors, data format and the processing method for the conversion of raw data into digital values. The spatial resolution and the dynamic range of the camera capture the essence of the above qualities.

Spatial resolution depends on the properties of the system forming the image. It measures how closely spaced lines (objects) can be resolved in an image. This depends on the lens quality of the camera, the sensitivity of the detector and the contrast differences between the various objects in the image. Finer spatial resolutions will result in better quality images.

The conversion of the raw data to an image in a standard format is affected by the dynamic range of the camera which in turn depends on the quality of the detector and the way in which the analog inputs are sampled. The effect of this is seen in the contrast that we see in the image. A large dynamic range for the camera or sensor will result in a better quality image.

For the purposes of measurements we look for an image that has a high spatial resolution and large dynamic range.

In general for the purposes of measurements we look for an image that has a high spatial resolution and large dynamic range. Since the imaging device is not under our control, the selection of the images for our study has to be made by visual inspection only.

Another aspect of image formation is the way the photograph is taken. Pictures taken from different angles give different perspectives. This can happen in two ways. In one way the camera views the object from top or from below. In the other way the camera views the object from the right or the left. In both these cases there will be distortions in the image as far as measurements are concerned.

If the camera is too close to the object and the object is fairly large, there will be further distortions in the image from scale effects also resulting in errors.

Images in the public domain will have different combinations of the above effects. Therefore if we need accurate measurements, the images have to be chosen for our study very carefully. Pictures 1, 2 and 3 are examples of images which are otherwise good but suffer from these geometric distortions.

In Picture 1 the launch vehicle is at an angle that shows the missile as being tilted into the plane of paper and therefore even though the picture appears to be of good quality with respect to the contrast levels, the measurement of lengths may not be accurate. In Picture 2, we observe that in addition to the effect of the angle, the quality of the image is not good. In images of this kind the edges are not clearly defined, so measurements of the diameter could pose a problem.

In Picture 3 the image is tilted into the paper as well as about the X – Y[15] plane.

Since we do not have any knowledge about the camera characteristics, the errors that are

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[15] The X-axis is the horizontal axis from left to right and the Y- axis is from the top to bottom. The Z axis is perpendicular to both of these and is into the paper.

Picture 1

Picture 2

Picture 3

described above could be avoided only by choosing the images carefully. Assuming that the images chosen have minimum inherent errors, we still have to account for the measurement errors.

In general the measurement error depends upon:

- Scale of the image or Pixel size.[16]
- The size of the calibration unit expressed in number of pixels.
- The size of the calibration unit[17] expressed in meters.
- The length of the object being measured expressed in meters.

The pixels size depends on the distance between the camera and the object. This is not the same as the spatial resolution mentioned earlier, though often in remote sensing literature the term is used that way. The farther away an object is from the camera, the coarser will be the resolution resulting in a larger pixel size. For the same diameter, a picture with a larger number of pixels covering the diameter would be considered to be a better picture for measurement purposes.

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[16] Pixel size refers to the size of pixel in units of length such as meters or centimeters. The pixel is a square. Thus if the pixel size is 2cm, then it means that an area of 2cm X 2cm is represented by one pixel.

[17] This is the diameter expressed in meters in our study

The measurement error E could be considered as a function of the above elements. Thus,

$E = f\ (p, d_p, d_m, L)$

where L is the measured length

p is the pixel size in meters

d_m is the diameter in meters and

d_p is the diameter in pixels.

The error is bound to be very sensitive to changes in the pixel size. It is also well known that the error increases exponentially with increase in the pixel size. Crapper[18] has shown that the accuracy of the area estimates from remote sensing images is a function of the logarithm of the pixel size. In the case of satellite images where pixel sizes from the sensor range from 1m to as much as 100m, it has been possible to assess the error in estimating the areas of well known features such as agricultural fields under different crops.

For a missile with a given diameter, the image having a larger number of pixels for this diameter will provide more accurate measurements than an image having a smaller number of pixels for the same diameter.

The second important factor is the size of the calibration unit. In our study the calibration unit is the diameter of the missile or rocket. The measurement error obviously decreases with an increase in the actual diameter say from one metre to two metres.

The measurement error is also dependent on the diameter of the missile measured in pixels. For a missile with a given diameter, the image having a larger number of pixels for this diameter will provide more accurate measurements than an image having a smaller number of pixels for the same diameter.

In addition to this, one would expect that the error will also increase as the lengths we measure become larger. Given that the diameter also influences the measurement error, we should expect that the Length to Diameter (L/D) ratio would play an important role in determining the measurement error.

Methodology

The following approach is used in this study.

- Images for assessing the measurement error are first selected carefully so that geometric and scale effects are minimal.
- Since characteristics such as diameter (D) and length are known for a missile, we use the known value of the diameter to determine the pixel size.
- We use ENVI 3.5 image processing software to measure the diameter. Enhancement of the image is some times necessary to make sure that the edges are clearly demarcated.
- With the value of the diameter in pixels and the known value of the diameter in units of actual length such as meters we determine the pixel size in units of actual length.
- Once the pixel size is known, the length of the missile or its relevant part in pixels is measured.
- Using the pixel size derived from the diameter measurement we convert this pixel length into a measured length L.

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[18] n.1

This measured length is compared with the actual length and the error is estimated.

Table 1 provides the diameter of the various Launch Vehicles/missiles selected for this exercise.

Annexure 1 and 2 provide the sources as well as other details about the images used in our study

A few sample images used in this study are shown in Annexure 3

The error is therefore largely contributed by three factors namely, quality of the image, angle at which the picture is taken and the bias of the person making the measurement.

The diameters for these vehicles range from 1m to 2.8m and their lengths vary from 10 m to 20 m.

Problems Associated with Measurements

Images used in the study have all been obtained from public domain. It must be emphasized here that all analysis of the capability of Pakistani and Chinese missiles are also based on the images available in public domain. So, basically the problems faced in this study are common to such studies. We list below the problems:

- quality of the image
- Angle at which the picture is taken
- Pixel aspect ratio[19] of the camera used
- Aspect ratio of the display monitor
- Bias / parallax error of the person making the measurement

Of these, the aspect ratio is not a serious problem and does not contribute to the error. The angle at which the picture is taken is a significant contributor, and this is overcome by being selective about the images used. As a result of this many of the images available in public domain could not be used. This is one of the reasons for the small sample size. Similarly the quality of the image as measured by the pixel size and image contrast contributes significantly to the error. If the diameter is not correctly measured because of lack of contrast at the edges, the errors can be significant. The choice of the images thus becomes very important. The error is therefore largely contributed by three factors namely, quality of the image, angle at which the picture is taken and the bias of the person making the measurement.

Table 1: Relevant characteristics of the vehicles used in the study

Launch vehicle / Missile	**Images**	**Diameter used for calibration (m)**
Black Arrow	Img1	2.00 m
Athena 1	Img2 & Img3	2.36 m
Minuteman III	Img4	1.67 m
PSLV 3	Img 5,6,7,8,9,10,11,12,13	1.00 m / 2.80 m
Agni II	Img14	1.00 m
Agni I	Img15 & Img16	1.00 m

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[19] Pixel aspect ratio describes a mathematical ratio between width and height of a pixel. The term may be applied to an imaging device such as a camera as well as a display monitor. Most modern digital imaging systems and display devices use square pixels.

Results

Detailed configurations of the selected missiles and launch vehicles are available in public domain. In all these images the diameter of the first stage is used as the calibration unit. These diameters vary from 1 m to 2.8 m. The lengths measured in the images vary depending on the vehicle configurations. In the case of PSLV, the length of the strap-on and the length of the first stage are both measured wherever possible. In these images the diameter of the first stage (2.8 m) and the diameter of the strap-on (1 m) were both used as the calibration unit where ever possible. This enabled us to compare the errors for the same length when different diameters (calibration units) were employed. In the case of the Minuteman, the first stage length was measured without the bottom shroud, using the first stage diameter as the calibration unit. In the case of Athena, Agni II and Black Arrow, the total length of the launch vehicle or missile was measured.

The measurements made on the sample data and the errors calculated are shown in Tables 2 and 3.

Table 2: Measurements made on the sample images

Image	Actual		Measured		
	Diameter (m)	Length (m)	Diameter (Pixels)	Length (Pixels)	Length (m)
Img1	2.00	12.98	30	194.00	12.94
Img2	2.36	18.86	35	277.00	18.67
Img3	2.36	18.86	19	146.00	18.13
Img4	1.67	7.95	37	176.00	7.94
Img5	1.00	10.22	14	142.50	9.83
Img5	2.80	10.22	38	142.50	10.55
Img5	2.80	20.21	38	273.00	20.24
Img6	1.00	10.22	7	68.00	9.72
Img6	2.80	10.22	19	68.00	10.00
Img7	1.00	10.22	7	75.50	10.80
Img8	2.80	10.22	22	85.00	10.54
Img9	2.80	10.22	24	94.00	10.77
Img9	2.80	20.21	25	273.00	20.58
Img10	2.80	10.22	27	98.00	10.14
Img11	2.80	10.22	23	85.00	10.43
Img11	2.80	20.21	23	171.00	20.86
Img12	2.80	20.21	35	255.00	20.40
Img13	1.00	12.40	7	92.00	12.88
Img13	2.80	12.40	21	92.00	12.24
Img14	1.00	20.00	21	402.00	19.14
Img15	1.00	15.00	83	1253.00	15.10
Img16	1.00	15.00	15	217.00	14.46

Table 3: Estimated Measurement Errors

Image	Pixel Size (cm)	Actual Length (m)	Measured Length (m)	Error (m)	Error (%)
Img1	6.70	12.98	12.94	-0.04	-0.31
Img2	6.70	18.86	18.67	-0.19	-1.00
Img3	12.40	18.86	18.13	-0.73	-3.90
Img4	4.50	7.95	7.94	-0.01	-0.15
Img5	6.90	10.22	9.83	-0.39	-3.90
Img5	7.40	10.22	10.55	0.32	3.10
Img5	7.40	20.21	20.24	0.03	0.15
Img6	14.30	10.22	9.72	-0.50	-4.90
Img6	14.70	10.22	10.00	-0.22	-2.20
Img7	14.30	10.22	10.80	0.58	5.70
Img8	12.40	10.22	10.54	0.32	3.20
Img9	11.40	10.22	10.77	0.55	5.40
Img9	11.40	20.21	20.58	0.37	1.83
Img10	10.40	10.22	10.14	-0.08	-0.80
Img11	12.20	10.22	10.43	0.21	2.10
Img11	12.20	20.21	20.86	0.65	3.24
Img12	8.00	20.21	20.40	0.19	0.95
Img13	13.30	12.40	12.88	-0.16	1.30
Img13	14.20	12.40	12.24	0.48	3.90
Img14	4.80	20.00	19.14	-0.86	-4.30
Img15	1.20	15.00	15.10	0.10	0.64
Img16	6.70	15.00	14.46	-0.54	-3.60

Figure 1 shows the actual and measured values for the sample images.

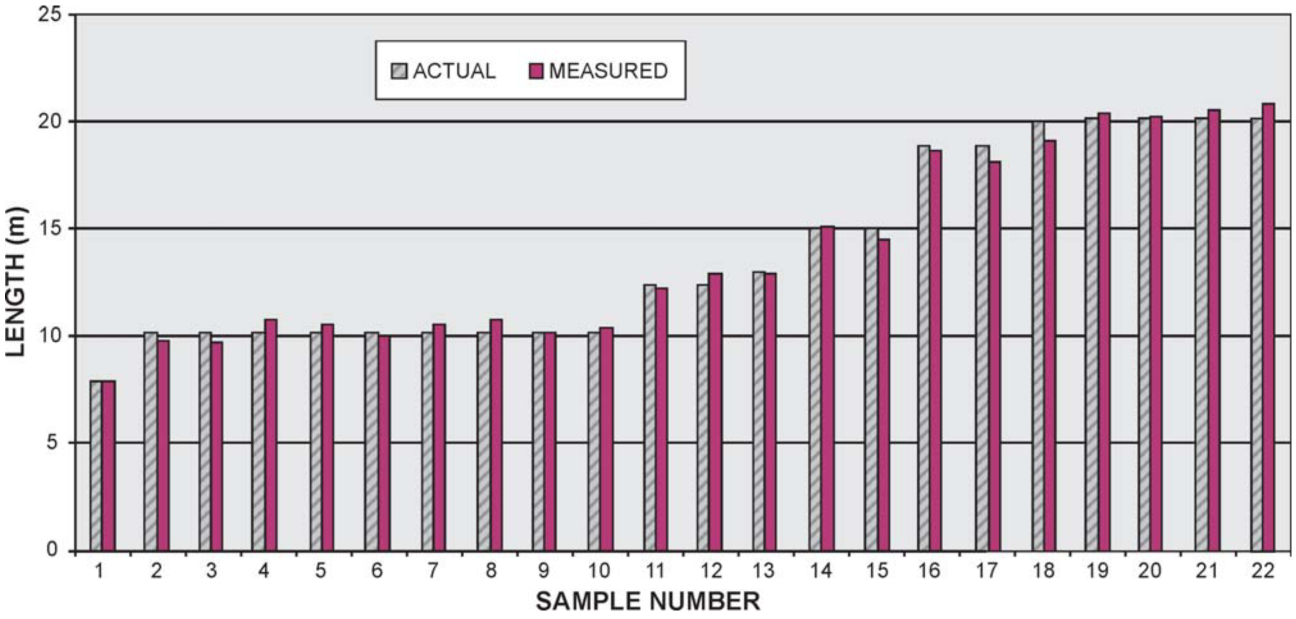

Figure 1: Comparison of actual and measured lengths

The absolute error ranges from 12 cm. to 73 cm. The maximum error is observed to be close to 6%. In measuring lengths up to 13 m, the average error is found to be 0.081 m with a standard deviation of 0.36 m. This is based on 13 samples. A 95% Confidence interval for the average error was found to be between 2.1 cm and 14.1 cm. This works out to be on the average an error of 1% with a standard deviation of 3.4%. This is particularly relevant to the analysis of Pakistani missiles[20] whose lengths are generally less than 13 m.

The absolute error ranges from 12 cm. to 73 cm. The maximum error is observed to be close to 6%.

Error and Pixel Size

Figure 2 shows the distribution of error (%) for different pixel sizes. In the figure the absolute error (%) is plotted against the pixel size. No special significance can be attributed to the underestimates and overestimates of the lengths. Hence in all our analysis we only use the absolute value of the error.

Pixel sizes of the images examined varied from 1.2 cm to 14.7 cm. One would expect that the

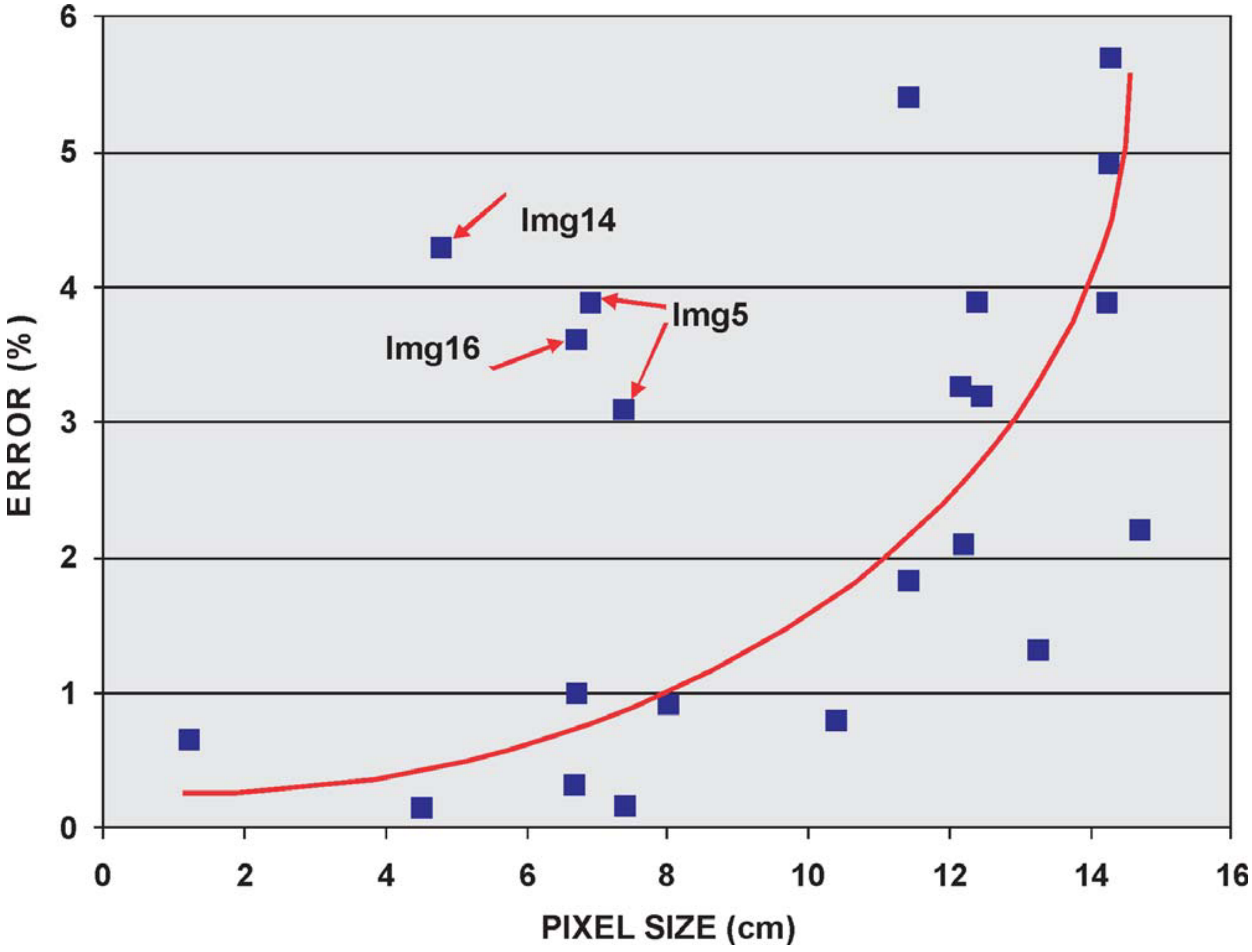

Figure 2: Error variation with pixel sizes

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[20] The Ghauri missiles are longer than 13 m.

error will increase with increasing pixel sizes. We can see that the error increases exponentially with increasing pixel size. The curve drawn on the figure is only a guide to the eye to show this trend and has no mathematical or statistical basis.

We can also see from Figure 2 that there are four points that are far away from the guided curve. These correspond to Strap-on lengths in Img5 and total lengths in Img14 and Img16.

In the case of Img5 the top end of the strap-on was not very clear. Hence irrespective of whether 1 m or 2.8 m was used as the calibration diameter, the length of the strap-on was measured with large error. In the same image the first stage length was measured with less than 1% error.

In the case of Img14 even though the pixel size was small (less than 5 cm) the error was more than 4%. The picture itself was quite good; and the large error was probably due to the large length to diameter (L/D) ratio which was 20 in this case.

In the case of Agni1, two images were available for measurements - Img15 and Img16. In the case where the pixel size was 1.2 cm the error was less than 1% and when the pixel size was 6.7 cm, the error was 3.6%.

Since several sample images had diameters 1 m and 2.8 m, the errors were analyzed separately for these cases. Figure 3 plots the percentage error against pixel size for these diameters.

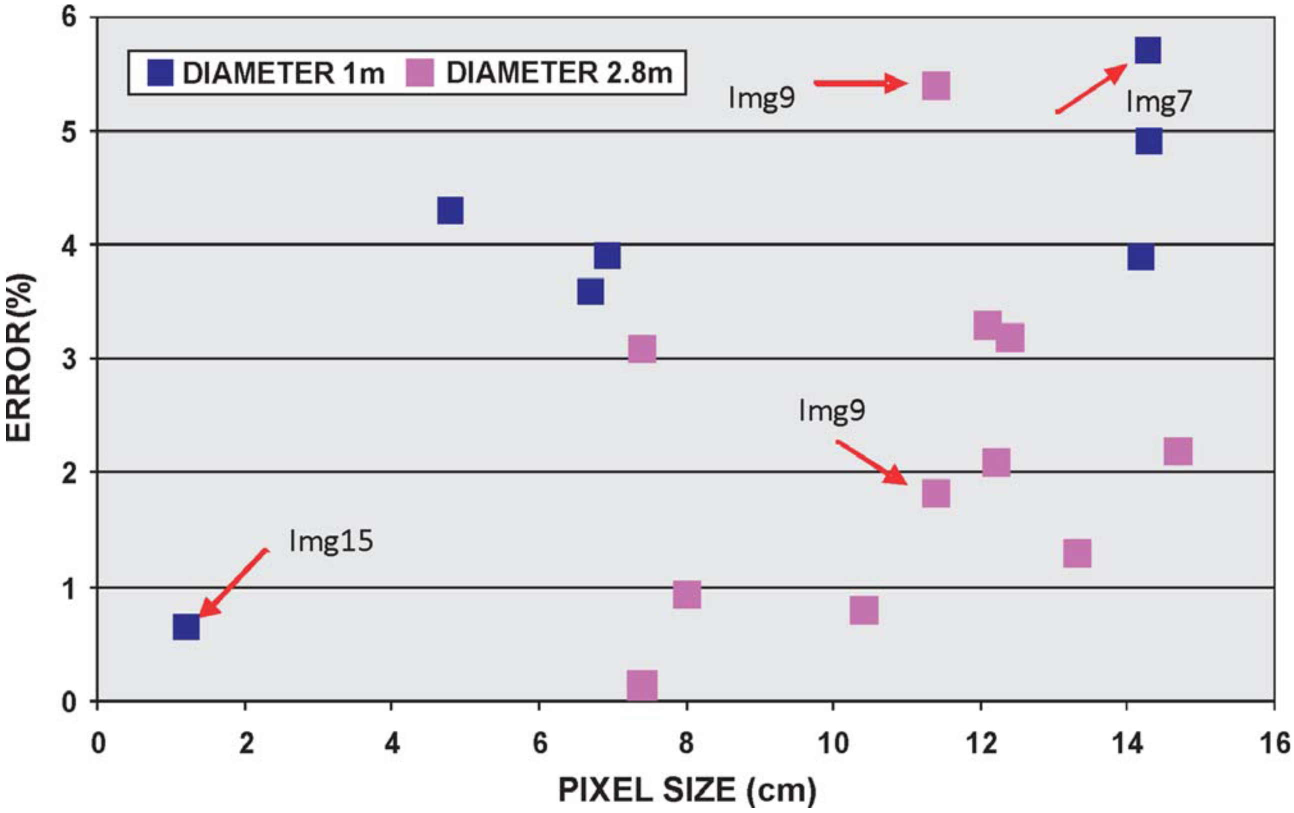

Figure 3: Error variation with pixel size for calibration diameters 1m and 2.8m

Except for one point (Img15), where the error was less than 1%, the error was between 3% and 6% when the calibration diameter was 1m. When the calibration diameter was 2.8 m, the error was between 0.2% and 3.2% except for one point (Img9) where the error was 5.6%. Visually Img9 appears to be very good and probably the slight haziness at the top end of the strap-on could have caused the large error. For the same image the length of the first stage was measured with an error of 1.8%.

In the case of Img6 the errors were 4.9% and 2.2% respectively for calibration diameters of 1 m and 2.8 m. The large error (4.9%) was because of the undefined edges of the strap-on diameter which could not be improved with enhancement.

The error was between 3% and 6% when the calibration diameter was 1m. When the calibration diameter was 2.8 m, the error was between 0.2% and 3.2%.

Img7 has the highest error (close to 6%). Close visual examination of the image suggests that the edges defining the diameter are not very clear.

Error and Calibration Unit

The effect of the calibration unit (expressed as number of pixels) on the error is shown in Figure 4. The number of pixels describing the calibration unit is a function of the actual diameter and the distance from which the image was taken. Thus for example, we note

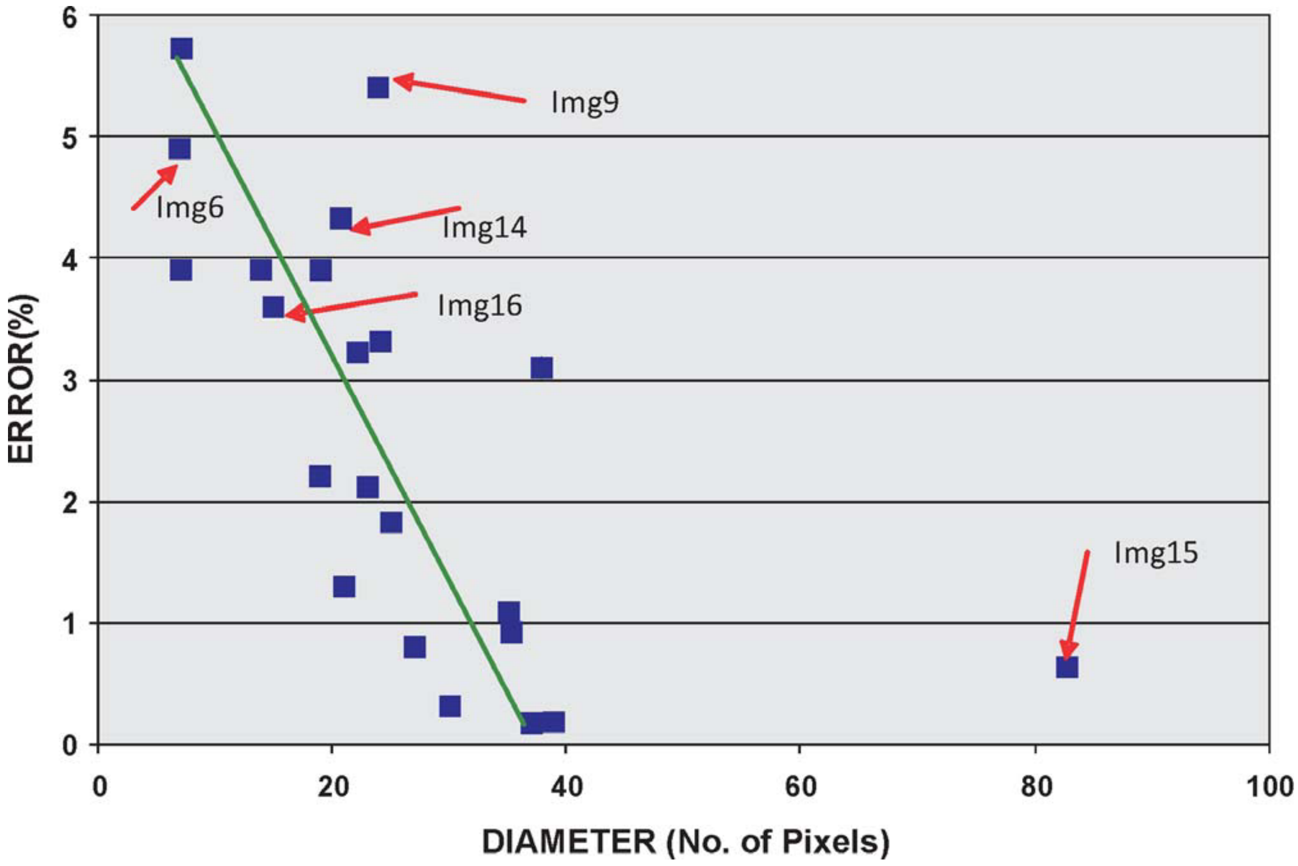

Figure 4: Distribution of error for varying calibration units

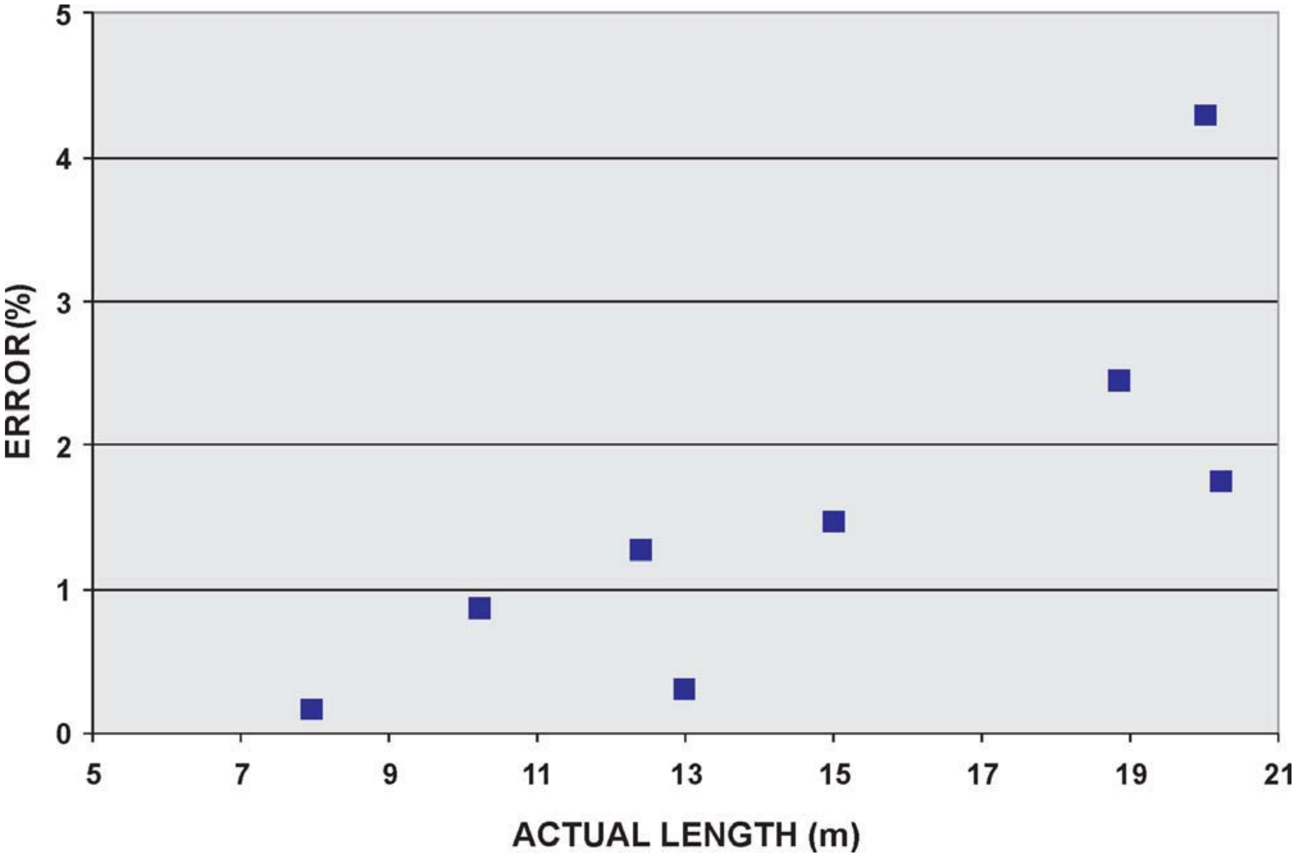

Figure 5: Error variation with length measured

that images Img15 and Img16 have the same diameter (1m) but the diameter in pixels for these images are 83 and 15 respectively. The effect of this on the measurement error is quite obvious in the figure.

In general, the error gets smaller if the number of pixels for the same diameter size increases. The line drawn is a guide to the eye. Img9 and Img14 shows large error compared to other data points having the same number of pixels describing the diameter.

Error and Length of the Object

The actual length is plotted against the average error. The average error was calculated as follows. Wherever more than one sample data was available for the same actual length, an average of the measurement error was computed. The percentage error was then calculated with this average error.

The absolute value of this error is plotted in the graph. Clearly the error increases as the length being measured gets larger. The maximum error was 4.5% when the length being measured was 20 m.

Regression Equation for Estimating the Measurement Error

In the previous section we had looked at the effect of pixel size, calibration unit, and length of the object being measured on the measurement error. For small pixel sizes, the measurement error is small, although we do observe a few outliers.

We also noted that a calibration diameter of 2.8 m gave smaller measurement error compared to a calibration diameter of 1 m. Whenever the image had a larger number of diameter pixels, the measurement errors were not very large.

In order to estimate the error involved in measurements made on images of missiles of unknown dimensions, we tried to establish an empirical relationship between the absolute error and the combined effect of these parameters. The results of the previous section indicate that some of the data points may be outliers and need to be eliminated while obtaining the regression equation. We observed that strap-on length measurements made on Img5 and Img6, as well as total length measured in Img14 and Img16 show errors incompatible with images having similar pixel size, number of diameter pixels, and measured length. These data points were therefore not included for obtaining the regression equation. Although Img7 and Img9 show large errors, the images were of good quality and the large error could not be attributed to any specific image related problems and were therefore included for the regression analysis.

Stepwise regression[21] of the absolute error (%) with the above parameters was performed using MYSTAT12. The method involved a backward selection process that included the variables - pixel size in cm, measured length in m, calibration diameter in m, calibration diameter expressed as number of pixels, reciprocal of the calibration diameter in meters and Length to Diameter ratio. The best fit regression equation with a p value[22] of 0.0004 and a coefficient of determination value of 0.80 was found to be,

$$Y = -4.346 + 0.068\, X_1 + 0.350\, X_2 + 3.475 X_3$$

Where,

Y = Absolute Error in (%)
X_1 = Measured length in m
X_2 = Pixel Size in cm
X_3 = Reciprocal of calibration size in m.

The pixel size in cm, reciprocal of the actual diameter and the measured length had significant influence on the measurement error. In the process of obtaining the regression equation, strap-on length measurement made on Img9 was found to be an outlier and hence was eliminated. The regression equation was thus based on 16 data points.

If the image of a missile is available, then the methodology described here can be used to determine the pixel size (X_2), measured length (X_1) and the reciprocal of the calibration diameter in meters (X_3). We can then use these values in the equation above to get an estimate of the % error. The length of the missile could then be expressed as (L-e, L+e), where e is the estimated error in meters and L is the measured length.

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[21] Stepwise regression removes and adds variables to the regression model for the purpose of identifying a useful subset of the influencing parameters also called predictors. This can be done by either forward selection or backward selection. In the forward selection, the predictor having the highest R squared value is introduced first and the predictor that increases the R squared value the most is introduced next. At each step a predictor is introduced and the process of introduction is stopped when no significant increase in the R Squared value is noted. In the backward selection, all the predictors are included in the equation and the predictor contributing the least to the R squared value is removed first. The predictors are removed one by one till the removal of a predictor significantly reduces the R squared value.

[22] A p-value is a measure of how much evidence we have against the null hypothesis. The smaller the p-value, the greater the evidence we have against the null hypothesis. In this case, our null hypothesis states that the variables included in our regression equation have zero co-efficients or in other words have no influence on the dependent variable Y. p=0.004 suggests that the three parameters X1, X2, X3 affect the measurement error Y significantly.

Application of the Methodology to Shaheen 2 Images

A series of Shaheen 2 missiles were tested by Pakistan during 2004-2008 and several images were available for making measurements. On visual examination some of the images were found to have poor contrast, low resolution or were tilted and no measurements could be made on these. The best quality images of the different launches of the Shaheen 2 missile were selected and measurements made on them. The first stage diameter of 1 m was used for calibration. The total length of the missile was then measured. These are shown in Table 4 for the best images. Note that the March 2004 and March 2005 lengths are very similar. The April 19, 2008 and April 21, 2008 lengths are also very similar. The April 2006 launch appears to have longer length compared to 2005 and 2008 launch. Although, there was a launch in 2007, we could not clearly and unambiguously locate an image corresponding to that date to make measurements.

In order to ascertain whether the differences we see in the lengths of these Shaheen2 launches was due to only measurement error, we applied the regression equation to the data and arrived at an interval estimate for the measured lengths.

The estimated errors are significantly smaller in the case of Image 18, Image 20 and Image 19B. These images were of launches in 2005, 2006 and 2008 respectively. We can clearly see that Shaheen 2 launched in 2006 is significantly longer than those launched in 2005 and 2008. The missiles tested in 2005 and 2008 also do not have the same length, the latter being 30cm shorter than the former.

We can clearly see that there are three different variants of the Shaheen2. The length of the missile changed from 11.89m in 2004 to 12.68m in 2006. This difference (0.791m) is much more than the estimated measurement error (0.24 m). Hence the missile launched in 2006 could be considered to be different from missile launched in 2004.

Similarly the differences in the lengths of the missiles launched in 2006 and that launched in 2008 is 1.03 m which is much more than the estimated measurement error for these images.

Table 4: Estimated Error in selected Shaheen 2 images

Image Name	Launch Date	Diameter* (m)	Diameter (Pixels)	Measured Length (m)	Estimated absolute error (m)	Interval estimate of length
Image 15	10 March 2004	1.0	17	11.89	0.24	11.65, 12.12
Image 18	19 March 2005	1.0	24	12.00	0.17	11.83, 12.17
Image 20	April 2006	1.0	34	12.68	0.13	12.55, 12.80
Image 19B	19 April 2008	1.0	38	11.65	0.13	11.52, 11.78
Image 21B	21 April 2008	1.0	18	11.45	0.21	11.23, 11.66

(*) The first stage diameter is known to be 1 m and is used as the calibration size for these measurements

The difference between the lengths of the missiles launched in 2004 and 2008 is 0.44m which is also larger than the estimated measurement error (0.24 m).

Thus we clearly see that there are three groups of missiles. Although no specific reason can be given for such variations in lengths of missiles of the same class, one can infer that some experimental development process is going on in Pakistan.

We clearly see that there are three groups of missiles. Although no specific reason can be given for such variations in lengths of missiles of the same class, one can infer that some experimental development process is going on in Pakistan.

Conclusion

It is possible to estimate with reasonable statistical confidence the dimensions of a missile system from its digital image using simple image processing tools. Such a conclusion is supported by the quantitative estimates given below.

The average error and the standard deviation of the error in measuring lengths up to 13 m from digital images is estimated to be 0.08 m and 0.36 m respectively. This leads to a 95% confidence interval of 2 cm and 14 cm for the estimated error.

When the calibration diameter is 1m, the measurement error is between 3% and 6% and when the calibration diameter is 2.8m, it is between 0.2% and 3.2%.

The pixel size in centimeters, reciprocal of the calibration diameter in meters, and the measured length in meters had significant influence on the measurement error. A regression equation to estimate this measurement error has been derived from a sample of 16 data points.

Using this equation on the measurements made on images of Shaheen 2 missiles launched between 2004 and 2008, indicate that there have been three versions of Shaheen 2 missiles during this period.

Annexure 1

Characteristics of the vehicles used in the study[23,24,25,26]

Launch vehicle/ MissileFile Name	Name	Diameter (m)
Black Arrow[24]	Img1	2.00
Athena 1[24]	Img2	2.36
Athena1_on_launch_pad[24]	Img3	2.36
Minuteman III[24]	Img4	1.67
PSLV 3[23]	Img5	1.00/2.80
PSLV Taking off[25]	Img6	1.00/2.80
PSLV_C7_on_pad[23]	Img7	1.00
PSLV 15[25]	Img8	2.80
PSLV_C2_1[23]	Img9	2.80
PSLV_C2_on_pad[23]	Img10	2.80
PSLV_C5_on_pad[25]	Img11	2.80
DSC_1082_small[23]	Img12	2.80
PSLV_C11_on_its_way_to_launch_pad[23]	Img13	1.00/2.80
Agni II[25]	Img14	1.00
Agni I_just_launched[26]	Img15	1.00
Agni I little after launch[26]	Img16	1.00

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[23] http://www.isro.gov.in/

[24] http://hometown.aol.de/b14643/space-rockets/Rest_World/India/Description/Frame.htm?f=fs

[25] http://www.bharat-rakshak.com/SPACE/

[26] DRDO Head Quarters, New Delhi, India.

Annexure 2

Brief Description of the Missiles / Launch Vehicles Used in the Study

Black Arrow:

A British Carrier rocket developed by Royal Aircraft Establishment, the Black Arrow was first launched on 28 June 1969. The last Black Arrow was launched on 28 October 1971. It was a three stage rocket weighing 18 tonnes with a LEO payload capacity of nearly 100Kg and placed into orbit the Prospero X-3 satellite. It was abandoned in 1971. It had a total length of 12.98 m and a diameter of 2 m.

Athena:

A three stage launch vehicle developed commercially by Lockheed Martin. Two images were available for the analysis. The diameter of Athena was 2.36 m and the length was 18.86 m.

LGM-30F Minuteman:

The Minuteman series of ICBMs have been operated by US Air Force from 1960. Minuteman-I and II operated till 1997. The LGM-30F Minuteman-III was first deployed in 1970. The total length is about 18m and the first stage diameter is 1.67m. For our study we have made measurements of only the length of the first stage without the nozzle.

AGNI II:

It is an Intermediate Range Ballistic Missile of India having a length of 20m and a first stage diameter of 1m.

PSLV:

The Polar Satellite Launch Vehicle (PSLV) is a heavy launcher designed and developed by the Indian Space Research Organisation (ISRO) to launch its satellites into sun synchronous orbits. The basic PSLV configuration is 44.4 m tall having four stages with a first stage diameter of 2.8 m and length of 20.203 m. There are six strap-on motors (PSOM) carrying solid propellants of which two or four are ignited on the ground depending on mission requirement. The length of these PSOMs from the nose cone tip to nozzle end base ring outer face is 10.223 m. Some variants of PSLV without the strap on motors have also been tested.

PSLV C11

PSLV C11 that recently launched Chandrayaan - 1 spacecraft is an updated version of ISRO's PSLV. In this case the first stage dimensions are the same as the other PSLVs but the six strap - on motors have an increased length of 12.4m.

Agni I

Agni I is a single stage Intermediate Range Ballistic Missile of India having a length of 15m and diameter of 1m.

Annexure 3

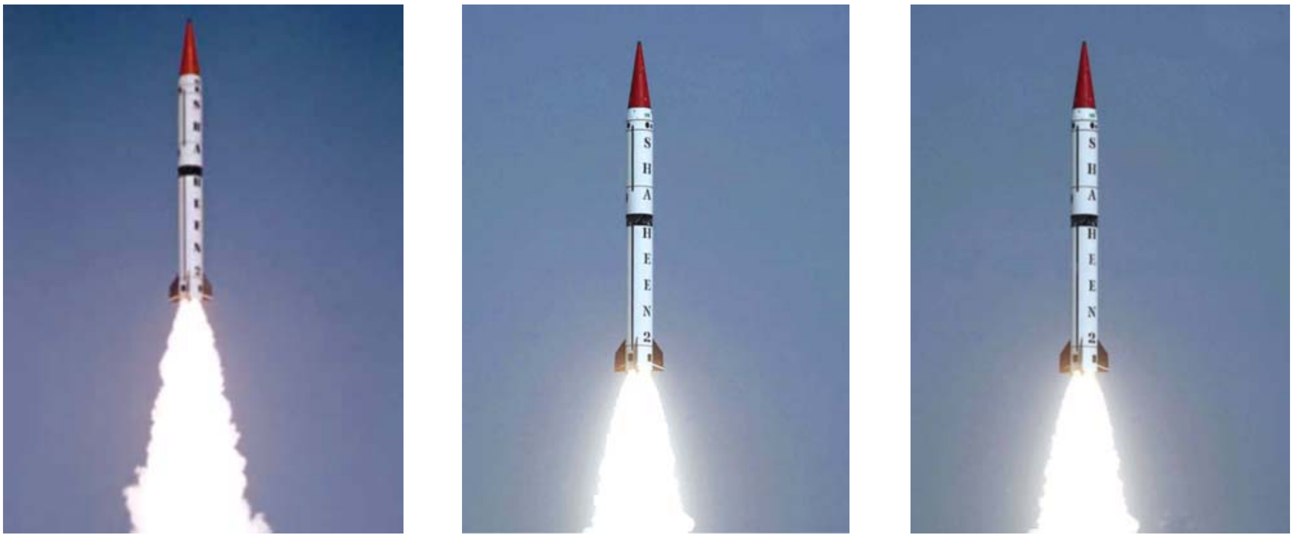
Typical images used in the study

Agni I

PSLV

Athena

AGNI II



Shaheen 2 Images

International Strategic & Security Studies Programme

The International Strategic & Security Studies Programme at NIAS promotes and conducts research that addresses the strategic and security concerns of India. It has over the years carried out a number of studies in technology dominated areas of international security - nuclear weapons and missiles. NIAS has also facilitated exchange of knowledge and views between interested groups working around the globe on issues related to international security. The programme focuses on the region with a special emphasis on China and Pakistan.

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