# Revolution or Gradualism: Optimal Strategy of a Leader

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**Abstract**

Leader wishes to confront/overthrow the present regime and every period chooses the nature of its opposition. Opposition can either be a non-political protest or a political protest. The non-political protest does not threaten the existence of the present regime. The sucess or failure of both the types of protest depends upon the unknown ability of the leader and mass participation. We find that for intermediate ranges of the ability of the leader, it is optimal for the leader to follow a strategy of gradualism in which it undertakes non political protest initially to favorably update the belief about his ability and mobilize a higher participation for the political protest. For very low and high values of the ability of the leader, it is optimal to do the political protest in the first period.

# 1 Introduction

*"Effective leadership is putting first things first. Effective management is discipline, carrying it out."*- Stephen Covey

A leader is an architect of change. Leaders or heads of organizations, be it political parties, corporates or any institution play an important role in choosing optimal actions and coordinate with the followers to bring about the desired change. We observe substantial variation in the outcomes of organizations depending upon the ability of the leader. Some leaders be it in business or politics are better able to manage resources and direct the followers effectively and hence achieve the desired change while others fail. Apart from an individual's leadership ability, one cannot be a leader without followers. The most important aspect of successful leadership in any organization or setting is to have a sufficient pool of dedicated followers. However the question is then how does a leader able to draw a set of dedicated followers to bring about a successful change.

It is widely agreed that the "Salt March" by Mahatma Gandhi in 1930 was the first shot that eventually brought down the British Empire in India. However, Gandhi's effectiveness in transforming a novel protest into a broad movement for change was also driven his ability to draw on a cadre of followers that he had attracted by this time (Dalton, 1993). The question is how was he able to draw this pool of followers. Looking back at history, Gandhi's first great experiment in *Satyagraha* came in 1917, in Champaharan in Bihar, followed by Kheda satyagraha (1918) and then the Ahmedabad Mill workers strike (1918) and none of these events were a direct revolt against the British regime and hence a threat to their existence. However Gandhi emerged as one of the most popular and acceptable figure in Indian politics by his technique of mass mobilization through smaller protests that he initially undertook after coming back to India in 1915. Turning to modern India, Arvind Kejriwal formed a new political party named the Aam Admi Party (AAP) and is now the chief minister of Delhi where his party swiped the assembly elections winning 67 seats out of 70 in 2015. However Kejriwal started his career as a leader with formation of a movement named "Parivartan" in December 1999 which addressed citizens' grievances related to Public Distribution System (PDS), led many other smaller protests by filing public interest litigation (PIL) demanding transparency in public dealings of the Income Tax department and then in 2011 joined several other activists to form the India Against Corruption (IAC) group. By this time he was successful in gathering enough momentum to have a dedicated pool of followers which he leveraged to contest the assembly elections in which his party won with a massive mandate. On the other hand, the Lok Satta party started by Jayaprakash Narayan in 2006 which wanted to project itself as an alternative in Indian politics has hardly been successful.

In the examples above on Gandhi and Arvind Kejriwal, the leaders took a strategy of gradualism through which they were successful in mobilizing the mass before attacking the regime directly. On the other hand leaders can also choose to attack the regime directly rather than following a process of gradualism. In this paper, we show that under what conditions it might be optimal to take a gradual path and then announce a revolution against the regime versus announcing a revolution against the regime immediately.

In this paper there are two types of leaders - a "political" leader and a "non-political" leader. The political leader, $P$, aims at overthrowing the present regime while the non-political leader, $NP$, is one who aims at protesting against social injustice and tries to bring about social reforms. We call the protest to overthrow the regime as a "revolution" and a protest against social injustice and reforms as "social protest". A leader with a political objective, despite her aim being to overthrow the present regime might still undertake social protest initially to favorably update the belief about his ability and mobilize a higher participation for the revolution. The underlying assumption is that revolution directly threatens the existence of the regime while any social protests do not directly threaten the existence of the regime.

In this model, there are three types of agents - the present regime or the Government, a Leader and a unit mass of citizens. We assume that there are two types of leaders who have different objectives or motives - a leader with a social objective ($NP$) who never intends to over throw the regime. However, a leader with a political motive ($P$) can choose to do so. A leader can also be of two different abilities, high and low. Given the same resources a high ability leader is able to manage more efficiently and hence has a higher probability of success in a small protest or revolution as compared to a low ability leader. The probability of success in a small protest or revolution depends upon the unknown ability of the political leader and mass participation. In this paper the leader is assumed to be inexperienced and does not know his own ability. However the objective is known to the leader privately. All players in the society have initial priors about the objective as well as about the ability of the leader. The political leader might still do a social protest because upon success in the social protest, the beliefs about his ability is revised upwards and hence helps her to mobilize more masses in future which ultimately helps in overthrowing the present regime by announcing a revolution. The mass is assumed to be myopic and enjoys some benefit from a successful small protest and revolution but also bears a cost of participation in either of the movements. We assume that the objective of the leader $P$ is aligned with the broader populace and wants to overthrow the present regime. Hence the mass enjoys a higher payoff from a successful revolution as compared to a successful social protest.[1]

The Government can exert force to suppress a revolution and also a social protest but

[1] In the background it is assumed that the leader has enforcement as well as persuasive powers.

is costly to do so. The problem that the present regime faces is that if there is a social protest, then it does not know with certainty whether it is by a leader, $NP$ or it is by a leader with a political objective, $P$ and is being used as a device to mobilize mass. If the case is the former and the government knows with certainty, then it does not need to exert any force while it would probably like to suppress the movement if it is by a leader with a political objective. In this paper we solve for a two period model and we characterize the equilibrium.

# 2 Related Literature:

# 3 Model

In the society there are three types of agents - Government ($G$), Leader of political movement ($L$) and a mass of citizens, ($m$). The leader does not belong to the government, and wants to bring about a change in the society by garnering sufficient support from the masses. The leader is endowed with a privately known efficiency or quality parameter, $\theta$ which can either be high, $\theta_H$ or low, $\theta_L$. At the beginning of the game, the common prior that the leader is of high type is $\alpha_1$ i.e., $\theta = \theta_H$. The leader is inexperienced, i.e. does not know his own quality. The actual quality parameter of the leader is not known to any of the agents in the society. A leader has private information about his motives or objectives, $\zeta$. A leader with a non-political objective, $\zeta = NP$, never intends to over throw the government. However, a leader with a political motive, $\zeta = P$ can choose to do so. $\beta_1$ is the prior probability that the leader has a non-political objective, $\zeta = NP$. We denote the type of the leader by $\tau = \theta \times \zeta \in \mathbb{T}$, where $\mathbb{T} = \{\theta_H, \theta_L\} \times \{NP, P\}$.

In this paper, we consider a two period model. The sequence of events in each period $t$ is as follows. At the beginning of each period, $t$, the leader makes an announcement about the nature of the movement it would lead/conduct. The movement can either be a revolution, $R$ or a non-political protest, $s$. Action of a leader in time period $t$ is $a_t \in \{R, s\}$. Upon hearing leader's announcement, the government and mass update their belief about the leader's quality, $\hat{\alpha_1}$ and about the objective of the leader, $\hat{\beta_1}$. The government announces the extent/level of force with which it wishes to combat the leader's movement, $g_t \in \mathbb{R}_+$. After hearing leader's nature of movement and government's force, each participant decides to partcipate ($f$) or not participate ($nf$) in the movement. Each participant bears a private cost of participating in the movement, $e_i \sim U[-e_L, e_H]$ and a cost equal to the force implemented by the government, $g_t$. There can be some individuals who draw satisfaction by being a part of the movement and hence might have negative private costs of participation.The total cost of participating in a movement for a particpant is

$c_i = e_i + g_t$. Let the number of particpants who choose to participate in the movement at anytime period be $m_t$. After the participants take a decision, nature determines the success or failure of the movement announced at time period $t$, i.e. $\gamma_t \in \{S, F\}$. The probability of a movement announced at $t$ being a success, $Pr(\gamma_t = S)$ depends upon the quality of the leader, $\theta$ and the mass of people that participate in the movement, $m_t$, i.e. $Pr(\gamma_t = S) = \theta m_t$

Let $h_t = (a_t, g_t, m_t, \gamma_t)$ be the public history at the beginning of time period $t$, with $h_0 = \phi$ and $\mathbb{H}_t$ as the set of all possible histories at time period $t$. At the end of each period, government, leader and masses observe the nature of the movement chosen by the leader, government's force, fraction of mass that participated and the success or failure of the movement in the period. All agents in the society use this information to update their beleif about the uncertain type of the leader, i.e. $\alpha_{t+1} = Pr(\theta = \theta_H \mid \gamma_t)$, $\beta_{t+1} = Pr(\zeta = NP \mid \gamma_t)$.

Upon the the success of the movement, each participant receives a benefit dependent on the nature of the movement, $V_{at} \in \{W, V_s\}$ which is common to all participants. We assume that $W \geq V_s$ because there is a greater benefit to a regime change. The mass dislike the present government and dismantling the present government gives them a huge benefit than any other social movement. The failure of the movement gives the participants a benefit of 0. We assume that participants are myopic and decide to participate in a movement at time period $t$ if their current period payoff is greater than the cost of doing so in the period. We assume $e_L > W$ and $e_H > \theta_H W$ where $W$ is the benefit or the rents enjoyed by the government in every period $t$ for being in power. Thus, at any time period there is always a mass of participants with $e_i \leq -W$ who would always participate in a movement and a mass of participants with $e_i > \theta_H W$ who would never participate in any movement.

Leader of type $\tau$ derives per period utility, $U^L(a_t, \tau, \gamma_t)$ from a movement of type $a_t$ in state $\gamma_t$.

$$
\begin{aligned}
U^L(a_t, \tau = (\theta, \zeta), \gamma_t) &= V_s \quad if\; a_t = s \quad \& \quad \zeta = NP, \gamma_t = S, \forall \theta \in \{\theta_H, \theta_L\} \\
&= W \quad if\; a_t = R \quad \& \quad \zeta = P \gamma_t = S, \forall \theta \in \{\theta_H, \theta_L\}
\end{aligned}
$$

In all other cases, $U^L(a_t, \tau = (\theta, \zeta), \gamma_t) = 0$. Conditional on the success of the movement the utility derived by the leader is independent of the quality of the leader. A leader that has non political objectives, $\zeta = NP$ derives positive payoff only from a successful non-political protest. It gains nothing from conducting a revolution that overthrows the government. However, a leader that has political objectives, $\zeta = P$ derives a positive payoff only from a successful revolution but gains nothing from a non-political protest. The cost of implementing a movement is zero irrespective of the type of the movement and the quality of the leader.

The government derives per period utility, $U^G(a_t, \gamma_t)$ from a movement of type $a_t$ in state $\gamma_t$.

$$\begin{aligned} U^G(a_t, \gamma_t) &= W \qquad if\, a_t = s \qquad \& \qquad \gamma_t = S/F \\ &= W \qquad if\, a_t = R \qquad \& \qquad \gamma_t = F \end{aligned}$$

In all other cases the government receives a payoff 0. $W$ is the benefit to the government from being in power. We assume that the government can be thrown out of power only if the movement is a successful revolution. The government also incurs a cost, $cg_t$ for implementing force $g_t$ against the movement. We assume that $g_t \in \{0, W\}$ for simplicity i.e., either the government puts no effort or puts all the effort. The leader and the government, discount the future with the same discount factor, $\delta$.

## 3.1 Strategies

In each period, $t \in \{1, 2\}$, the leader knows its own choice of nature of movement, extent of force exerted by the government to reduce the extent of mass participation and hence increase the probability of failure and the fraction of mass that participated in movements of all previous periods. A pure strategy of the leader of type $\tau \in \mathbb{T}$ at time period $t$ is a function $\sigma_t : \mathbb{H} \times \mathbb{T} \to \{R, s\}$ that maps for every type of leader, $\tau$and every history, $h_t$ to a nature of movement, $a_t$ at time period $t$. Similarly a pure strategy of the government at time period $t$ is a function $\Gamma_t : \mathbb{H} \to \mathbb{R}$ that maps for every history, $h_t$ the level of force exerted by the government, $g_t$ at time period $t$. The pure strategy of an individual from the mass is given by $\Omega_t : \{R, s\} \times \{0, W\} \to \{f, nf\}$

We will now solve for the two period model.We assume that the game ends whenever the leader calls for a revolution. We solve for pure strategy Perfect Bayesian Equilibrium (PBE) for this game.

# 4 Analysis

We first consider the decision of an individual to partcipate or not partcipiate in a movement annouced by the leader in period $t$. Upon hearing leader's announcement, the participants update their belief about the leader's quality, $\hat{\alpha}_t$ and about the objective of the leader, $\hat{\beta}_t$. Expected payoff of each participant of type $e_i$ from participating in a movement at time period $t$ is given by

$$Pr[\gamma_t = S \mid a_t, g_t, \hat{\alpha}_t, \hat{\beta}_t]V_{at} - c_i$$

where where $c_i = e_i + g_2$ is the cost of participation in a movemen and $V_{at} \in \{W, V_s\}$ depenending upon the nature of the movement. The probability of success of a movement is given by

$$\begin{aligned} Pr[\gamma_t = S \mid a_t, g_t, \hat{\alpha}_t, \hat{\beta}_t] &= \sum_{\theta \in \{\theta_H, \theta L\}} \sum_{\zeta \in \{P, NP\}} [Pr(\theta \mid a_t, g_t) Pr(\zeta \mid a_t, g_t) Pr(\gamma_t = S \mid \theta; a_t, g_t)] \\ &= [(1-\hat{\beta_t})(1-\hat{\alpha_t})\theta_L + [(1-\hat{\beta_t})\hat{\alpha}_t + \hat{\beta}_t]\theta_H]m_t \end{aligned}$$

Hence we know that an individual $i$ will partcipate only if

$$Pr[\gamma_t = S \mid a_t, g_t, \hat{\alpha}_t, \hat{\beta}_t]V_{at} - c_i \geq 0$$

Hence the equilibrium level of participation in a movement of type $a_t$ given that the government announces force $g_t$ is given by

$$m_t^*(g_t, a_t, \hat{\alpha}_t, \hat{\beta}_t) = \frac{e_L - g_t}{(e_H + e_L) - [(1-\hat{\beta}_t)(1-\hat{\alpha}_t)\theta_L + [(1-\hat{\beta}_t)\hat{\alpha}_t + \hat{\beta}_t]\theta_H]V_{at}} \quad (1)$$

It is clearly seen from above that the equilibrium level of mass participation depends on multiple things. The equilibrium level of mass participation in period $t$ decreases as government increases its effort level, i.e.

$$m_t^*(g_t = 0, a_t, \hat{\alpha}_t, \hat{\beta}_t) > m_t^*(g_t = W, a_t, \hat{\alpha}_t, \hat{\beta}_t)$$

Also the equilibrium level of mass participation increases as the quality of the leader increases i.e., $m_t^*$ increases with $\hat{\alpha}_t$ for any given $\hat{\beta}_t$.

## 4.1 Second Period

Now we start solving from the game from the last period. First we can argue that since there is no cost of implementing a movement, a leader with a political objective, i.e. $\zeta = P$ will always announce a revolution in the second period. He has no incentive to announce for a non-political movement because his payoff will be 0 whereas from a revolution he enjoys positive payoff. By a similar argument, a leader with a non-political objective, i.e. $\zeta = NP$ will always call for a non-political protest in the second period. In fact, a leader with a non-political objective will never call for a revolution in either the first or the second period.

Now, consider the decision of the government that observes the nature of announcement by the leader in period 2, $a_2$. Upon hearing the leader's announcement, the government updates their belief about the leader's quality, $\hat{\alpha_2}$ and about the objective of the leader, $\hat{\beta}_2$. Since, $U^G(a_2 = s, \gamma_t = S) = U^G(a_2 = s, \gamma_t = F)$, then optimal government force announced when $a_2 = s$ will be zero. However if the leader with a political objective calls for a revolution i.e., $a_2 = R$ then the government loses power if the revolution is a success and in that case receives a payoff of zero while it will remain in power and obtain a payoff of $W$ if the revolution is a failure. Hence the government chooses $g_2^*$ in order to maximize the following expected payoff

$$Max_{g_2} Pr[\gamma_2 = F \mid a_2 = R; \hat{\alpha_2}, \hat{\beta}_2] U^G(a_2 = R, \gamma_2 = F) - cg_2$$

The government can choose $g_2 \in \{0, W\}$. Upon hearing about a revolution in period 2, i.e. $a_2 = R$, the belief about the objective of the leader is revised to $\hat{\beta}_2 = 0$ because given the payoff structure the leader with non-political objective will never call for a revolution. We can now find the optimal level of force i.e. $g_2^*$ that will be exerted by the government given that it hears about a revolution. The payoff to the government given that it exerts a force $g_2$ is given by

$$U^G(g_2, a_2 = R, \gamma_2 = F; \hat{\alpha_2}) = [(1 - \hat{\alpha_2})(1 - \theta_L m_2(g_2)) + \hat{\alpha_2}(1 - \theta_H m_2(g_2))]W - cg_2$$

Given that the government has to choose between 0 or $W$, substituting for optimal $m_2$ from above we can write

$$U^G(g_2 = 0, \hat{\alpha_2}) - U^G(g_2 = W, \hat{\alpha_2}) = \frac{-[(1 - \hat{\alpha_2})\theta_L + \hat{\alpha_2}\theta_H]W^2}{[(e_H + e_L) - [(1 - \hat{\alpha_2})\theta_L + \hat{\alpha_2}\theta_H]W} + cW \quad (2)$$

The government will choose $g_2 = 0$ if equation2 is greater than zero while it exert $g_2 = W$ if equation2 is less than zero. Hence setting equation2 equal to zero we can get a cutoff value $\bar{\alpha}$ such that that if $\hat{\alpha_2} < \bar{\alpha}$, then the government exerts $g_2 = 0$ while it exerts an effort $g_2 = W$ if $\hat{\alpha_2} > \bar{\alpha}$. The value of $\bar{\alpha}$ is given by

$$\bar{\alpha} = \frac{1}{(\theta_H - \theta_L)}[\frac{c(e_H + e_L)}{W + cW} - \theta_L]$$

Given the assumptions on the parameters above, we obtain that $0 < \bar{\alpha} < 1$.

**Lemma 1.** *In the second period, the leader with political objective i.e., $\zeta = P$ will always announce a revolution($a_2 = R$)while a leader with a non-political objective i.e., $\zeta = NP$ will always annouce a non-political protest ($a_2 = s$). Upon hearing, $a_2 = s$, the* *governement will put effort* $g_2 = 0$ *while if* $a_2 = R$*, then the governement's effort is* $g_2 = W$ *if* $\hat{\alpha_2} > \bar{\alpha}$ *while* $g_2 = 0$*,* $\forall \hat{\alpha_2} \leq \bar{\alpha}$*. The level of mass participartion is given by equation1*

## 4.2 First Period

Since the participants in a movement are myopic, therefore their problem remains the same as in the last period. Hence $m_1^*(a_1,\, g_1,\, \hat{\alpha_1}, \hat{\beta_1})$ is determined in the same way as in the last period.

Now we have to first determine the government's optimal action in the first period. The government's problem remains exactly the same as in the second period if the leader announces $a_1 = R$ in the first period. Hence it will follow the threshold startegy as described earlier. We have to now determine the optimal action of the governement in the first period when the leader announces $a_1 = s$. The non-political protest can either be announced by a leader with a non-political objective in which case there is no threat to the government's existence. While the leader with a political objective might also call for a small protest in the first period and then try to improve his reputation, increase the mass participation in the second period and call for a revolution in the final period. In this case the government faces a threat to existence and hence may like to crush the non-political protest in the first period itself.

Before analysing the government's strategy in the first period, we need to define the updating rules about the the quality of the leader at the end of the first period. Upon success in the first period of a social movement, the updated belief about the quality of the leader in the second period is given by

$$\alpha_2^S = \frac{\theta_H \alpha_1}{\theta_H \alpha_1 + \theta_L (1 - \alpha_1)} \tag{3}$$

On the other hand, if the movement is a failure in the first period, then the updated belief about the quality of the leader in the second period is given by

$$\alpha_2^F(g_1) = \frac{\alpha_1[1 - \theta_H m_1(g_1)]}{\alpha_1[1 - \theta_H m_1(g_1) + (1 - \alpha_1)[1 - \theta_L m_1(g_1)]} \tag{4}$$

It is interesting to note that $\alpha_2^S$ is independent of the level of mass participation while $\alpha_2^F(g_1)$ depends on the level of mass participation which in turn depends on the level of governement's force in period 1. From the second period analysis, we know that the government will put force on hearing a revolution if and only if $\hat{\alpha_2} > \bar{\alpha}$. Hence to determine the first period government's strategy we need to find out whether the updated belief about the quality of the leader upon a successful non-political protest crosses $\bar{\alpha}$ or not.

Let us define $\alpha_1^S$, $\alpha_1^F(g_1 = W)$ and $\alpha_1^F(g_1 = 0)$ such that $\alpha_1^S$ satisfies the first equation, $\alpha_1^F(g_1 = W)$ satisfies the second equation while the third equation is satisfied by $\alpha_1^F(g_1 = 0)$.

$$\alpha_2^S(\alpha_1^S) = \bar{\alpha}$$

$$\alpha_2^F(\alpha_1^F(g_1 = W)) = \bar{\alpha}$$

$$\alpha_2^F(\alpha_1^F(g_1 = 0)) = \bar{\alpha}$$

**Lemma 2.** *If* $\frac{\partial \alpha_2^F(g_1=W)}{\partial \alpha_1} > 0$ *and* $\frac{\partial \alpha_2^F(g_1=0)}{\partial \alpha_1} > 0$*, then* $\alpha_1^S < \bar{\alpha} < \alpha_1^F(g_1 = W) < \alpha_1^F(g_1 = 0)$

*Proof.* We first show that $\alpha_2^S - \alpha_2^F(g_1)$ is always positive for any given $\alpha_1$

$$\alpha_2^S - \alpha_2^F(g_1) = \frac{\alpha_1(1-\alpha_1)(\theta_H - \theta_L)}{[\theta_H\alpha_1 + \theta_L(1-\alpha_1)][(1-\theta_H m_1)\alpha_1 + (1-\alpha_1)(1-\theta_L m_1)]}$$

The denominator is always positive and given that $\theta_H > \theta_L$, we know that $\alpha_2^S > \alpha_2^F(g_1)$ Since $\frac{\partial \alpha_2^F}{\partial m_1} < 0$ and $\frac{\partial m_1}{\partial g_1} < 0$, then we have $\frac{\partial \alpha_2^F}{\partial g_1} = \frac{\partial \alpha_2^F}{\partial m_1}\frac{\partial m_1}{\partial g_1} > 0$. Hence we have

$$\alpha_2^S > \alpha_2^F(g_1 = w) > \alpha_2^F(g_1 = 0)$$

We know that $\frac{\partial \alpha_2^S}{\partial \alpha_1} > 0$ and assume that $\frac{\partial \alpha_2^F(g_1=W)}{\partial \alpha_1} > 0$ and $\frac{\partial \alpha_2^F(g_1=0)}{\partial \alpha_1} > 0$. Since these functions are increasing and $\alpha_2^S > \alpha_2^F(g_1 = w) > \alpha_2^F(g_1 = 0)$, then we have $\alpha_1^S < \alpha_1^F(g_1 = w) < \alpha_1^F(g_1 = 0)$. Since $\theta_H > \theta_L$, we can easily show that $\alpha_1^S < \bar{\alpha} < \alpha_1^F(g_1 = w) < \alpha_1^F(g_1 = 0)$. □

Now we will analyze the strategy of the government in the first period. First we assume that the leader with a political objective will have a threshold policy and then try and analyze the government's optimal action in period 1. Later we show that in equilibrium, the leader does indeed have a threshold policy. By threshold policy of the leader ($\zeta = NP$) we mean, that there exists a $0 < \alpha_L < 1$ such that $\forall \alpha_1 < \alpha_L$, the leader's action in period 1 is $a_1 = R$ while there exists a $0 < \alpha_H < 1$, such that $0 < \alpha_L < \alpha_H < 1$ and $\forall \alpha_1 > \alpha_H$, the leader's optimal action in period 1 is $a_1 = R$, while for all intermediate values of $\alpha_1$, the political leader does non-political protest, $a_1 = s$ in the first period and then announces a revolution,$a_2 = R$ in the second period.

We start with the assumption $0 < \alpha_L < \alpha_1^S < \alpha_1^F(g_1 = W) < \alpha_1^F(g_1 = 0) < \alpha_H < 1$. One important thing to note is that the thresholds are dependent on $\beta_1$ and we will discuss about how these thresholds move with respect to $\beta_1$ later in the paper. Now we already know the government's optimal strategy when the leader announces $a_1 = R$. Hence we have to determine the optimal strategy for the four ranges in between $\alpha_L$ and $\alpha_H$. Now the four ranges are, Range I - $\alpha_1 \in [\alpha_L, \alpha_1^S]$, Range II-$\alpha_1 \in [\alpha_S, \alpha_1^F(g_1 = W)]$, Range III-$\alpha_1 \in [\alpha_1^F(g_1 = W), \alpha_1^F(g_1 = 0)]$ and Range IV -$\alpha_1 \in [\alpha_1^F(g_1 = 0), \alpha_H]$. An important point to keep in mind at this juncture is that the optimal action of the government in each region doesn't depend on the position of $\alpha_L$ and $\alpha_H$. For example if $\alpha_L = \alpha_S$, then the optimal action in any of the other regions, like Region II or Region III is unaffected. This is for the simple reason that the expected payoff of the government in each region does not depend upon the postion of $\alpha_L$ and $\alpha_H$. Hence we consider the most general case where $0 < \alpha_L < \alpha_1^S < \alpha_1^F(g_1 = W) < \alpha_1^F(g_1 = 0) < \alpha_H < 1$. We will first write the expected payoffs of the government for each region and then analyze the optimal action in each region.

We first write the expected payoff of the government when the leader announces $a_1 = s$ and government exerts an effort $g_1$.

$$\begin{aligned}
EU^G(g_1, a_1 = s, \hat{\alpha_2}, \hat{\beta}_2) = \quad & W - cg_1 \\
& +\delta[Pr(\zeta = P)[Pr(\gamma_1 = S \mid \zeta = P, a_1 = s) \\
& *[Pr(\gamma_2 = S \mid a_2 = R, \zeta = P, \gamma_1 = S, g_2, \hat{\beta}_2 = 0)(-cg_2)] \\
& +[Pr(\gamma_2 = F \mid a_2 = R, \zeta = P, \gamma_1 = S, g_2, \hat{\beta}_2 = 0)(W - cg_2)]] \\
& +[Pr(\gamma_1 = F \mid \zeta = P, a_1 = s) \\
& *[Pr(\gamma_2 = S \mid a_2 = R, \zeta = P, \gamma_1 = F, g_2, \hat{\beta}_2 = 0)(-cg_2)] \\
& +[Pr(\gamma_2 = F \mid a_2 = R, \zeta = P, \gamma_1 = F, g_2, \hat{\beta}_2 = 0)(W - cg_2)]] \\
& +Pr(\zeta = NP)W]
\end{aligned} \tag{5}$$

In each region, the government has a choice between $g_1 = 0$ or $g_1 = W$. Hence for each region we write the difference of the expected payoffs of the government from exerting $g_1 = 0$ and $g_1 = W$, i.e., $EU^G(g_1 = 0) - EU^G(g_1 = W)$.

*Range I:* $\alpha_1 \in [\alpha_L, \alpha_1^S]$
In this range, $\alpha_1$ is such that even when the non-political protest is succesful, then the updated belief, i.e. $\alpha_2 S$ is still less than $\bar{\alpha}$ which means that it will attract zero effort

when the leader announces revolution in the second period.

$$\begin{aligned} EU^G(g_1 = 0) - EU^G(g_1 = W) = \; & cW + \delta W(1-\beta_1)[(\alpha_1\theta_H + (1-\alpha_1)\theta_L)* \\ & [1 - (\alpha_2^S\theta_H + (1-\alpha_2^S\theta_L)m_2(g_2 = 0, \hat{\beta}_2 = 0, \alpha_2^S)]* \\ & [m_1(g_1 = 0) - m_1(g_1 = W)] \\ & +[1 - (\alpha_1\theta_H + (1-\alpha_1)\theta_L)m_1(g_1 = 0)]* \\ & [1 - (\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)m_2(g_2 = 0, \hat{\beta}_2 = 0, \alpha_2^F(0)] \\ & -[1 - (\alpha_1\theta_H + (1-\alpha_1)\theta_L)m_1(g_1 = W)]* \\ & [1 - (\alpha_2^F(W)\theta_H + (1-\alpha_2^F(W))\theta_L)m_2(g_2 = 0, \hat{\beta}_2 = 0, \alpha_2^F(W)] \end{aligned} \tag{6}$$

*Range II:* $\alpha_1 \in [\alpha_S, \alpha_1^F(g_1 = W)]$

In this range, the leader's quality $\alpha_1$ is such that if the non-political protest is successful then the updated belief, i.e., $\alpha_2^S$ is now greater than $\bar{\alpha}$ which means that the government will exert effort $g_2 = W$ in case of a revolution while if the non-political protest is unsuccessful then the updated belief is less than $\bar{\alpha}$ and government's effort in case of revolution is $g_2 = 0$.

$$\begin{aligned} EU^G(g_1 = 0) - EU^G(g_1 = W) = \; & cW + \delta W(1-\beta_1)[(\alpha_1\theta_H + (1-\alpha_1)\theta_L)* \\ & [[1 - (\alpha_2^S\theta_H + (1-\alpha_2^S\theta_L)m_2(g_2 = 0, \hat{\beta}_2 = 0, \alpha_2^S)] - c]* \\ & [m_1(g_1 = 0) - m_1(g_1 = W)] \\ & +[1 - (\alpha_1\theta_H + (1-\alpha_1)\theta_L)m_1(g_1 = 0)]* \\ & [1 - (\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)m_2(g_2 = 0, \hat{\beta}_2 = 0, \alpha_2^F(0)] \\ & -[1 - (\alpha_1\theta_H + (1-\alpha_1)\theta_L)m_1(g_1 = W)]* \\ & [1 - (\alpha_2^F(W)\theta_H + (1-\alpha_2^F(W))\theta_L)m_2(g_2 = 0, \hat{\beta}_2 = 0, \alpha_2^F(W)] \end{aligned} \tag{7}$$

*Range III:* $\alpha_1 \in [\alpha_1^F(g_1 = W), \alpha_1^F(g_1 = 0)]$

In this range, similar to range II, the updated belief in the second period is greater than $\bar{\alpha}$ and hence government's effort in period 2 is $g_2 = W$. However if the non-political protest is unsuccesful, then the updated belief goes beyond $\bar{\alpha}$ if the government's effort in period 1, i.e.,$g_1 = 0$ while it remains below $\bar{\alpha}$ if $g_1 = W$. Hence the government's effort in the second period will depend accordingly.

$$\begin{aligned} EU^G(g_1=0)-EU^G(g_1=W) = \; & cW+\delta W(1-\beta_1)[(\alpha_1\theta_H+(1-\alpha_1)\theta_L)* \\ & [[1-(\alpha_2^S\theta_H+(1-\alpha_2^S\theta_L)m_2(g_2=0,\hat{\beta}_2=0,\alpha_2^S)]-c]* \\ & [m_1(g_1=0)-m_1(g_1=W)] \\ & +[1-(\alpha_1\theta_H+(1-\alpha_1)\theta_L)m_1(g_1=0)]* \\ & [1-(\alpha_2^F(0)\theta_H+(1-\alpha_2^F(0))\theta_L)m_2(g_2=0,\hat{\beta}_2=0,\alpha_2^F(0)] \\ & -[1-(\alpha_1\theta_H+(1-\alpha_1)\theta_L)m_1(g_1=W)]* \\ & [1-c-(\alpha_2^F(W)\theta_H+(1-\alpha_2^F(W))\theta_L)m_2(g_2=W,\hat{\beta}_2=0,\alpha_2^F(W)] \end{aligned} \tag{8}$$

*Range IV:* $\alpha_1 \in [\alpha_1^F(g_1=0), \alpha_H]$

In this range the quality of the leader is sufficiently high such that even if the non-political protest is unsuccessful in the first period, the updated belief is always greater than $\bar{\alpha}$ and hence government's effort in the second period is always $g_2 = W$ on observing a revolution.

$$\begin{aligned} EU^G(g_1=0)-EU^G(g_1=W) = \; & cW+\delta W(1-\beta_1)[(\alpha_1\theta_H+(1-\alpha_1)\theta_L)* \\ & [[1-(\alpha_2^S\theta_H+(1-\alpha_2^S\theta_L)m_2(g_2=0,\hat{\beta}_2=0,\alpha_2^S)]-c]* \\ & [m_1(g_1=0)-m_1(g_1=W)] \\ & +[1-(\alpha_1\theta_H+(1-\alpha_1)\theta_L)m_1(g_1=0)]* \\ & [1-c-(\alpha_2^F(0)\theta_H+(1-\alpha_2^F(0))\theta_L)m_2(g_2=W,\hat{\beta}_2=0,\alpha_2^F(0)] \\ & -[1-(\alpha_1\theta_H+(1-\alpha_1)\theta_L)m_1(g_1=W)]* \\ & [1-c-(\alpha_2^F(W)\theta_H+(1-\alpha_2^F(W))\theta_L)m_2(g_2=W,\hat{\beta}_2=0,\alpha_2^F(W)] \end{aligned} \tag{9}$$

The following lemma describes the strategy of the government on observing a non-political protest in the first period. The strategy of the government is crucially dependent on the value of $c$, which can be interpreted as the marginal cost of exerting effort for the government.

**Lemma 3.** *Upon observing $a_1 = s$, the government's effort is $g_1 = 0$ for all the ranges if $c > \bar{c}$ and when $c < \bar{\bar{c}}$, then $g_1 = W$, $\forall \alpha_1 \in [\alpha_1^S, \alpha_1^F(g_1 = 0)]$ and $g_1 = 0$ for all other ranges.*

*Proof.* First we refer to region I. We redefine equation6 as

$$EU^G(g_1=0)-EU^G(g_1=W) = cW+\delta W(1-\beta_1)[A(\alpha_1)+B(\alpha_1)-C(\alpha_1)]$$

Where

$$A(\alpha_1) = (\alpha_1\theta_H+(1-\alpha_1)\theta_L)[1-(\alpha_2^S\theta_H+(1-\alpha_2^S\theta_L)m_2(g_2=0,\hat{\beta}_2=0,\alpha_2^S)][m_1(g_1=0)-m_1(g_1=W)]$$

$$B(\alpha_1) = [1-(\alpha_1\theta_H+(1-\alpha_1)\theta_L)m_1(g_1=0)][1-(\alpha_2^F(0)\theta_H+(1-\alpha_2^F(0))\theta_L)m_2(g_2=0,\hat{\beta}_2=0,\alpha_2^F(0))]$$

$$C(\alpha_1) = [1-(\alpha_1\theta_H+(1-\alpha_1)\theta_L)m_1(g_1=W)][1-(\alpha_2^F(W)\theta_H+(1-\alpha_2^F(W))\theta_L)m_2(g_2=0,\hat{\beta}_2=0,\alpha_2^F(W)]$$

We can verify that $A(\alpha_1) + B(\alpha_1) - C(\alpha_1)$ is always positive for all values of $\alpha_1$. Hence it means that in range I, $EU^G(g_1 = 0) - EU^G(g_1 = W) > 0$ i.e. the optimal strategy of the government is $g_1 = 0$.

Now we refer to range II. As before we can redefine equation7 as

$$EU^G(g_1 = 0) - EU^G(g_1 = W) = cW + \delta W(1-\beta_1)[\bar{A}(\alpha_1) + B(\alpha_1) - C(\alpha_1)]$$

We can verify that the expression $\bar{A}(\alpha_1) + B(\alpha_1) - C(\alpha_1)$ is monotone in $\alpha_1$. Evaluating the expression $EU^G(g_1 = 0) - EU^G(g_1 = W)$ at $\alpha_1 = 0$ gives an equation in $c$. The equation is given by

$$EU^G(g_1=0)-EU^G(g_1=W) = cW+\delta W(1-\beta_1)\frac{W\theta_L}{e_H+e_L-(\beta_1\theta_H+(1-\beta_1)\theta_L)V_s}[\frac{\theta_L W}{e_H+e_L-\theta_L W}-c]$$

The above expression is increasing in $c$ and hence we can define a value of $c = c^1$ where $EU^G(g_1 = 0) - EU^G(g_1 = W) = 0$ evaluated at $\alpha_1 = 0$. Hence $\forall c > c^1$, $EU^G(g_1 = 0) - EU^G(g_1 = W) > 0$ evaluated at $\alpha_1 = 0$. Similarly $\forall c < c^1$, $EU^G(g_1 = 0) - EU^G(g_1 = W) < 0$ evaluated at $\alpha_1 = 0$.
Evaluating the expression $EU^G(g_1 = 0) - EU^G(g_1 = W)$ at $\alpha_1 = 1$ gives an equation which is given by

$$EU^G(g_1=0)-EU^G(g_1=W) = cW+\delta W(1-\beta_1)\frac{W\theta_H}{e_H+e_L-(\beta_1\theta_H+(1-\beta_1)\theta_L)V_s}[\frac{\theta_H W}{e_H+e_L-\theta_H W}-c]$$

The above expression is increasing in $c$ and hence we can define a value of $c = c^2$ where $EU^G(g_1 = 0) - EU^G(g_1 = W) = 0$ evaluated at $\alpha_1 = 1$. Hence at $\alpha_1 = 1$, $\forall c > c^2$, $EU^G(g_1 = 0) - EU^G(g_1 = W) > 0$ and $\forall c < c^2$, $EU^G(g_1 = 0) - EU^G(g_1 = W) < 0$.
Hence $\forall c > max\{c^1, c^2\}$, $EU^G(g_1 = 0) - EU^G(g_1 = W) > 0$ evaluated at $\alpha_1 = 0$ and $\alpha_1 = 1$ and given the expression $\bar{A}(\alpha_1) + B(\alpha_1) - C(\alpha_1)$ is monotone in $\alpha_1$ we can then claim that $EU^G(g_1 = 0) - EU^G(g_1 = W) > 0$, $\forall\alpha_1 \in [0, 1]$. Hence the optimal response is $g_1 = 0$. By similar reasoning $\forall c < min\{c^1, c^2\}$, $EU^G(g_1 = 0) - EU^G(g_1 = W) < 0$, $\forall\alpha_1 \in [0, 1]$. Hence the optimal response is $g_1 = W$.
Now we refer to range III. As stated before we can again redefine equation8 as

$$EU^G(g_1 = 0) - EU^G(g_1 = W) = cW + \delta W(1-\beta_1)[\bar{A}(\alpha_1) + B(\alpha_1) - \bar{C}(\alpha_1)]$$

We can verify that the expression $\bar{A}(\alpha_1) + B(\alpha_1) - \bar{C}(\alpha_1)$ is monotone in $\alpha_1$. Evaluating the expression $EU^G(g_1 = 0) - EU^G(g_1 = W)$ at $\alpha_1 = 0$ gives an equation in $c$ which is

increasing in $c$. Hence we can define a value of $c = c^3$ where $EU^G(g_1 = 0) - EU^G(g_1 = W) = 0$ evaluated at $\alpha_1 = 0$. Hence $\forall c > c^3$, $EU^G(g_1 = 0) - EU^G(g_1 = W) > 0$ evaluated at $\alpha_1 = 0$. Similarly $\forall c < c^3$, $EU^G(g_1 = 0) - EU^G(g_1 = W) < 0$ evaluated at $\alpha_1 = 0$.
We can then define a value of $c = c^4$ where $EU^G(g_1 = 0) - EU^G(g_1 = W) = 0$ evaluated at $\alpha_1 = 1$. Hence at $\alpha_1 = 1$, $\forall c > c^4$, $EU^G(g_1 = 0) - EU^G(g_1 = W) > 0$ and $\forall c < c^4$, $EU^G(g_1 = 0) - EU^G(g_1 = W) < 0$.
Hence $\forall c > max\{c^3, c^4\}$, $EU^G(g_1 = 0) - EU^G(g_1 = W) > 0$ evaluated at $\alpha_1 = 0$ and $\alpha_1 = 1$ and given the expression $\bar{A}(\alpha_1) + B(\alpha_1) - \bar{C}(\alpha_1)$ is monotone in $\alpha_1$ we can then claim that $EU^G(g_1 = 0) - EU^G(g_1 = W) > 0$, $\forall \alpha_1 \in [0, 1]$. Hence the optimal response is $g_1 = 0$. By similar reasoning $\forall c < min\{c^3, c^4\}$, $EU^G(g_1 = 0) - EU^G(g_1 = W) < 0$, $\forall \alpha_1 \in [0, 1]$. Hence the optimal response is $g_1 = W$.
We now define a value of $c = \bar{c} = max\{c^1, c^2, c^3, c^4\}$,such that $\forall c > \bar{c}$, $g_1 = 0$ for range II and III. Similarly we can define a value of $c = \bar{\bar{c}} = min\{c^1, c^2, c^3, c^4\}$such that $\forall c < \bar{\bar{c}}$, $g_1 = W$.
Similarly we can refer to range IV and redefine equation9 as

$$EU^G(g_1 = 0) - EU^G(g_1 = W) = cW + \delta W(1 - \beta_1)[\bar{A}(\alpha_1) + \bar{B}(\alpha_1) - \bar{C}(\alpha_1)]$$

We can verify that the expression $\bar{A}(\alpha_1) + \bar{B}(\alpha_1) - \bar{C}(\alpha_1)$ is always positive for all values of $\alpha_1$. Hence it means that in range IV, $EU^G(g_1 = 0) - EU^G(g_1 = W) > 0$ i.e. the optimal strategy of the government is $g_1 = 0$. □

The above lemma suggests that when it is sufficiently costly for the government to exert effort then the costs of putting effort are lower than the expected benefits, hence the government doesn't put any effort upon observing a non-political protest. On the other hand when the marginal costs of exerting effort are sufficiently low, then the government exerts effort for intermediate ranges of $\alpha_1$.
**Proposition 1.** *If $\delta > \bar{\delta}$ and $c > \bar{c}$, the leader with a political objective i.e.,$\zeta = P$ in the first period will announce a revolution ($a_1 = R$), $\forall \alpha_1 \in [0, \bar{\alpha}_1)$ and $\forall \alpha_1 \in [\alpha_1^F(g_1 = 0), 1]$ and a non-political protest ($a_1 = s$), $\forall \alpha_1 \in [\bar{\alpha}_1, \alpha_1^F(g_1 = 0))$. The leader with a non-political objective i.e.,$\zeta = NP$ will always announce non-political protest ($a_1 = s$) irrespective of $\alpha_1$. The government will exert a force $g_1 = 0$ upon observing a non-political protest in the first period.*

*Proof.* As stated in lemma3, the government will exert an effort $g_1 = 0$ given the value of $c$. Now we write the expected payoffs of the leader, $\zeta = P$ for different actions it take in period 1 and the value of $\alpha_1$ and $\alpha_2$. The expected payoff when the leader announces a revolution and $\alpha_1 < \bar{\alpha}_1$ is given by

$$\begin{aligned} H_0(\alpha_1, \beta_1 = 0) &= EU^R(\alpha_1, g_1 = 0) \\ &= \frac{[\alpha_1\theta_H + (1-\alpha_1)\theta_L]e_L W}{[e_H + e_L - [\alpha_1\theta_H + (1-\alpha_1)\theta_L]W]} \end{aligned}$$

The expected payoff when the leader announces a revolution and $\alpha_1 > \bar{\alpha}_1$ is given by

$$\begin{aligned} \bar{H}_0(\alpha_1, \beta_1 = 0) &= EU^R(\alpha_1, g_1 = W) \\ &= \frac{[\alpha_1\theta_H + (1-\alpha_1)\theta_L](e_L - W)W}{[e_H + e_L - [\alpha_1\theta_H + (1-\alpha_1)\theta_L]W]} \end{aligned}$$

The expected payoff when the leader announces $a_1 = s$ and $\alpha_1 < \alpha_1^S$ is given by

$$\begin{aligned} H_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = 0, \hat{\beta}_1 = \beta_1, g_2 = 0) \\ &= \delta W K(\alpha_1, \beta_1)\frac{(\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)e_L}{e_H + e_L - (\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)W} \\ &+ \delta W[1 - K(\alpha_1, \beta_1)]\frac{(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)e_L}{e_H + e_L - (\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)W} \end{aligned}$$

where $K(\alpha_1, \beta_1) = \frac{[\alpha_1\theta_H + (1-\alpha_1)\theta_L]e_L}{e_H + e_L - [\beta_1\theta_H + (1-\beta_1)(\theta_H\alpha_1 + (1-\alpha_1)\theta_L)]W}$

Now we calculate the expected payoff when the leader announces $a_1 = s$ and $\alpha_1^S \leq \alpha_1 < \alpha_1^F(0)$. In this range, upon success of the non-political protest in the first period, the updated $\alpha$ at the start of the second period is above $\bar{\alpha}$ while on failure it is below $\bar{\alpha}$.

$$\begin{aligned} \bar{H}_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = 0, \hat{\beta}_1 = \beta_1, g_2) \\ &= \delta W K(\alpha_1, \beta_1)\frac{(\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)W} \\ &+ \delta W[1 - K(\alpha_1, \beta_1)]\frac{(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)e_L}{e_H + e_L - (\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)W} \end{aligned}$$

Now we calculate the expected payoff when the leader announces $a_1 = s$ and $\alpha_1 \geq \alpha_1^F(0)$. In this range, irrespective of success or failure of the non-political protest in the first period, the updated $\alpha$ is always greater than $\bar{\alpha}$ which means the government will put effort in the second period.

$$\begin{aligned} \hat{H}_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = 0, \hat{\beta}_1 = \beta_1, g_2 = W) \\ &= \delta W K(\alpha_1, \beta_1)\frac{(\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)W} \\ &+ \delta W[1 - K(\alpha_1, \beta_1)]\frac{(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)W} \end{aligned}$$

It is easy to verify that $H_0(\alpha_1, \beta_1 = 0)$, $\bar{H}_0(\alpha_1, \beta_1 = 0)$, $H_1(\alpha_1, \beta_1)$, $\bar{H}_1(\alpha_1, \beta_1)$ and $\hat{H}_1(\alpha_1, \beta_1)$ are all increasing in $\alpha_1$. We are considering the equilibrium where $\forall \alpha_1 \in [0, \alpha_L)$ and $\forall \alpha_1 \in [\alpha_H, 1]$, the leader does revolution in the first period, $a_1 = R$ while $\forall \alpha_1 \in [\alpha_L, \alpha_H)$, the leader does non-political protest, $a_1 = s$. Now we will show that $\alpha_L = \bar{\alpha}$ and $\alpha_H = \alpha_1^F(0)$. We show by the method of eliminating different cases.

Let us assume that $\alpha_L < \alpha_1^S$. For this to be true we need the condition that $\forall \alpha_1 \in [\alpha_L, \alpha_1^S)$, the following holds, $H_0(\alpha_1, \beta_1 = 0) < H_1(\alpha_1, \beta_1)$. However we can show that $H_0(\alpha_1 = 0, \beta_1 = 0) > H_1(\alpha_1 = 0, \beta_1)$ and also $H_0(\alpha_1 = 1, \beta_1 = 0) > H_1(\alpha_1 = 1, \beta_1)$. Since $H_0(\alpha_1, \beta_1 = 0)$ and $H_1(\alpha_1, \beta_1)$ are increasing in $\alpha_1$, this implies that $H_0(\alpha_1, \beta_1 = 0) > H_1(\alpha_1, \beta_1) \forall \alpha_1$. Thus the condition does not hold and hence $\alpha_L \nless \alpha_1^S$.

Let us assume that $\alpha_L = \alpha_1^S$. For this to hold, we need the condition that $\forall \alpha_1 \in [\alpha_1^S, \bar{\alpha})$, the following holds, $H_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)$. Now one can easily show that $H_1(\alpha_1, \beta_1) > \bar{H}_1(\alpha_1, \beta_1)\ \forall \alpha_1$ and since we have proved that $H_0(\alpha_1, \beta_1 = 0) > H_1(\alpha_1, \beta_1) \forall \alpha_1$, therefore we have $\forall \alpha \in [0, 1]$, $H_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1)$. Hence the necessary condition does not hold and therefore $\alpha_L \neq \alpha_1^S$.

Now lets assume that $\alpha_L \in (\alpha_1^S, \bar{\alpha})$. For this to be true we need the condition that $\forall \alpha_1 \in [\alpha_L, \bar{\alpha})$, the following holds, $H_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)$. However we have proved that this condition cannot hold and hence $\alpha_L \notin (\alpha_1^S, \bar{\alpha})$.

Next we show that $\alpha_H \ngtr \alpha_1^F(0)$. Let us assume that $\alpha_H > \alpha_1^F(0)$. For this to hold, we need that $\forall \alpha_1 \in [\alpha_1^F(0), \alpha_H)$, the following condition holds, $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \hat{H}_1(\alpha_1, \beta_1)$. However we can show that $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \hat{H}_1(\alpha_1 = 0, \beta_1)$ and also $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \hat{H}_1(\alpha_1 = 1, \beta_1)$. Since $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\hat{H}_1(\alpha_1, \beta_1)$ are increasing in $\alpha_1$, this implies that $\bar{H}_0(\alpha_1, \beta_1 = 0) > \hat{H}_1(\alpha_1, \beta_1) \forall \alpha_1$. Thus the condition does not hold and hence $\alpha_H \ngtr \alpha_1^F(0)$.

Now the only possibility therefore we have is that $\alpha_L, \alpha_H \in [\bar{\alpha}, \alpha_1^F(0)]$. We consider the case where $\alpha_L = \bar{\alpha}$ and $\alpha_H = \alpha_1^F(0)$. For this to hold we need the following conditions to be true

1. $\forall \alpha_1 < \alpha_1^S : H_0(\alpha_1, \beta_1 = 0) > H_1(\alpha_1, \beta_1 = 1)$
2. $\forall \alpha_1 \in [\alpha_1^S, \bar{\alpha}) : H_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1 = 1)$
3. $\forall \alpha_1 \geq \alpha_1^F(0) : \bar{H}_0(\alpha_1, \beta_1 = 0) > \hat{H}_1(\alpha_1, \beta_1 = 1)$
4. $\forall \alpha_1 \in [\bar{\alpha}, \alpha_1^F(0)) : \bar{H}_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)$

As proved previously conditions 1, 2 and 3 holds true. Now we consider the last condition and check whether it holds or not. We evaluate $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1, \beta_1)$ at $\alpha_1 = 0$ and $\alpha_1 = 1$. We can find that if $\delta > \delta_1 = \frac{e_L - W}{e_L[1-\frac{\theta_L W}{e_H + e_L - \theta_H W}]}$, then $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 0, \beta_1)\forall\beta_1$. We can also find another $\delta = \delta_2$ such that if $\delta < \delta_2 = \frac{e_L - W}{e_L[1-\frac{\theta_L W}{e_H + e_L - \theta_L W}]}$, then $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 0, \beta_1)\forall\beta_1$.
Now comparing $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1 = 1, \beta_1)$, we can show that there exists a $\delta = \delta_3 = \frac{1}{[1-\frac{\theta_L W}{e_H + e_L - \theta_L W}]}$ such that if $\delta > \delta_3$, then $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 1, \beta_1)$ and if $\delta < \delta_3$, then $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 1, \beta_1)$.
We can then define $\bar{\delta} = max\{\delta_1, \delta_3\}$ such that if $\delta > \bar{\delta}$ then $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 0, \beta_1)$ and $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 1, \beta_1)$. Since $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1, \beta_1)$ are increasing in $\alpha_1$, then $\bar{H}_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)\forall\alpha_1, \beta_1$ and therefore condition 4 follows.

Now we can have three other cases, i.e. *Case 1*: $\alpha_L > \bar{\alpha}, \alpha_H < \alpha_1^F(0)$, *Case 2*: $\alpha_L = \bar{\alpha}, \alpha_H < \alpha_1^F(0)$ and *Case 3*: $\alpha_L > \bar{\alpha}, \alpha_H = \alpha_1^F(0)$. In all these cases we need the condition $\bar{H}_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1 = 1)$ to be satisfied for some range of $\alpha_1$ which varies according to the case considered. However given the range of $\delta$ we consider the above condition can never hold and hence we prove that $\alpha_L = \bar{\alpha}$ and $\alpha_H = \alpha_1^F(0)$.

The non-political leader always have a positive expected payoff by announcing $a_1 = s$ and hence calls for a non-political protest. □

**Proposition 2.** *If $\delta < \hat{\delta}$ and $c > \bar{c}$, the leader with a political objective i.e.,$\zeta = P$ in the first period will announce a revolution ($a_1 = R$), $\forall\alpha_1 \in [0, 1]$ and the game ends. The leader with a non-political objective i.e.,$\zeta = NP$ will always announce non-political protest ($a_1 = s$) irrespective of $\alpha_1$. The government will exert a force $g_1 = 0$ if $\alpha_1 < \bar{\alpha}$ and exert a force $g_1 = W$ if $\alpha_1 \geq \bar{\alpha}$ upon observing a revolution $a_1 = R$ exert a force $g_1 = 0$ if it observes $a_1 = s$.*

*Proof.* This follows from the proof of previous proposition. Now suppose we define $\hat{\delta} = min\{\delta_2, \delta_3\}$. Then we can claim that $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 0, \beta_1)\forall\beta_1$ and $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 1, \beta_1)\forall\beta_1$. Since we know that $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1, \beta_1)$ are increasing in $\alpha_1$, then $\bar{H}_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1)\forall\alpha_1, \beta_1$. Under this circumstances, we cannot sustain a non-political protest by the political leader in equilibrium and her expected payoff is always higher from $a_1 = R$ for all values of $\alpha_1$. Given that the leader announces $a_1 = R$, then the government's action is the same as in lemma1. □

**Proposition 3.** *If $\delta > \bar{\bar{\delta}}$ and $c < \bar{\bar{c}}$, the leader with a political objective i.e.,$\zeta = P$ in the first period will announce a revolution ($a_1 = R$), $\forall\alpha_1 \in [0, \bar{\alpha}_1)$ and $\forall\alpha_1 \in [\alpha_1^F(g_1 = W), 1]$*

*and a non-political protest* ($a_1 = s$)*,* $\forall \alpha_1 \in [\bar{\alpha}_1, \alpha_1^F(g_1 = W))$*. The leader with a non-political objective i.e.,*$\zeta = NP$ *will always announce non-political protest* ($a_1 = s$) *irrespective of* $\alpha_1$*. Upon observing a non-political protest in the first period, the government will exert a force* $g_1 = W$*,* $\forall \alpha_1 \in [\alpha_1^S, \alpha_1^F(g_1 = 0)]$ *and* $g_1 = 0$ *for all other ranges.*

*Proof.* As stated in lemma 3, given the value of $c$, the governnment will exert an effort $g_1 = W$, $\forall \alpha_1 \in [\alpha_1^S, \alpha_1^F(g_1 = 0)]$ and $g_1 = 0$ for all other ranges of $\alpha_1$. Now we write the expected payoff of the leader, $\zeta = P$ for different actions it takes in period 1 and the value of $\alpha_1$ and $\alpha_2$. The expected payoff when the leader announces a revolution and $\alpha_1 < \bar{\alpha}_1$ is given by

$$\begin{aligned} H_0(\alpha_1, \beta_1 = 0) &= EU^R(\alpha_1, g_1 = 0) \\ &= \frac{[\alpha_1\theta_H + (1-\alpha_1)\theta_L]e_L W}{[e_H + e_L - [\alpha_1\theta_H + (1-\alpha_1)\theta_L]W]} \end{aligned}$$

The expected payoff when the leader announces a revolution and $\alpha_1 > \bar{\alpha}_1$ is given by

$$\begin{aligned} \bar{H}_0(\alpha_1, \beta_1 = 0) &= EU^R(\alpha_1, g_1 = W) \\ &= \frac{[\alpha_1\theta_H + (1-\alpha_1)\theta_L](e_L - W)W}{[e_H + e_L - [\alpha_1\theta_H + (1-\alpha_1)\theta_L]W]} \end{aligned}$$

The expected payoff when the leader announces $a_1 = s$ and $\alpha_1 < \alpha_1^S$ is given by

$$\begin{aligned} H_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = 0, \hat{\beta}_1 = \beta_1, g_2 = 0) \\ &= \delta W K(\alpha_1, \beta_1)\frac{(\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)e_L}{e_H + e_L - (\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)W} \\ &+ \delta W[1 - K(\alpha_1, \beta_1)]\frac{(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)e_L}{e_H + e_L - (\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)W} \end{aligned}$$

where $K(\alpha_1, \beta_1) = \frac{[\alpha_1\theta_H + (1-\alpha_1)\theta_L]e_L}{e_H + e_L - [\beta_1\theta_H + (1-\beta_1)(\theta_H\alpha_1 + (1-\alpha_1)\theta_L)]W}$
Now we calculate the expected payoff when the leader announces $a_1 = s$ and $\alpha_1^S \leq \alpha_1 < \alpha_1^F(W)$. In this range, upon success of the non-political protest with government effort $g_1 = W$ in the first period, the updated $\alpha$ at the start of the second period is above $\bar{\alpha}$ while on failure it is below $\bar{\alpha}$.

$$\begin{aligned} \bar{H}_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = W, \hat{\beta}_1 = \beta_1, g_2) \\ &= \delta W \bar{K}(\alpha_1, \beta_1)\frac{(\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)W} \\ &+ \delta W[1 - \bar{K}(\alpha_1, \beta_1)]\frac{(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)e_L}{e_H + e_L - (\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)W} \end{aligned}$$

where $\bar{K}(\alpha_1, \beta_1) = \frac{[\alpha_1\theta_H + (1-\alpha_1)\theta_L](e_L - W)}{e_H + e_L - [\beta_1\theta_H + (1-\beta_1)(\theta_H\alpha_1 + (1-\alpha_1)\theta_L)]W}$

Now we calculate the expected payoff when the leader announces $a_1 = s$ and $\alpha_1 \leq \alpha_1^F(W) < \alpha_1^F(0)$. In this range, irrespective of success or failure of the non-political protest in the first period and the government exerting $g_1 = W$, the updated $\alpha$ is always greater than $\bar{\alpha}$ which means the government will put effort in the second period in case of $a_2 = R$.

$$\begin{aligned}
\hat{H}_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = W, \hat{\beta}_1 = \beta_1, g_2 = W) \\
&= \delta W \bar{K}(\alpha_1, \beta_1) \frac{(\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)W} \\
&+ \delta W[1 - \bar{K}(\alpha_1, \beta_1)] \frac{(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)W}
\end{aligned}$$

At last we need to calculate the expected payoff when the leader announces $a_1 = s$ and $\alpha_1 \geq \alpha_1^F(0)$. In this range, the government doesn't exert force in the first period, i.e., $g_1 = 0$ and irrespective of success or failure of the non-political protest in the first period, he updated $\alpha$ is always greater than $\bar{\alpha}$ which means the government will put effort in the second period in case of $a_2 = R$.

$$\begin{aligned}
\tilde{H}_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = 0, \hat{\beta}_1 = \beta_1, g_2 = 0) \\
&= \delta W K(\alpha_1, \beta_1) \frac{(\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)W} \\
&+ \delta W[1 - K(\alpha_1, \beta_1)] \frac{(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)W}
\end{aligned}$$

where $K(\alpha_1, \beta_1) = \frac{[\alpha_1\theta_H + (1-\alpha_1)\theta_L]e_L}{e_H + e_L - [\beta_1\theta_H + (1-\beta_1)(\theta_H\alpha_1 + (1-\alpha_1)\theta_L)]W}$
It is easy to verify that $H_0(\alpha_1, \beta_1 = 0)$, $\bar{H}_0(\alpha_1, \beta_1 = 0)$, $H_1(\alpha_1, \beta_1)$, $\bar{H}_1(\alpha_1, \beta_1)$, $\hat{H}_1(\alpha_1, \beta_1)$ and $\tilde{H}_1(\alpha_1, \beta_1)$are all increasing in $\alpha_1$. We are considering the equilibrium where $\forall \alpha_1 \in [0, \alpha_L)$ and $\forall \alpha_1 \in [\alpha_H, 1]$, the leader does revolution in the first period, $a_1 = R$ while $\forall \alpha_1 \in [\alpha_L, \alpha_H)$, the leader does non-political protest, $a_1 = s$. Now we will show that $\alpha_L = \bar{\alpha}$ and $\alpha_H = \alpha_1^F(W)$. We show by the method of eliminating different cases.

Let us assume that $\alpha_L < \alpha_1^S$. For this to be true we need the condition that $\forall \alpha_1 \in [\alpha_L, \alpha_1^S)$, the following holds, $H_0(\alpha_1, \beta_1 = 0) < H_1(\alpha_1, \beta_1)$. However we can show that $H_0(\alpha_1 = 0, \beta_1 = 0) > H_1(\alpha_1 = 0, \beta_1)$ and also $H_0(\alpha_1 = 1, \beta_1 = 0) > H_1(\alpha_1 = 1, \beta_1)$. Since $H_0(\alpha_1, \beta_1 = 0)$ and $H_1(\alpha_1, \beta_1)$ are increasing in $\alpha_1$, this implies that $H_0(\alpha_1, \beta_1 = 0) > H_1(\alpha_1, \beta_1) \forall \alpha_1$. Thus the condition does not hold and hence $\alpha_L \not< \alpha_1^S$.

Let us assume that $\alpha_L = \alpha_1^S$. For this to hold, we need the condition that at $\alpha_1 = \alpha_1^S$, the following holds, $H_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)$. Now one can easily show that $H_1(\alpha_1, \beta_1) > \bar{H}_1(\alpha_1, \beta_1)$ $\forall \alpha_1$ and since we have proved that $H_0(\alpha_1, \beta_1 = 0) > H_1(\alpha_1, \beta_1) \forall \alpha_1$, therefore we have $\forall \alpha \in [0,1]$, $H_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1)$. Hence the necessary condition does not hold and therefore $\alpha_L \neq \alpha_1^S$.

Now lets assume that $\alpha_L \in (\alpha_1^S, \bar{\alpha})$. For this to be true we need the condition that $\forall \alpha_1 \in [\alpha_L, \bar{\alpha})$, the following holds, $H_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)$. However we have proved that this condition cannot hold and hence $\alpha_L \notin (\alpha_1^S, \bar{\alpha})$.

Next we show that $\alpha_H \not> \alpha_1^F(0)$. Let us assume that $\alpha_H > \alpha_1^F(0)$. For this to hold, we need that $\forall \alpha_1 \in [\alpha_1^F(0), \alpha_H)$, the following condition holds, $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \tilde{H}_1(\alpha_1, \beta_1)$. However we can show that $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \tilde{H}_1(\alpha_1 = 0, \beta_1)$ and also $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \tilde{H}_1(\alpha_1 = 1, \beta_1)$. Since $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\tilde{H}_1(\alpha_1, \beta_1)$ are increasing in $\alpha_1$, this implies that $\bar{H}_0(\alpha_1, \beta_1 = 0) > \tilde{H}_1(\alpha_1, \beta_1) \forall \alpha_1$. Thus the condition does not hold and hence $\alpha_H \not> \alpha_1^F(0)$.

Let us now assume that $\alpha_H \in (\alpha_1^F(W), \alpha_1^F(0))$. For this to be true we need the condition that $\forall \alpha_1 \in [\alpha_1^F(W), \alpha_H)$, the following condition holds, $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \hat{H}_1(\alpha_1, \beta_1)$. However we can show that $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \hat{H}_1(\alpha_1 = 0, \beta_1)$ and also $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \hat{H}_1(\alpha_1 = 1, \beta_1)$. Since $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\hat{H}_1(\alpha_1, \beta_1)$ are increasing in $\alpha_1$, this implies that $\bar{H}_0(\alpha_1, \beta_1 = 0) > \hat{H}_1(\alpha_1, \beta_1) \forall \alpha_1$.

Now let us assume that $\alpha_H = \alpha_1^F(0)$. Now for this to be true, we need the condition that for $\forall \alpha_1 \in [\alpha_1^F(W), \alpha_H)$, the following condition holds, $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \hat{H}_1(\alpha_1, \beta_1)$. However we have already shown that $\bar{H}_0(\alpha_1, \beta_1 = 0) > \hat{H}_1(\alpha_1, \beta_1) \forall \alpha_1$ and hence $\alpha_H \neq \alpha_1^F(0)$.

Now the only possibility therefore we have is that $\alpha_L, \alpha_H \in [\bar{\alpha}, \alpha_1^F(W)]$. We consider the case where $\alpha_L = \bar{\alpha}$ and $\alpha_H = \alpha_1^F(W)$. For this to hold we need the following conditions to be true

1. $\forall \alpha_1 < \alpha_1^S : H_0(\alpha_1, \beta_1 = 0) > H_1(\alpha_1, \beta_1 = 1)$
2. $\forall \alpha_1 \in [\alpha_1^S, \bar{\alpha}) : H_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1 = 1)$
3. $\forall \alpha_1 \in [\alpha_1^F(W), \alpha_1^F(0)) : \bar{H}_0(\alpha_1, \beta_1 = 0) > \hat{H}_1(\alpha_1, \beta_1 = 1)$
4. $\forall \alpha_1 \geq \alpha_1^F(0)) : \bar{H}_0(\alpha_1, \beta_1 = 0) > \tilde{H}_1(\alpha_1, \beta_1)$
5. $\forall \alpha_1 \in [\bar{\alpha}, \alpha_1^F(W)) : \bar{H}_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)$

As proved previously conditions 1, 2, 3 and 4 holds true. Now we consider the last condition and check whether it holds or not. We evaluate $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1, \beta_1)$ at $\alpha_1 = 0$ and $\alpha_1 = 1$. We can find that if $\delta > \delta_1 = \frac{1}{[\frac{e_L}{e_L - W} - \frac{\theta_L W}{e_H + e_L - \theta_H W}]}$, then $\bar{H}_0(\alpha_1 =$

$0, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 0, \beta_1) \forall \beta_1$. We can also find another $\delta = \delta_2$ such that if $\delta < \delta_2 = \frac{1}{[\frac{e_L}{e_L - W} - \frac{\theta_L W}{e_H + e_L - \theta_L W}]}$, then $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 0, \beta_1) \forall \beta_1$.

Now comparing $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1 = 1, \beta_1)$, we can show that there exists a $\delta = \delta_3 = \frac{1}{[\frac{e_L}{e_L - W} - \frac{\theta_H W}{e_H + e_L - \theta_H W}]}$ such that if $\delta > \delta_3$, then $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 1, \beta_1)$ and if $\delta < \delta_3$, then $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 1, \beta_1)$.
We can then define $\bar{\bar{\delta}} = max\{\delta_1, \delta_3\}$ such that if $\delta > \bar{\bar{\delta}}$ then $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 0, \beta_1)$ and $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 1, \beta_1)$. Since $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1, \beta_1)$ are increasing in $\alpha_1$, then $\bar{H}_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1) \forall \alpha_1, \beta_1$ and therefore condition 4 follows.

The values of $\delta_1$, $\delta_2$ and $\delta_3$ lies between 0 and 1 as long as $\theta_H < 1$ and $\theta_L < 1$ which is the case in our model.Now we can have three other cases, i.e. *Case 1*: $\alpha_L > \bar{\alpha}, \alpha_H < \alpha_1^F(W)$, *Case 2*: $\alpha_L = \bar{\alpha}, \alpha_H < \alpha_1^F(W)$ and *Case 3*: $\alpha_L > \bar{\alpha}, \alpha_H = \alpha_1^F(W)$. In all these cases we need the condition $\bar{H}_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1 = 1)$ to be satisfied for some range of $\alpha_1$ which varies according to the case considered. However given the range of $\delta$ we consider the above condition can never hold and hence we prove that $\alpha_L = \bar{\alpha}$ and $\alpha_H = \alpha_1^F(W)$. The non-political leader always have a positive expected payoff by announcing $a_1 = s$ and hence calls for a non-political protest. □

**Proposition 4.** *If* $\delta < \tilde{\delta}$ *and* $c < \bar{\bar{c}}$*, the leader with a political objective i.e.,*$\zeta = P$ *in the first period will announce a revolution* ($a_1 = R$)*,* $\forall \alpha_1 \in [0, 1]$ *and the game ends. The leader with a non-political objective i.e.,*$\zeta = NP$ *will always announce non-political protest* ($a_1 = s$) *irrespective of* $\alpha_1$*. The government will exert a force* $g_1 = 0$ *if* $\alpha_1 < \bar{\alpha}$ *and exert a force* $g_1 = W$ *if* $\alpha_1 \geq \bar{\alpha}$ *upon observing a revolution* $a_1 = R$ *exert a force* $g_1 = 0$ *if it observes* $a_1 = s$ *,*$\forall \alpha_1 \in [\alpha_1^S, \alpha_1^F(g_1 = 0)]$ *and* $g_1 = 0$ *for all other ranges.*

*Proof.* This follows from the proof of previous proposition. Now suppose we define $\tilde{\delta} = min\{\delta_2, \delta_3\}$. Then we can claim that $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 0, \beta_1) \forall \beta_1$ and $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 1, \beta_1) \forall \beta_1$. Since we know that $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1, \beta_1)$ are increasing in $\alpha_1$, then $\bar{H}_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1) \forall \alpha_1, \beta_1$. Under this circumstances, we cannot sustain a non-political protest by the political leader in equilibrium and her expected payoff is always higher from $a_1 = R$ for all values of $\alpha_1$. Given that the leader announces $a_1 = R$, then the government's action is the same as in lemma1. □